Table: 6.1.2: Harmonization of Consumer Price Indices
(Council Regulations [A], Commission Regulations [B], Guidelines)

Type	Number, date	Title, comments
1) A	No **2494/95*** 23 Oct. 1995	defines aim, **comparability**; timetable, procedure etc. of harmonization but no details of compilation of indices
2) B	No **1749/96*** 9 Sept. 1996	on initial implementing measures for Council Regulation No 2494/95; this Regulation defines • initial coverage of goods and services, • practices for updating the coverage and inclusion of **newly significant goods and services**, • minimum standards for the procedures of quality adjustment (for example automatic linking was banned), • minimum standards for sampling and for the prices used, • formula for compiling price indices for elementary aggregates amended by the Council Regulation [A] No 1687/98 of 20 July 1998; see row 5 below
3) B	No 2214/96 of 20 Nov. 96	concerning transmission and dissemination of sub-indices of the HICP; defines breakdown of HICP in sub-indices. Amended by: Commission Regulation [B] No 1749/99 of 23. July 1999 (see row 8 below)
4) B	No **2454/97*** of 10 Dec. 97	concerning **minimum standards for the quality of HICP weights** • defines a maximum age of weights (7 years) and, • requires an annually checking of "critical" weights
5) A	No 1687/98 of 20 July 98	coverage of the HICP; defines • additional coverage of HICP for 1999, • use of expenditure concept
6) A	No 1688/98 of 20 July 98	concerning the geographic and population coverage of the HICP • defines use of domestic concept, • inclusion of expenditures of people living in institutional households
7) B	No 2646/98 of 9 Dec. 98	concerning the treatment of tariffs in the HICP; defines obligation of respondents to provide structural information of tariffs and data related to consumption pattern to the NSOs
8) B	No 1749/99 of 23. July 99	• adopts the final version of COICOP, • takes regard of extended coverage (Regulation No 1687/98 see row 5 above)
9) B	No 1617/99 of 23. July 99	concerning treatment of insurances in the HICP; defines • use of service charges for calculation of weights and • use of gross premiums for compilation of prices supplemented by Guidelines for the implementation of the insurance regulation (problem of index-linked contracts)
10) A	No 2166/99 of 8. Oct. 99	laying down detailed rules for the implementation of Council Regulation No 2494/95 (see row 1 above) as regards standards for the treatment of goods and services in the education, health and social protection services in the HICP, decides • which reimbursements should be deducted from the "price", • how to handle "income related" prices (or reimbursements)
11) B	No 2601/2000 17. Nov. 2000	time of entering purchaser prices into the HICP (prices ought to be included when they are observed [goods] or when the service has been rendered)
12) B	No 2602/2000 17. Nov. 2000	treatment of price reductions or discounts (they should be taken into account when they are non-discriminatory and their impact on quantities purchased is substantial)
13) B	No 1920/2001 28. Sept. 2001	Financial Services, service charges proportional to transaction values (i.e. prices depending on the volume of a transaction)**

14) B	No 1921/2001 28. Sept. 2001	Standards for revisions of the HICP (revisions have to be approved and there is no quantitative assessment of the impact of revisions unless a revision affects the results by more than 1 per thousand)
15) B	No 1708/2005 19. Oct. 2005	Index reference period, amending No 3 (2214/96 temporal coverage of price collections), introducing consumption "**segments**" with far reaching implications for quality adjustment and replacement strategy
16) B	No 330/2009 26. April 2009	Minimum standards for seasonal products (items with seasonally varying prices) implementing regulations and amending regulation No 2494/95 (row 1)

* More details in Tab. 6.1.3

** The problem with *financial services* is not only its scope but also the fact that many prices (e.g. fees for bank services) depend on the value or the frequency of transactions.

Guidelines for the treatment of

extreme price observations in HICPs (automatic corrections not permitted); data processing equipment and especially PCs in HICPs; clothing in HICPs; rules for inclusion of seasonal and fashion clothing, replacements of items , problems of comparing (special) sales prices with regular prices, dealing with rejected prices, microcomputers etc., rules for reporting of rebates

***Forthcoming regulations** concerning*

minimum standards for sampling, owner occupied housing, minimum standards for sub-indices, (new rules concerning) treatment of newly significant goods and services etc.

Table 6.1.3: Some texts of Council Regulations concerning the HICP

	Extract (epitome) of the Regulation text
No **2494/95** row 1 in Tab. 6.1.2	**Comparability** "HICPs shall be considered to be comparable if they reflect only differences in price changes or consumption patterns between countries. HICPs which differ on account of differences in the concepts, methods or practices used in their definition and compilation shall not be considered comparable. The Commission (Eurostat) shall adopt rules to be followed to ensure the comparability of HICPs under the procedure laid down in Article 14. Differences in concepts, methods or practices, which would affect the change in the HICPs by more than 0.1 percentage point on average over one year against the previous year cannot be accepted."
No. **1749/96** row 2	**Inclusion of newly significant goods (NSG) and services** ***Definition, sales volume of 1 per 1000 of total consumers' expenditure*** "Newly significant goods and services are defined as those goods and services the price changes of which are not explicitly included in a Member State's HICP and the estimated consumers' expenditure on which has become at least one part per thousand of the expenditure covered by that HICP." ***Compulsory checks (once a Member State reports NSG) and adjustments*** "Member States shall: (a) systematically seek to identify newly significant goods and services and (b) check the significance of goods and services reported to be newly significant in other Member States. The HICP shall be compiled to include the price changes of a newly significant good or service ... This shall be accomplished within 12 months of their identification either by adjusting the weights of or within the relevant category of COICOP/HICP classification ... or by assigning part of the weight specifically to the newly significant good or service." **Price indices for elementary aggregates** accepts formulas of Dutot (ratio of average prices) or Jevons (geometric mean of price relatives) but bans formula of Carli (arithmetic mean of price relatives).

No **2454/97** row 4 in Tab 6.1.2	**Minimum Standards for the Quality of HICP Weights** ***1) maximum age of weights*** "Each month Member States shall produce HICPs using weightings which reflect consumers' expenditure patterns in a weighting reference period ending *no more than seven years* before." (there is currently much debate on this provision and a tendency to have shorter and more uniform intervals; see below "projects, ongoing activities") ***2) frequency of revision*** "Each year, Member States shall carry out a review of weightings in order to ensure that they are sufficiently reliable and relevant to meet the comparability requirement." the annual review of weights is requested at the level of sub-indices and their major components. ***3) object of revision*** "In the review, Member States shall check whether or not there have been any important changes since the weighting reference period in current use regarding price developments of each major component index relative to the HICP, or sustained market developments in each major component group." ***4) obligatory adjustment of weights*** "Where reliable evidence shows ... [that a weighting change] ... would affect the change in the HICP by more than 0.1 percentage point on average over one year against the previous year, Member States shall adjust the weightings of the HICP appropriately."

Timeliness

Flash estimates for the current month released at the end of the month, full release two weeks later

Classification

COICOP Clasification[36]

01 Food and non-alcoholic beverages	07 Transport
02 Alcoholic beverages, tobacco and narcotics	08 Communication
03 Clothing and footwear	09 Recreation and culture
04 Housing, water, electricity, gas and other fuels	10 Education
05 Furnishings, household equipment and routine household maintenance	11 Restaurants and hotels 12 Miscellaneous goods and services
06 Health	Individual consumption expenditure other sectors*

13: of non-profit institutions serving households (NPISHs), 14: of general government

Projects, ongoing activities:

- CENEX[37]-HICP project (on the basis of regulation 1749/96) on quality adjustment (compilation of a manual, assessment and standardization of methods, recommendations for standards in the case of specific products such as PCs, books, TV sets, washing machines, new and used cars, notebooks and goods for telecommunication, rents (housing) and social protection, furthermore research work on the measurement of "quality" in health service and insurance.
- Speedier and more uniform (tighter standards for the) revision of weights: the present practice allows weights of an age up to seven years; the practice of Member States (MS) is widely different. While the majority of MS review annually HICP sub-index weights on the basis National Accounts (problems: such weights are less detailed then those derived from household expenditure surveys [HES], and they are liable to [repeated] revisions) other countries such as Austria, Bel-

[36] More details in http://unstats.un.org/unsd/cr/registry/regcst.asp?Cl=5

[37] CENEX stands for Center and Network of Excellence, participating (know-how contributing) countries were in this case: Germany (chair), Belgium, Ireland, the Netherlands, Austria, Portugal and Sweden.

gium, Cyprus, Denmark, Finland, Germany, Greece, Ireland and Malta conduct a general update of volumes underlying (the only price updated) HICP weights only at three to five years intervals. HES data as the only reliable source for detailed weights are not annually available. More frequent updates are found necessary esp. in the case of fast evolving markets (information and communication technology for example)

The relevance of a as speedy as possible update of weights seem to be a bit exaggerated (see also chapter 7): according to the German National CPI the difference between annual inflation rates for 2006 and 2007 was only about 0.1 percentage points depending on whether weights of the year 2000 or of the year 2005 were used.

Further remarks

- Treatment of the so called "***income related prices***" frequently encountered in the field of social and welfare activities.[38]
- As to the inclusion of *insurance services* it is an open question to define[39] the part of the gross ***insurance premiums*** that can be regarded as the price for the service taken in isolation as well as the definition of weights and the treatment of changes in the specified amount (e.g. 100.000 €) insured (more quantity or improved "quality"?).

Chain index formula of the HICP

December [month 12] of year t-1 is the linking month of this chain-linked Laspeyres type index. For this purpose weights (of National Accounts) are "price updated" only (as a rule volumes are less frequently updated) and normalized (in order to sum up to unity)

$$H_{0t,m} = \frac{\sum p_{t,m} q_{t-2}}{\sum p_{t-1,12} q_{t-2}} \frac{\sum p_{t-1,12} q_{t-3}}{\sum p_{t-2,12} q_{t-3}} \left(\frac{\sum p_{t-2,12} q_{t-4}}{\sum p_{t-3,12} q_{t-4}} \cdots \right), \text{ and}$$

$$H_{0,t-1,m} = \frac{\sum p_{t-1,m} q_{t-3}}{\sum p_{t-2,12} q_{t-3}} \left(\frac{\sum p_{t-2,12} q_{t-4}}{\sum p_{t-3,12} q_{t-4}} \cdots \right).$$

The expressions in brackets will cancel out when a ratio of two price indices, both for a month m is formed. Such a ratio of two indices is given by

(6.1.1) $$H_{0t,m} / H_{0,t-1,m} = \frac{\sum p_{t,m} q_{t-2}}{\sum p_{t-1,12} q_{t-2}} \Bigg/ \frac{\sum p_{t-1,m} q_{t-3}}{\sum p_{t-1,12} q_{t-3}}$$

Hence the annual growth rate $H_{0t,m} / H_{0,t-1,m} - 1$ describing the change from month m in t - 1 to month m in year t is a function of quantities (weights) relating to **two** years, t - 2 and t - 3,

The MUICP[40] (initially based on 1996 = 100), denoted M, is given by

(6.1.2) $$M_{05} = \left(\sum c_{m0} H_{m01}\right)\left(\sum c_{m1} H_{m12}\right)\left(\sum c_{m2} H_{m23}\right)\left(\sum c_{m3} H_{m34}\right)\left(\sum c_{m4} H_{m45}\right)$$

where the summation in each bracket takes place over a (possibly varying) number of countries m = 1, 2, ...M and c_{mt} denotes the (updated) country weights of the Member Country m.

[38] Such prices are linked to one or more income thresholds as for example in the following case: persons with an income exceeding 3000 €, pay for example 50 € while those having a lower income only pay 20 € or so. To ensure a pure price comparison now would not only require to keep the consumption pattern of households (i.e. the type of goods and their weights) constant but also to keep the income level and other socio-economic characteristics of the households under consideration constant.

[39] The estimated pure service charge is often defined as gross insurance premiums plus premium supplements minus claims minus changes in the actuarial reserves.

[40] Monetary Union Index of Consumer Prices

Note that M_{05} which is supposed to compare the price level in two periods 2001 (t = 5) and 1996 (t = 0) makes use of no less than 5M different national baskets as well as prices in 1997, ..., 2000 in addition to the prices in 1996 and 2001. Hence results like for example $M_{05} = 1.2$ may well be produced by a number of very different factors:

- the *prices* in each member country in each period, and
- changing weights of the *commodities* in each country and each period, as well as
- the change of weights of the *countries*, and
- the *path* of the index since the index is a chain index always depending on its "history".

since aggregations are made over commodities, countries, and time (sub-) intervals (due to the chain approach). Furthermore all sorts of changes in the "domain of definition" that is in the selection of goods and outlets have an impact.

6.2. Some controversial issues in inflation measurement

a) Core inflation	c) Owner occupied dwelling (OOD)
b) Asset inflation	

a) Core inflation

The notion of core inflation became popular in the 1970´s when a number of sector specific disturbances, the most prominent of which was the oil-price-crisis engendered turbulences and volatilities. Since these days it appeared desirable to identify and eliminate the more volatile and only temporary influences on "inflation" because they are not of primary concern to central bankers and might be viewed as only clouding and disturbing the statistical picture of inflation. A distinction therefore is common to all concepts of core-inflation, i.e. the distinction between a "core" - inflation component Π_{0t} and a distortion (or contamination by "idiosyncrasies") ε_{0t} .

The methods proposed to identify and eliminate the idiosyncratic component of the "headline" (observed) CPI) and thereby to isolate the core inflation can be broadly classified into

method	**T** methods time series-based methods	**C** methods cross-section-based methods
I (methods using individual price relatives)	Dynamic Factor (DF-) Index	Exclusion Method (EM) Trimmed Means (TM) Variability adjusted Means (VA)
G (global methods)	Smoothing techniques (ST)[1] VAR-methods[2]	

1) Moving averages, filtered series
2) vector autoregressive methods

b) Asset inflation

The starting point of those who argue in favour of asset inflation (AI) is that inflation is defined as a fall in the value of money irrespective of how (for which purposes) the money is spent (*"total" inflation*). Hence inflation should also mean a rise of prices of non-financial and financial assets. Following this reasoning there would no longer be a need for a distinction between various sorts of indices such as the CPI, PPI etc. or to recourse to a specific basket.

Problems: Asset boundary, discontinuous high priced acquisitions of durables, and (ambiguous) theoretical justification.

c) Owner occupied dwelling (OOD)

Fig. 6.2.2: Methods to deal with Owner Occupied Dwelling (OOD)

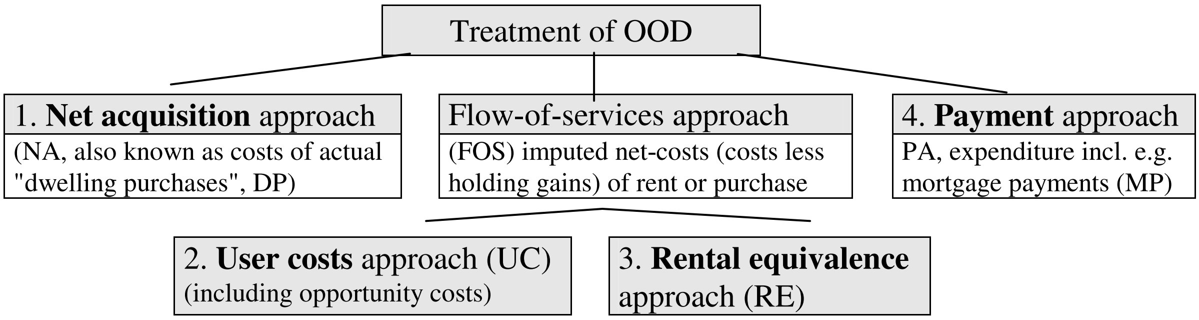


Table 6.2.1: Comparison between various methods of dealing with OOD

Method	General idea	Comments
Net acquisition approach (NA)	Value of OOD is estimated on the basis of observed prices of (new) houses. Weights are derived from construction statistics.	Current costs (e.g. for repairs), inheritance in addition to purchase of housing, quality changes and holding gains should also be taken into account.
User costs approach (UC)	Net operating and opportunity costs caused by owner occupation plus net holding gains (imputed rather than actually observed costs)	UC comprises in addition to actually observed costs (e.g. mortgage costs, MC) "costs" which need a more or less complicated estimation or imputation
Rental equivalence approach (RE)	Observed rents spent by tenants are used to impute what owner-occupiers would pay if they were tenants rather than homeowners.	The existence of well established and intact rental housing markets facilitates estimation of imputed prices. RE is also in line with the practice of NA*.
Payment approach (PA)	Actual payments made by owners due to having incurred liabilities (e.g. mortgage payments, MP)	Makes the price of a service flow dependant on factors which are more or less unrelated to this service.

* National Accounts

Using dwelling purchases as a proxy for shelter costs is to treat houses like cars and furniture or other consumer durables. The job of the statisticians then consists simply in observing *prices* for new as well as existing houses and flats.

Table 6.2.2(abridged): Aspects to be considered in the choice of a method to deal with OOD

1. The relevant data are readily available
2. Comparability of shelter quality [1]
3. Definition and statistical source of "quantities" (i.e. "weights" in the index formula).
4. Expenditures incurred vs. imputations
5. OOD should not unduly affect the CPI by *volatility*
6. Avoid treating comparable situations too differently [2]
7. Method should be defensible from an *economic-theory* point of view[3]

1. to which the observed expenditures are related
2. This is for example not the case when simply dwelling **purchases** are observed, because unlike buyers of houses those who inherited houses from dead relatives are treated as if their housing were completely free.
3. As to this criterion the UC approach clearly is an admirable concept but unfortunately none of the UC- components is directly observable (without certain corrections) and the sum of these components does not correspond to actual expenditures. Nothing reflects any actual transaction.

6.3. Producer Price Indices (PPI)

a) Price indices in agriculture	c) Price indices for construction
b) Producer prices for the production industries	

a) Price indices in agriculture

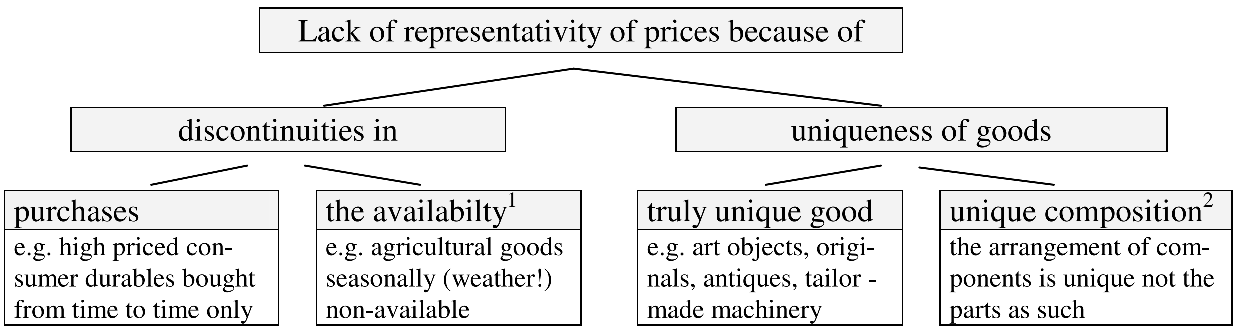
Coverage, weights, price collection, seasonal absence of sales

There are two concepts in use guiding the selection of activities and sales/purchases to be covered by Producer Price Indices (PPIs) in agriculture, viz. the so-called

- "*average farm concept*" (*AFC*) according to which all sales of farmers are supposed to be covered irrespective of the type of buyer (in particular this "gross sector approach" also covers all intrasectoral purchases of farmers), or
- the alternative concept known as "*federal farm concept*" (*FFC*) which may be viewed as a sort of "net sector approach".

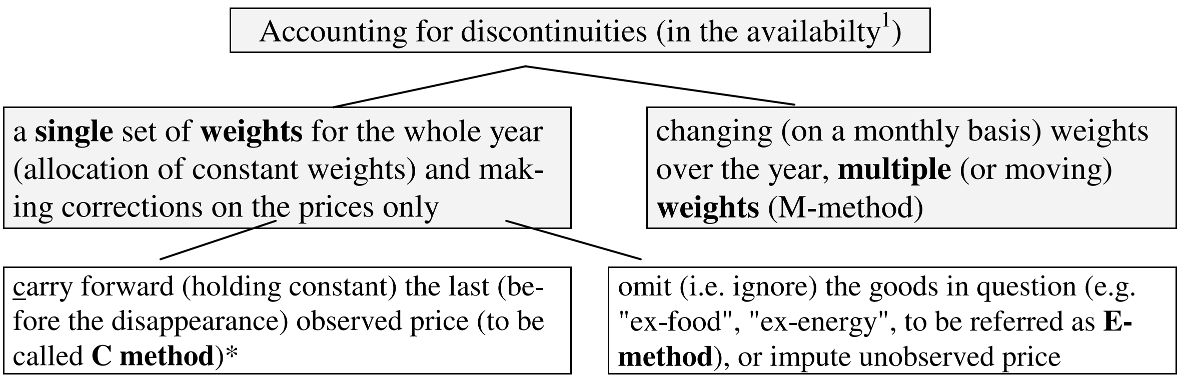
The effect of non-availability, or more general of a changing degree of representativity is to impair comparability over time. In this respect the situation is similar to quality change, but unlike quality change reduced representativity (due to fewer sales in certain months) is usually not taken into account, at least not by making adjustments on observed prices.

Figure 6.3.1: Various types of reduced representativity



1 (temporary) nonexistence of a commodity is a limiting case of reduced of purchases
2 in fig. 6.33 this case will be further subdivided in model pricing and specification pricing

Figure 6.3.2: Methods to deal with (seasonal) non-availability of (agricultural) products



* if there is a trend of rising or declining prices on the market the "price freezing" clearly creates a downward or upward bias respectively. It is therefore in general preferable to impute the average movement in prices for those goods the prices of which are temporarily not available

Remarks to each method see next page

The **E-method** is tantamount to assuming a price movement of the omitted commodity precisely equals the average price movement of the remaining commodities: Suppose commodity no. 1 is seasonally not available and the basket also contains goods 2 and 3 with price relatives (or if 2 and 3 denote *groups* of commodities this will be sub-indices rather than price relatives) P^2, P^3, and weights w_2 and w_3 respectively. The E-method procedure is equivalent to assuming the implicit unknown price relative (sub-index) P^1 equals the price index P_{0t} derived from the commodities no. 2 and 3, that is

(*) $P_{0t}^2 \frac{w_2}{w_2+w_3} + P_{0t}^3 \frac{w_2}{w_2+w_3} = P_{0t}$, on the other hand P_{0t} is by definition

(**) $w_1 P_{0t}^1 + w_2 P_{0t}^2 + w_3 P_{0t}^3 = P_{0t}$. Upon substitution of P_{0t} for P^1 in equation (**)[41] we get $w_2 P_{0t}^2 + w_3 P_{0t}^3 = P_{0t}(1-w_1) = P_{0t}(w_2+w_3)$ giving equation (*). ♦

Figure 6.3.2 (continued)

Name of the method	Description
exclusion (E-method)	Calculate the index without the prices of non available (though in other months available) commodities after having adjusted the weights of the remaining commodities such they add up to unity [1]. This is equivalent to the assumption that the price movement of the non available products equals the average price movement, i.e. the price index as calculated on the basis of the remaining products.
Continuation [2] **(C-method)**	To continue the calculation of the index with the last reported price of the commodity in question during the period of its not being traded until a new price is able to replace the old one.
Multiple weights (M- method)[3]	Provide different weighting schemes each month (hence a variable "basket") according to variations in supply and demand over a year.

1 Otherwise (without redefining the weights) the calculation would implicitly treat the price relatives of missing goods as if they were zero.
2 Because the method requires continuing with (carry forward) the last available price
3 also called Roswell index[42]

b) Producer prices for the Production Industries

c) Price indices for the Construction sector

The fundamental problems in pricing of construction: as structures (roads, buildings, dwellings, bridges etc.) do not come in mass-produced goods, sold at the same location and at the same time, the standard approach to collect prices of (more or less narrowly specified in order to make sure that prices are comparable) finished products fails.

Figure 6.3.3: Methods to compile output-price indices in construction

Pricing of construction (new buildings)

Method 1	Method 2	Method 3
Buildings treated as **finished goods**, incomparability is viewed as a problem of quality adjustment [1]	**Model pricing**: Producers calculate (hypothetical) costs of some well defined (or "model") buildings	**Specification pricing**: as weighted [2] sum of prices for certain standard construction works

1) treating different products as differences in quality; the method is applicable in particular in the case of pre-manufactured buildings.
2) using weights from actual contracts

41 using $w_2+w_3=1-w_1$.

42 Index results referring to successive months will not reflect a pure price change but also to an unknown extent differences in the weights which on their part are not designed to reflect variations in prices but rather variations in supply and demand.

6.4. Price indices and unit value indices, foreign trade and wages indices

a) Definition and properties of unit values	c) Relations between unit value and price indices
b) The notion of a "unit value index"	d) Decomposition of the UV-bias

a) Definition and properties of unit values

The unit value (a kind of average price) of sub collection of goods (commodity number) is

(6.4.1) $$\tilde{p}_{k0} = \frac{\sum p_{kj0} q_{kj0}}{\sum q_{kj0}} = \sum_{j=1}^{n_k} p_{kj0} \frac{q_{kj0}}{Q_{k0}} = \sum p_{kj0} m_{kj0}$$ and

(6.4.1a) $$\tilde{p}_{kt} = \frac{\sum p_{kjt} q_{kjt}}{\sum q_{kjt}} = \sum_{j=1}^{n_k} p_{kjt} \frac{q_{kjt}}{Q_{kt}} = \sum p_{kjt} m_{kjt}$$

where the summation takes place over the j = 1, …, n_k ($n_k < n$) goods of a CN. Only in the case of a commodity number (CN), like the k-th CN sums $Q_{k0} = \sum_{j=1}^{n_k} q_{kj0}$ or $Q_{kt} = \sum q_{kjt}$ of quantities have a meaningful interpretation. It is in general not possible to summate over all n = Σn_k commodities, that is to calculate $Q_t = \sum_k \sum_j q_{kjt}$ (Q_0 correspondingly) and thus compile

(1.2.2) $$P_{0t}^{UD} = \frac{\sum_k \sum_j p_{kjt} q_{kjt} / \sum_k \sum_j q_{kjt}}{\sum_k \sum_j p_{kj0} q_{kj0} / \sum_k \sum_j q_{kj0}} = \frac{\tilde{p}_t}{\tilde{p}_0}$$

which is Drobisch's index, an index, however, unfortunately also often called "unit value index". The index in fact calculated in some countries' official statistics differ from 1.2.2 in that unit values are established only for CNs (for example for the k-the CN: $\tilde{p}_{kt}$, $\tilde{p}_{k0}$), not for all commodities covered by the index in an all-items unit value-($\tilde{p}_t$ and $\tilde{p}_0$).

A unit-value is not reflecting a pure price movement it is also affected by the quantities involved. According to eq. 6.4.1/1a $\tilde{p}_{kt}$ ay rise (decline) compared to $\tilde{p}_{k0}$ even if no price within the aggregate is changing. It all depends on the structure of quantities in 0 and t (that is on the coefficients $m_{kjt} \neq m_{kj0}$. Assume only two commodities in group (CN) k with constant prices $p_{k10} = p_{k1t} = p$ and $p_{k20} = p_{k2t} = \lambda p$ and quantity shares $m_{k10} = m_{k20} = ½$. Then the difference of unit values of this k-th CN is depending on $\mu = m_{k2t}/0.5 = 2m_{k2t}$ ($0 \leq \mu \leq 2$)

(6.4.1b) $$\Delta = \tilde{p}_{kt} - \tilde{p}_{k0} = p(1+\mu\lambda-\mu-\lambda)/2 = \frac{p}{2}(\lambda-1)(\mu-1)$$

such that we have for a positive p four quadrants as follows

$\lambda > 1$	II	$\lambda > 1$ and $\mu < 1 \rightarrow \Delta < 0$ less of the more expensive good 2 unit value ***declining***	I	$\lambda > 1$ and $\mu > 1 \rightarrow \Delta > 0$ more of the more expensive good unit value ***rising***
$\lambda < 1$	III	$\lambda < 1$ and $\mu < 1 \rightarrow \Delta > 0$ less of the cheaper good 2 unit value ***rising***	IV	$\lambda < 1$ and $\mu > 1 \rightarrow \Delta < 0$ more of the cheaper good 2 unit value ***declining***
	$\mu < 1$		**$\mu > 1$**	

Also the ratio of unit values $\tilde{p}_{kt}/\tilde{p}_{k0}$ is not a mean value of price relatives p_{kjt}/p_{kj0} (as shown in the case of Drobisch's index) the weights are $\frac{p_{kj0} q_{kjt}}{\tilde{p}_{k0} Q_{kt}}$ summing up to $\frac{Q_{k0}}{Q_{kt}} \cdot Q_{0t}^{L(k)}$.

b) The notion of a "unit value index"

Unit values can take the part of prices in both price- and quantity indices; hence we have unit value indices on the level of price and of quantity indices respectively (the latter is less common, however) and furthermore in both forms, Laspeyres and Paasche.

Figure 6.4.1: The structure of indices on the basis of unit values*

Indices on the basis of

unit values (UV)		prices	
UV price indices	UV quantity indices	true price indices	true quantity indices
summation over k = 1, ... ,K groups $\sum_k$		summation over n commodities $\sum_k \sum_j$	
formula of Laspeyres		formula of Laspeyres	
$PU_{0t}^{L} = \frac{\sum \tilde{p}_t Q_0}{\sum \tilde{p}_0 Q_0}$	$QU_{0t}^{L} = \frac{\sum Q_t \tilde{p}_0}{\sum Q_0 \tilde{p}_0}$	$P_{0t}^{L} = \frac{\sum\sum p_t q_0}{\sum\sum p_0 q_0}$	$Q_{0t}^{L} = \frac{\sum\sum q_t p_0}{\sum\sum q_0 p_0}$
formula of Paasche		formula of Paasche	
$PU_{0t}^{P} = \frac{\sum \tilde{p}_t Q_t}{\sum \tilde{p}_0 Q_t}$	$QU_{0t}^{P} = \frac{\sum Q_t \tilde{p}_t}{\sum Q_0 \tilde{p}_t}$	$P_{0t}^{P} = \frac{\sum\sum p_t q_t}{\sum\sum p_0 q_t}$	$Q_{0t}^{P} = \frac{\sum\sum q_t p_t}{\sum\sum q_0 p_t}$

* The universe of n commodities is partitioned into K groups (sub-collections) of related commodities; the subscript k = 1, 2, ..., K denotes the number of the group and the subscript j the j-th commodity of the k-th group.

In Germany there exists a unit value index of exports and imports of the Paasche form in addition to genuine Laspeyres price indices of export and import respectively. From the practical point of view unit value indices have many advantages; they have, however, also serious disadvantages from a theoretical point of view, in particular because they do not comply with the principle of pure price comparison (see table 6.4.1/2 below).

Use of unit values in some countries (according top the Internet)

Canada The export/import price index (= International Merchandise Trade Price index IMTPI) makes use of both unit values processed by the International Trade Division (on the basis of customs data) and when unit values are not accurate (heterogeneous aggregates) or unavailable price data provided by other (Canadian and foreign, e.g. the BLS of the USA) sources. Both direct index formulas, Laspeyres and Paasche are used. For internal use also a chained Fisher index is being compiled.

Italy an export Unit Value Index (based 2004) is compiled in collaboration with the Italian Customs as a chained Fisher index Weights "are the previous year value for the Laspeyres links and the current monthly value for the Paasche links"

Finland also uses unit values for a proxy of a price index (Laspeyres "unit value index") and as by-product a Paasche (unit value) volume index. All indices are "calculated as chain indices, and the year previous to the year calculated is always used as comparison."

c) The formal relations between price and unit value indices

Consider now K groups (k = 1, ..., K), each containing n_k commodities such that there are n = Σn_k commodities altogether. The value index then is

$$V_{0t} = \frac{\sum_k \tilde{p}_{kt} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{k0}} = \frac{\sum_k^K \sum_j^{n_k} p_{kjt} q_{kjt}}{\sum_k^K \sum_j^{n_k} p_{kj0} q_{kj0}} = \frac{V_t}{V_0} \qquad (6.4.2)$$

Table 6.4.1: Comparison of true price and unit value (price) indices

	Price index	Unit value index (UV [price [1]] index)
What is measured?	how the prices of ideally the same products of a given (fixed) collection of products are developing over time	unit value (average value) of all products of a certain type (e.g. all exported goods) at two points in time
Merits	guarantees pure price comparison by keeping the selection constant and making adjustments for quality changes	satisfies "representativity" by inclusion of *all* products (complete coverage [3] instead of a selection)
Demerits	reliability is said to depend on representativity of the selection [2]; a lot more demanding as far as price collection, empirical derivation of weights and quality adjustment is concerned	influenced by changes in the composition of the products such that a structural change is reflected in the UV (price) index rather than in the quantity (volume) dimension [4]

1 Note that UVs instead of prices can be used in the compilation of a sort of price index as well as a sort of quantity index such that there is UV-price index (PU_{0t}) as well as a UV-quantity index (QU_{0t}).
2 and (allegedly) impaired by lack of representativity (an argument advanced in particular by advocates of the chain-index approach).
3 In some countries such exhaustive data files are available, for example in the case of wages, and it will be difficult *not* to make use of them by compiling a UV-wage-index because of preferring a wage-index to the true price index type.
4 A mere switch from cheaper to more expensive products within a group of commodities for which a unit value (UV) is established is reflected as a rise in the UV (and thus in the price dimension which thereby is overstated since prices remained unchanged); using PU_{0t} (instead of P_{0t}) as deflator therefore may overstate price and understate volume change.

Table 6.4.2: Indices of prices in foreign trade (export and import) in Germany

	Price index	Unit value index (UVI)
Data	Survey based (monthly), sample; more demanding (weights!)	Customs based (by-product), census, in the case of Intrastat a survey
Formula	Laspeyres	Paasche
Prices, aggregates	Prices of specific goods at time of contracting (lead of price index?)	Average value of CNs; time of crossing border (lag of UVI?)
New/disappearing goods	Included only with a new base period; vanishing goods replaced by *similar* ones constant selection of goods *	Immediately included; price quotation of disappearing goods is simply discontinued variable universe of goods
Quality	Quality adjustment	No quality adjustment (not feasible?)

* All price determining characteristics are deliberately kept constant

(6.4.4) $$PU_{0t}^{P} = \frac{\sum_k \tilde{p}_{kt} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}} = \frac{\sum_k^K \sum_j^{n_k} p_{kjt} q_{kjt}}{\sum_k^K \left(\sum_j^{n_k} \frac{p_{kj0} q_{kj0}}{Q_{k0}} \right) Q_{kt}} = \frac{V_t}{\sum_k V_{k0} \frac{Q_{kt}}{Q_{k0}}}$$

(6.4.5) $$V_{0t} = PU_{0t}^{L} QU_{0t}^{P} = PU_{0t}^{P} QU_{0t}^{L} = \frac{\sum p_t q_t}{\sum p_0 q_0} \text{ (just like } V_{0t} = P_{0t}^{L} Q_{0t}^{P} = P_{0t}^{P} Q_{0t}^{L}\text{).}$$

d) Decomposition of the UV-bias (or "discrepancy")

In order to establish a relationship explaining the discrepancy between PU_{0t}^{P} and P_{0t}^{L} it is useful to make a recourse to the famous equation of L. v. Bortkiewicz (eq. 1.3.13) according to which the covariance C between price and quantity relatives is given by $C = Q_{0t}^{L}\left(P_{0t}^{P} - P_{0t}^{L}\right)$. In combination with eq. 6.4.5 this leads to the multiplicative decomposition

(6.4.6) $$D = \frac{PU_{0t}^{P}}{P_{0t}^{L}} = \left(\frac{C}{Q_{0t}^{L}P_{0t}^{L}} + 1\right)\left(\frac{Q_{0t}^{L}}{QU_{0t}^{L}}\right) = \frac{P_{0t}^{P}}{P_{0t}^{L}} \cdot \frac{PU_{0t}^{P}}{P_{0t}^{P}} = L \cdot S$$

or the additive decomposition

(6.4.6a) $$D^* = D - 1 = \frac{PU_{0t}^{P}}{P_{0t}^{L}} - 1 = \left(\frac{C}{QU_{0t}^{L}P_{0t}^{L}}\right) + \left(\frac{Q_{0t}^{L}}{QU_{0t}^{L}} - 1\right) = L^* + S^* \quad (S^* = S-1,\ L^* = (L-1)S).$$

The term L (or L* respectively) is referred to as Laspeyres- or simply ***L-effect*** reflecting the fact that $P^P \neq P^L$. A negative covariance ($P^P < P^L$) may arise from rational substitution among goods in response to price changes on a given (negatively sloped) demand curve. A second component of the discrepancy is coming into play which may well reinforce but also counteract the L-effect. This factor is called structural component or ***S-effect*** for short and refers to changing quantities within a group of goods k = 1, …, K (for which unit values are established). In order to understand the meaning of the S term recall the structural coefficients in eq. 6.4.1 $m_{kj0} = q_{jk0}/Q_{k0}$ ·and m_{kjt} correspondingly. Since

$QU_{0t}^{L} = \sum_k Q_{kt}\sum_j m_{jk0}p_{jk0} \big/ \sum_k\sum_j q_{jk0}p_{jk0}$ and $Q_{0t}^{L} = \sum_k Q_{kt}\sum_j m_{jkt}p_{jk0} \big/ \sum_k\sum_j q_{jk0}p_{jk0}$ the ratio S = Q_{0t}^{L}/QU_{0t}^{L} is given by

(6.4.6a) $$S = \frac{Q_{0t}^{L}}{QU_{0t}^{L}} = \frac{\sum_k Q_{kt}\sum_j m_{jkt}p_{jk0}}{\sum_k Q_{kt}\sum_j m_{jk0}p_{jk0}}$$

With $\tilde{Q}_{0t}^{(k)} = Q_{kt}/Q_{k0} = \sum_j q_{kjt} \big/ \sum_j q_{kj0}$ and $Q_{0t}^{L(k)} = \sum_j q_{kjt}p_{kj0} \big/ \sum_j q_{kj0}p_{kj0}$ as two different ways of measuring the development of quantities within the k^th CN and expenditure shares $s_{k0} = Q_{k0}\tilde{p}_{k0} \big/ \sum_k Q_{k0}\tilde{p}_{k0} = \sum_j p_{kj0}q_{kj0} \big/ \sum_k\sum_j p_{kj0}q_{kj0}$ it can easily be seen that

(6.4.7) $QU_{0t}^{L} = \tilde{Q}_{0t}^{(k)}s_{k0}$ and (6.4.7a) $Q_{0t}^{L} = Q_{0t}^{L(k)}s_{k0}$.

The S-effect would vanish (S = 1) if

- $n_k = 1$ (a perfectly homogenous CN), such that $m_{kjt} = m_{kj0} = 1$ (unlike the L-effect the S effect only exists when commodities are grouped together in CNs)
- for all j = 1, …, n_k holds $m_{kjt} = m_{kj0}$ (no structural change within a CN), or
- all n_k base period prices of a CN k are equal $p_{kj0} = \tilde{p}_{k0}\ \forall j = 1,\dots,n_k$, because in this case

$$\tilde{Q}_{0t}^{k} = \sum_j \frac{q_{kjt}}{q_{kj0}}\frac{q_{kj0}}{\sum_j q_{kj0}} = Q_{0t}^{L(k)} = \sum_j \frac{q_{kjt}}{q_{kj0}}\frac{q_{kj0}p_{kj0}}{\sum_j q_{kj0}p_{kj0}}$$

Both indices are linear indices suggesting an application of Bortkiewicz's generalized theorem of fig. 3.2.2, p. 43 in order to show which CN contributes positively (if $Q_{0t}^{L(k)} > \tilde{Q}_{0t}^{k}$) or negatively (if $Q_{0t}^{L(k)} < \tilde{Q}_{0t}^{k}$) to $S = \frac{Q_{0t}^{L}}{QU_{0t}^{L}} = \frac{\sum_k Q_{0t}^{L(k)}s_{k0}}{\sum_k \tilde{Q}_{0t}^{k}s_{k0}} = \sum_k \frac{Q_{0t}^{L(k)}}{\tilde{Q}_{0t}^{k}}\frac{\tilde{Q}_{0t}^{k}s_{k0}}{\sum_k \tilde{Q}_{0t}^{k}s_{k0}}$.

A covariance $s_{xy}^{(1)}$ explaining $\frac{X_t}{X_0} = 1 + \frac{s_{xy}^{(1)}}{\overline{X}\cdot\overline{Y}} = \frac{Q_{0t}^{L(k)}}{\tilde{Q}_{0t}^{k}}$. the k^th CN contribution to S is eg

$$s_{xy}^{(1)} = \sum \left(\frac{q_{kjt}}{q_{kj0}} - \tilde{Q}_{0t}^{k} \right) \left(p_{kj0} - \tilde{p}_{k0} \right) \frac{q_{kj0}}{\sum q_{kj0}} ,$$

and the covariance $s_{xy}^{(2)}$ explaining $\frac{X_t}{X_0} = 1 + \frac{s_{xy}^{1}}{\overline{X} \cdot \overline{Y}} = \frac{\tilde{Q}_{0t}^{k}}{Q_{0t}^{L(k)}}$ and therefore S^{-1} (instead of S) is

$$s_{xy\ xy}^{(2)} = \sum \left(\frac{q_{kjt}}{q_{kj0}} - Q_{0t}^{L(k)} \right) \left(\frac{1}{p_{kj0}} - \frac{1}{\tilde{p}_{k0}} \right) \frac{p_{kj0} q_{kj0}}{\sum p_{kj0} q_{kj0}} .$$

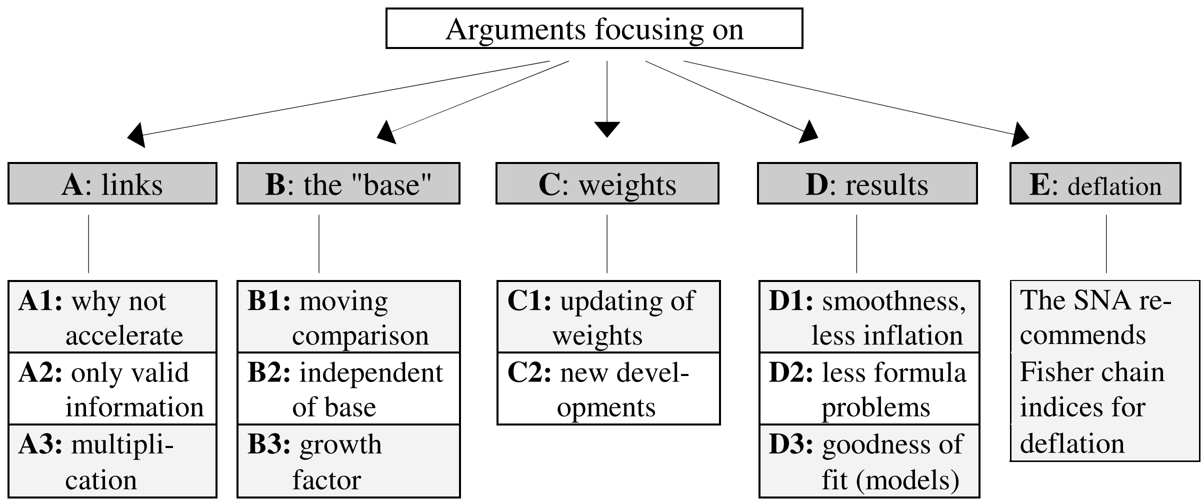
Chapter 7: Chain indices

7.1. Chain indices, arguments pro and con

a) Overview of pro-arguments	c) Arguments in the SNA
b) Some arguments in detail	d) Logical status of the arguments

a) Overview of arguments in favour of chain indices

Figure 7.1.1: Twelve arguments in favour of chain indices, an overview



* this argument also comprises the idea that chain indices provide valuable additional information because of making better use of all time series data

b) Some arguments in detail

A1*: *The "why not" or "limiting case" argument: Allen 1975, p. 177 has put it: "why not accelerate and go for annual chaining? There is no reason why not."[43]

Assume a new base at t = 5 then $\overline{P}_{09}^{LC}$ is the product of nine factors, and <u>19 vectors</u> affecting the result $\overline{P}_{09}^{LC} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \frac{\Sigma p_2 q_1}{\Sigma p_1 q_1} \ldots \frac{\Sigma p_8 q_7}{\Sigma p_7 q_7} \frac{\Sigma p_9 q_8}{\Sigma p_8 q_8}$, while $P_{09}^{L*} = P_{05}^{L} P_{59}^{L}$ is product of *two* factors only, and influenced by <u>5 vectors</u> $\mathbf{p}_0$, $\mathbf{p}_5$, $\mathbf{p}_9$, $\mathbf{q}_0$ and $\mathbf{q}_5$.

[43] or: "In effect, the underlying issue is not whether to chain or not but how often to rebase. Sooner or later the base year for fixed weight Laspeyres ... indices ... has to be updated because the prices of the base year become increasingly irrelevant ... Long runs of data therefore almost inevitably involve some form of chain indices. Annual chaining is simply the limiting case in which rebasing is carried out each year instead of every five or ten years." (SNA para 16.77).

Furthermore: "Chainers" find the idea convincing that annual chaining is better than chaining at five year intervals; however, monthly chaining is *not* better than annual chaining.

A2: The "only valid information" argument (accuracy issue)

Given that links $P_1^{LC}, P_2^{LC}, \ldots$ were in fact the only meaningful measures whereas P_{0t} is not valid, why then should $\overline{P}_{0t}^{LC}$ be a useful measure, and P_{0t}^{L} not?

A3: The multiplication mystery (Martini's theory of indirect comparison): we are able to compare validly *indirectly* (by chaining) things that are totally incomparable *directly* (by linking partial comparisons with a certain overlap as visualized in **fig. 7.1.2**)

It appears inconsistent to take the link for the whole chain, and to ignore the aspect of multiplying links, as done in argument **A2**, and at the same time (as argued under **A3**) to derive anew some "advantages"[44] of the chain approach from the simple fact that links are multiplied to form a chain.

A3 ignores the different nature of the result of direct and indirect comparison: path dependence for example in the indirect method, is unknown to the direct method. Conspicuously there is no limit for the length of a chain. *Indirectly* "comparable" is virtually everything with everything.

B1: Moving comparison, additional information: Chain indices provide a different type of comparison {"moving", "run", "rolling"} base

It is only the "base" of the link, that is t-1 in P_t^{LC}, which is "moving", but of course *not* the base 0 of the chain $\overline{P}_{0t}^{LC}$. Why is the sequence $\overline{P}_{01}^{LC}$, $\overline{P}_{02}^{LC}$, ... called a "run", while P_{01}^{L}, P_{02}^{L}, ... is not?

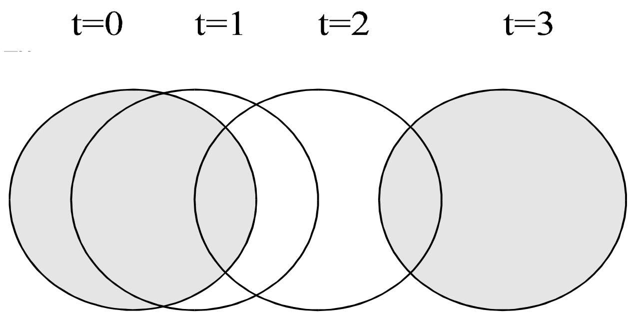
The interesting feature, is not the "moving" base of the links but rather the path dependence of the resulting chain. **B1** also comprises the somewhat arcane idea that chain indices provide *valuable additional information* because of making better use of all time series data.

To infer

more *data input* → more *information output*

is blatantly erroneous: statistics affected by *many* (possibly uncorrected) influences are in general *inferior* rather than superior to statistics determined by a few known factors only, and in general much is done to purge a statistic from irrelevant, contaminating influences.

Figure 7.1.2: Successive comparisons of partially overlapping circles



[44] The alleged advantages consist in: more accuracy (**A3**), in making long distance comparisons possible which are otherwise (i.e. directly) impossible, in utilizing the information represented in the time series data in a more efficient way, and thereby also in providing valuable "additional information" (argument **B1**), the contents of which yet remains to be made clear, however.

B2: Independence of a base: there is no need to bother with choosing an appropriate base period.

The interpretation given to[45]

(7.1.1a) $\overline{P}_{04}^{C}/\overline{P}_{03}^{C} = \overline{P}_{14}^{C}/\overline{P}_{13}^{C} = \overline{P}_{24}^{C}/\overline{P}_{23}^{C} = P_{4}^{C}$

is usually this: A chain index is no more tied to one base than to another, and the result P_{34} is the same irrespective of whether 0, 1, or 2 is the base[46]. In chain indices the *reference* base (RB) is deemed irrelevant On the other hand increased attention given to the *weight* base (WB), the up-to-date-ness of which is of utmost importance.

However the irrelevance of the RB is *not* inherent in the data but rather resulting from the construction of the index, i.e. derived from the fact that the chain is *defined* by multiplication of links. It is ensuing from restrictive assumptions of proportionality imposed on the time series P_{01}, P_{02}, ... (see pages 21/22 above). Moreover different subdivisions of a given interval between 0 and t will yield different results of $\overline{P}_{0t}^{C}$ (see below).

In P^L the price level in period t is said to be measured *in terms of* (in units of, in percent of) the level in 0. The base year matters[47].This does *not* apply to $\overline{P}_{0t}^{LC}$. Once a decision is made on the starting point (RB) the sequence of weights (WB) is also *uniquely* determined. There is simply nothing left which might make the choice of a base period important and at the same time difficult.

B3: More relevant growth factor: the growth factor of a volume (real value) should be expressed in terms of the most recent (most "relevant", most "representative") prices

What (allegedly) makes the growth factor

(7.1.2a) $\frac{\overline{Q}_{0t}^{LC}}{\overline{Q}_{o,t-1}^{LC}} = Q_t^{LC} = \frac{\sum q_t p_{t-1}}{\sum q_{t-1} p_{t-1}}$ "better" than (7.1.3a) $\frac{Q_{0t}^{L}}{Q_{0,t-1}^{L}} = \frac{\sum q_t p_0}{\sum q_{t-1} p_0}$.

is the price vector $\mathbf{p}_{t-1}$. However $\frac{Q_{0t}^{P}}{Q_{0,t-1}^{P}} = \frac{\sum q_t p_t}{\sum q_0 p_t} \frac{\sum q_0 p_{t-1}}{\sum q_{t-1} p_{t-1}}$. And there is also a need for consistent comparisons of annual growth rates over a number of successive years (over a business cycle of four, five or six years or so for example).

The growth rate of Norway's "real GDP" alternatively calculated using fixed prices of 1984 (row A) on the one hand, and calculated using previous year prices (row B) on the other hand was given by:

		1987	1988	1989
A	constant base period prices	4.9	3.0	5.2
B	previous year prices	3.9	1.8	0.9

The problem is not (or not only) to take a single growth rate in isolation, but to make comparisons over a number of periods in a consistent manner. However again the aspect of comparability (by keeping weights constant) is dwarfed by "relevance" of weights.

45 The symbol C is used to denote that this equation applies to a chain index of any type, not only the Laspeyres chain index (LC).

46 Note that this is simply implied by the *definition* of a "chain" (or the operation of "*chaining*" as opposed to the property of "*chainability*"). Moreover there is no reason given in **B2**, why P_{04}/P_{03} should equal P_{14}/P_{13}, and P_{24}/P_{23}. Taken in isolation the relationship given by eq. 7.1.1 is not desirable, though in a sense it guarantees irrelevance of the reference base.

47 The price level in period 0 acts as a sort of yardstick

Digression on annual growth rates of figures compiled monthly

In the comparison of the value a (chain) price index takes in a certain month m in year t to the value of the corresponding month of the previous year t-1 may also be a function of weights of *two* years, i.e. in a comparison of sub-annual intervals (months for example)[48] Compare

Prices	Jan. 99	...	May 99	...	Dez. 99	Jan. 00
Weights	∅ 98	...	∅ 98	...	∅ 98	∅ 99

with

Prices	Jan. 98	...	May 98	...	Dez. 98	Jan. 99
Weights	∅ 97	...	∅ 97	...	∅ 97	∅ 98

C1: Most frequent update of weights: The SNA in particular is praising emphatically chain indices as "indices whose weighting structures are as up-to-date and relevant as possible".

The common feature of this group (**C**) of arguments is the contention that chain indices are able to solve almost insurmountable problems involved in fixed basket indices and at the same time these arguments give chain indices an image of a much greater suitability to modern needs.[49]

Note this argument again compares a direct index with a chain index (or rather a link), as if they both had a single weighting scheme only. The implicit assumption, $\overline{P}_{0t}^{LC}$ has a *single* (and better) WB just like a direct index P_{0t}^{L} has one WB only, is simply false. Moreover there is no clear concept to define, or measure the *degree* of "representativity" or "relevance", hence the argument in itself cannot give any hint concerning the best frequency of a renewal of weights.

C2: Less problems with new developments, quality adjustment less difficult[50]

Instead of "quality adjustments less difficult" the argument **C2** should rather read: "quality adjustments less necessary"(there is no need for an index to be comparable across more than just two adjacent periods, t-1 and t).

Group D arguments referring to expected and desired results of index calculations

Smoother development, and possibly smaller inflation rates, only "if individual prices and quantities tend to increase or decrease monotonically over time" (SNA 93, para. 16.44). "The main advantage of the chain system is that under normal conditions, chaining will reduce the spread between the Paasche and the Laspeyres indices" (PPI/CPI manual): smaller Laspeyres-Paasche-gap (LPG)

c) Arguments in the SNA (group E)

Group E: Chain-index of Fisher is recommended for deflation (SNA)

In the opinion of the SNA for index formulas there is few if anything of equal importance to the up-to-dateness of weights. The SNA also found essential to avoid terms like $\Sigma p_0 q_t$ (as opposed to $\Sigma p_0 q_0$ or to $\Sigma p_t q_t$) in which prices and quantities refer to different periods.

48 This applies to the (Harmonized) European Consumer Price Index (HICP).

49 That is: a more modern and flexible index design with weights constantly updated (**C1**) *and* at the same time an elegant solution of a burdensome problem we always had to cope with in the case of the old design of a fixed basket (**C2**).

50 The chain principle is said to facilitate (or accelerate) the adaptation to new developments by making use of the most recent weights, and to handle better the withdrawal of old and the entry of new commodities.

The SNA recommendations read as follows:

1. the preferred measure of year to year movement of real GDP is a Fisher volume index, changes over longer periods being obtained by chaining: that is, by cumulating the year to year movements;
2. the preferred measure of year to year inflation for GDP is therefore a Fisher price index, price changes over long periods being obtained by chaining the year to year price movements: the measurement of inflation is accorded equal priority with the volume measurements;
3. chain indices that use Laspeyres volume indices to measure movements in real GDP and Paasche price indices to year to year inflation provide acceptable alternatives to Fisher indices

In short the recommendations express a general belief of

- the chain principle being superior to the direct binary comparison and
- being universally applicable for both, measurement of price levels as well as deflating aggregates, and finally a firm belief in
- Fisher's "ideal" index being better than traditional formulas (Laspeyres, Paasche), which are qualified as second best solutions only.

d) Logical status of the arguments

1. Justification of chain indices not theory-driven[51], inconsistency (unit value indices![52], uniqueness-theorem of Funke) and one-sidedness (no disadvantages of chaining mentioned);
2. "Advantages" of chain indices (e.g. more relevant [up to date] weights) are mainly derived from a critique of the *fixed basket* (direct *Laspeyres*) approach; they do not apply to certain "superlative indices" like P^F;

   The idea of updating weights not necessarily requires a chain index, a "superlative index" (like P^F or P^T) in which also the current year weights q_{it} enter (in addition to q_{i0}) will serve the same purpose (without the disadvantage of path dependency, violating axioms etc.).
3. "Solution vs. dissolution": Some problems purportedly "solved" by chain indices are not really solved but rather "dissolved" (e.g. choice of base period, quality adjustment)

   In the chain index approach there no longer is anything which makes the choice of the base period a problem. Once the fixed basket concept is abandoned, there is no need for comparability over more than just two adjacent periods, t-1 and t.
4. $\overline{P}_{0t}^{LC}$ should be advantageous especially in those cases in which direct indices P_{0t}^{L} fail. However, this is conspicuously not the case. It is said that the chain approach is not commendable
   - when comparisons over *long* intervals in time rather than short ones are wanted, and
   - whenever consumption patterns change *rapidly* and *fundamentally* rather than smoothly.
5. It is far from clear that the most recent weights are also the most "relevant" and most "representative" weights. The following two assumptions are (mostly tacitly) made

   a) the actually observed consumption structure is the result of *voluntary* decisions made by consumers, enjoying a real income by and large the same in 0 and in t, and
   b) the choice is *not restricted* by activities on the *supply* side, there should be no doubt that consumers at time t have chosen qualities and quantities q_t instead of quantities q_{t-1} of t-1 because q_t was *preferred* to q_{t-1} and not because q_{t-1} was no longer available.

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[51] unlike for example the COLI-approach. Chain indices do not add a new index theory: they are based on the same basket-concept as P^L, however, it is primarily the updating of the basket that matters.

[52] They were rightly rejected by the SNA as being "affected by changes in the mix of items as well as by changes in their prices. Unit value indices cannot therefore be expected to provide good measures of average price change over time" (para 16.13).

7.2. Properties of chain indices

a) Mean of relatives and ratio of expenditures	f) Inconsistency in aggregation
b) Axioms apply to links only	g) No structural consistency of volumes
c) Cyclical movement of prices/quantities	h) Determinants of the drift
d) No transitivity but path dependence	i) Justification of chainlinking, Theorem of Funke
e) Nonlinearity (of increase/decrease)	j) Aspects of official statistics, acceptance etc.

Table 7.2.1: Summary of (ten) shortcomings of chain indices

Problem, property	part*
theoretical justification and interpretation	
Interpretation in terms of mean value (of relatives) and ratio of expenditures	a
Determinants of the drift (temporal correlation, growth factors)	h
Justification of chainlinking, inconsistency with constant adjustment of weights	i
Acceptance, understandability, room for manipulation[2]	j
axioms, aggregation over time	
No identity and monotonicity, axioms apply to links only (not to the chain)	b
Cyclical movement of prices/quantities	c
No transitivity, path dependence versus circularity	d
Nonlinearity (the determinants of an increase/decrease of the price level)	e
performance of chain indices in deflation and aggregation	
Inconsistency in aggregation	f
No structural consistency (of volumes) and proportionality in quantities	g

*) of sec. 7.2

a) Chain price indices and the two traditional interpretations of a price index: No mean of relatives and ratio of expenditures interpretation

While P^L (as well as P^P for example) has the advantage of allowing both interpretations (ratio of expenditures and mean of relatives), none of the interpretations applies to the direct Fisher index P^F nor to chain indices of whichever sort.

"Ratio of expenditures" (7.2.1) $$\bar{P}_{0t}^{LC} = \frac{\sum p_t q_0^{LC}}{\sum p_0 q_0} \text{ where } q_{i,0}^{LC} = \frac{q_{i,t-1}}{\bar{Q}_{0,t-1}^{PC}}.$$

No "mean of relatives" interpretation Consider a chain of two links only and two commodities with weights (expenditure shares) as follows: a and 1-a for good 1 and 2 respectively at period 0, and correspondingly b and 1-b at period 1. The direct Laspeyres index obviously is given by

(7.2.3) $P_{02}^{L} = \frac{p_{12}}{p_{10}} a + \frac{p_{22}}{p_{20}}(1-a) = m_1 a + m_2(1-a)$, where m_1 and m_2 denote price relatives. $\bar{P}_{02}^{LC} = \left[\frac{p_{11}}{p_{10}} a + \frac{p_{21}}{p_{20}}(1-a)\right] \cdot \left[\frac{p_{12}}{p_{11}} b + \frac{p_{22}}{p_{21}}(1-b)\right]$, or in terms of price relatives

(7.2.4) $$\bar{P}_{02}^{LC} = m_1 a\left[b + g(1-b)\right] + m_2(1-a)\left[(1-b) + b/g\right] = m_1 a\, f_1 + m_2(1-a)\, f_2$$

where $g = \frac{p_{11}p_{22}}{p_{12}p_{21}} = \frac{p_{22}/p_{21}}{p_{12}/p_{11}}$.[53] Note that weights af_1 and $(1\text{-}a)_{f2}$ do in general not add up to unity. Hence the mean of relative's interpretation does not apply to a chain index. A chain index can violate identity, monotonicity, and also the mean value property.[54] This will be demonstrated in the following numerical examples

Example 7.2.1

Given the following prices and quantities of n = 2 commodities (i = 1, 2)

	period 0		period 1		period 2	
i	prices	quantities	prices	quantities	prices	quantities
1	2	10	12	3	12	
2	5	4	7	10.29	14	

Quantities in period 2 are irrelevant. The direct index P_{02}^{L} is given by eq. 7.2.3 $\frac{12}{2}0.5 + \frac{14}{5}0.5 = 4.4$ such that $14/5 = 2.8 < P_{02}^{L} = 4.4 < 12/2 = 6$, whereas $\overline{P}_{02}^{LC}$ yields

$$\overline{P}_{02}^{LC} = \left[\frac{p_{11}}{p_{10}}a + \frac{p_{21}}{p_{20}}(1-a)\right]\left[\frac{p_{12}}{p_{11}}b + \frac{p_{22}}{p_{21}}(1-b)\right], \text{ or } \overline{P}_{02}^{LC} = \left(6\cdot\frac{1}{2} + \frac{7}{5}\cdot\frac{1}{2}\right)\left(1\cdot\frac{1}{3} + 2\cdot\frac{2}{3}\right)$$

$= 6.167 > 6.$ Hence $af_1 = 0.833$ and $(1\text{-}a)f_2 = 0.4167$ such that $af_1 + (1\text{-}a)f_2 = 1.25 \neq 1$, and $6 \cdot 0.833 + 2.8 \cdot 0.4167 = 6.167$, g =2. ♦

It is not only a theoretical possibility that chain indices may fail the mean value test: results of the Canadian Consumer Price Index (a chain index) March 1978 were as follows

Goods	171.1
Services	171.4
Goods and Services	170.8

b) No identity and monotonicity, axioms apply to links only (not to the chain)

Only the link is an index in the sense of satisfying or violating certain "axioms". A chain is *not* an index and can violate axioms, despite being "made" of links that satisfy them all.

Identity applied to situations 0 and 2 requires $\overline{P}_{02} = 1$ to hold whenever prices in 0 and 2 are equal. ***Monotonicity*** requires $\overline{P}_{02}$ to differ from unity in case a single price p_{i2} differs from p_{i0} all other prices being equal. This can easily be demonstrated in an example.

Example 7.2.2

Given the following prices and quantities of n = 2 commodities (i = 1, 2) (Note that prices in 0 and 2 are the same)

[53] The term g denotes a ratio of price relatives. In our book on chain indices we gave a more general representation of the relationship in matrix notation.

[54] Thus it may well exceed the greatest individual price relative or can be smaller than the smallest price relative. The idea of chain indices is to use the most recent and thus (!) most *"representative" weights in each link* (period), however, *the chain is by no means* necessarily *representative* in the sense of typical or average *price relative* (over the whole time interval).

	period 0		period 1		period 2	
i	prices	quantities	prices	quantities	prices	quantities
1	8	6	6	10	8	
2	12	4	15	5	12	

Direct index: P^L_{02} = 1 due to identity of prices in 0 and 2, the weights (expenditure shares) in notation of **ex. 7.2.1** are in a = 0.5 and b = 4/9 for commodity 1 in period 0 and 1.
Chain index: $P^{LC}_1 = P^L_{01} = 1$ and $P^{LC}_2 = 1.037$ and therefore $\overline{P}^{LC}_{02} = P^{LC}_1 P^{LC}_2 = 1.037$ indicating a rise in prices $\overline{P}^{LC}_{02} = 1\cdot\frac{1}{2}\cdot\frac{7}{9} + 1\cdot\frac{1}{2}\cdot\frac{35}{27} = \frac{28}{27} = 1.037 \neq 1$ which simply is the sum of the "new weights", a f_1 and (1-a) f_2. Consider modifications of the example as follows

variant a: prices in		
i	period 0	period 2
1	8	8
2	12	11

variant b: prices in		
i	period 0	period 2
1	8	7
2	12	12

All other prices and quantities remain unchanged

Variant a shows that monotonicity is violated: $\overline{P}^{LC}_{02}$ = 1 although the price of commodity 2 is clearly declining (P^L_{02} = 92/96 = 0.9583). The point in ***Variant b*** is that a chain index though in line with monotonicity may well be smaller than the smallest individual price relative $m_1 = p_{12}/p_{10} = 7/8 = 0.875$ (whilst m_2 remains unity). We get there $P^{LC}_1 = 1$ and P^{LC}_2 = 155/180 = 0.86111 while P^L_{02} = 90/96 = 0.9375.

Proportionality[55] is not met either because $P^{LC}_1 P^{LC}_2 = \frac{\sum p_1 q_0}{\sum p_0 q_0}\frac{\sum \lambda p_0 q_1}{\sum p_1 q_1}$ will not in general result in λ. ***Linear homogeneity*** requires $P(\mathbf{p}_0, \lambda\mathbf{p}_2) = \lambda P(\mathbf{p}_0, \mathbf{p}_2)$ and is fulfilled.

c) Cyclical movement of prices

The following **Ex. 7.2.3** shows that violation of axioms may also occur in the case of an unweighted chain index, as e.g. the formula of Carli such that it is in general the principle of chaining itself, not the existence of weights which causes the violation of the axiom. Moreover we cannot in general assume that a chain index will show less inflation or a smoother development. It rather can "explode" in the case of *cyclical* price movement.

Example 7.2.3

Consider again two commodities. The price of the first will be redoubled in 1, such that $p_{11}/p_{10} = 2$. and thereafter return to the original level such $p_{12}/p_{11} = 0.5$ or $p_{12} = p_{10}$. The opposite process will be assumed for the other (second) commodity. The Carli chain index now is simply $\overline{P}^{CC}_{02} = \frac{1}{2}(2+0.5)\cdot\frac{1}{2}(0.5+2) = (1.25)^2 = 1.56 > 1$.

Assume now the process will carry on endlessly such that we get the following prices

commodity	0	1	2 (= 0)	3 (= 1)	4 (= 0)
no. 1	6	12	6	12	6
no. 2	8	4	8	4	8

55 $P(\mathbf{p}_0, \lambda\mathbf{p}_0) = \lambda$.

Due to identical prices we get $P_{02}^{L} = P_{04}^{L} = P_{06}^{L} = ... = 1$. Likewise $P_{01}^{L} = P_{03}^{L} = ... = 1.25$. However $\overline{P}_{04}^{CC} = (1.56)^2 = 2.44$, $\overline{P}_{06}^{CC} = (1.56)^3 = 3.81$ and so on. If weights are introduced the situation is basically the same. ♦

when the relative prices in the first and last periods (0,t)	a chain index
1) are very different from each other and chaining involves linking periods in which prices and quantities are intermediate between those of 0 and t	should be used
2) are similar to each other (and very different to an intermediate period 0 < t* < t); example: seasonal variation	should **not** be used no indirect comparison via t*

d) No transitivity but path dependence and no pure price comparison

Many writers erroneously conclude from a chain index being defined as a product that the chain index is transitive (path *in*-dependent), or in other words, it can be *consistently aggregated over time* since $\overline{P}_{0t} = \overline{P}_{0k}\overline{P}_{kt}$. However transitivity (chainability) requires that *each* chaining over the *same* interval in time should yield the same result[56], and precisely this property is not given in the case of chain indices:

Example 7.2.4:

t=0		t=1		t=2		t=3		t=4	
p	q	p	q	p	q	p	q	p	q
2	10	4	12	3	20	1	16	2	10
5	20	3	15	4	10	4	12	5	20

The direct index is of course $P_{04}^{L} = 1$ because all prices (and also quantities) in 4 equal those in 0 (indicated by shadows). The chain index not only violates identity but also yields different results:

(a) $\overline{P}_{04}^{LC}(a) = P_{02}^{LC} P_{24}^{LC} = \frac{110}{120}\frac{90}{100} = 0.825$, with only *two* intervals (0, 2) and (2, 4), but

(b) $\overline{P}_{04}^{LC}(b) = P_{1}^{LC} P_{2}^{LC} P_{3}^{LC} P_{4}^{LC} = \left(\frac{100}{120}\frac{96}{93}\right)\left(\frac{60}{100}\frac{92}{64}\right) = \frac{88.89}{120}\frac{86.25}{100} = 0.7419$

upon dividing the *same* interval into *four* subintervals (0, 1), ... , (3, 4). The situations 0 and 4 are not uniquely compared. The result depends on how the interval is subdivided. ♦

Being "**path-dependent**" chain indices depend on how an interval is subdivided, they provide a summary description of a process rather than a comparison of two situations taken in isolation. A sequence of chain indices is not a consistent *temporal* aggregation.

Given two different points in time t and τ. In the case of chain indices $\overline{P}_{0t} = \overline{P}_{0\tau}$ does not imply identical prices nor $\overline{P}_{0t} \neq \overline{P}_{0\tau}$ different prices in t and τ. The principle of "pure price comparison" requires that a price index should exclusively reflect the price movement and therefore $P_{0t} = P_{0\tau}$ should hold if prices in t equal prices in τ otherwise $P_{0t} \neq P_{0\tau}$.

[56] that is indirect comparisons of *all* sorts (for whatever partition of the interval) should be consistent with direct comparison and also consistent among themselves.

e) Nonlinearity (the determinants of an increase/decrease of the price level)

Due to the linearity (in the prices of period t) of the direct Laspeyres index an equal amount of change in prices denoted by $\Delta p_t = p_t - p_{t-1}$ in period t and $t^* \neq t$ has the same effect as long as t or t* are periods in the range of an index of the same base 0 (0 < t* < t). We get

(7.2.8) $$P_{02}^{L} = 1 + \frac{\sum q_0 \Delta p_1}{\sum q_0 p_0} + \frac{\sum q_0 \Delta p_2}{\sum q_0 p_0} = P_{01}^{L} + \frac{\sum q_0 \Delta p_2}{\sum q_0 p_0} .$$

Table 7.2.2: Relation between changes of individual prices and the price index

t	direct Laspeyres	chain Laspeyres
1	$P_{01}^{L} = 1 + \sum \Delta p_1 \frac{q_0}{\sum q_0 p_0} = 1 + K_1^0$	$\overline{P}_{01}^{LC} = 1 + \sum \Delta p_1 \frac{q_0}{\sum q_0 p_0} = P_{01}^{L} = 1 + K_1^0$
2	$P_{02}^{L} = P_{01}^{L} + \sum \Delta p_2 \frac{q_0}{\sum q_0 p_0}$ $= 1 + K_1^0 + K_2^0 = P_{01}^{L} + K_2^0$	$\overline{P}_{02}^{LC} = \overline{P}_{01}^{LC}\left(1 + \sum \Delta p_2 \frac{q_1}{\sum q_1 p_1}\right)$ $= \left(1 + K_1^0\right)\left(1 + K_2^1\right) = \overline{P}_{01}^{LC}\left(1 + K_2^1\right)$
3	$= 1 + K_1^0 + K_2^0 + K_3^0 = P_{02}^{L} + K_3^0$	$= \left(1 + K_1^0\right)\left(1 + K_2^1\right)\left(1 + K_3^2\right) = \overline{P}_{02}^{LC}\left(1 + K_3^2\right)$

In the direct index case the same changes in prices at different times has the same effect on the price index. The corresponding relationship in the case of a Laspeyres *chain* index is much more complicated even if prices change to the same extent in consecutive periods (see **ex. 7.2.5**):

- P_{0t}^{L} is a linear combination of changes Δp_t with constant "weights" $q_0/\Sigma p_0 q_0$ such that in the case of a constant change ($\Delta p_1 = \Delta p_2 = ...$) in all periods a constant term is added, and the result is independent of how the interval (0, t) is subdivided into sub-intervals;
- in $\overline{P}_{0t}^{LC}$ changes of individual prices Δp_{i1}, Δp_{i2} are *multiplied* by weights which are in general *not* constant. Even the effect of a constant change will not be independent on when it takes place unless also all quantities remain constant such that $q_0 = q_1 = ... = q_{t-1}$.

For example the term to be added to $\overline{P}_{0t}^{LC}$ in order to get $\overline{P}_{0,t+1}^{LC}$

(7.2.14) $$\sum_i \Delta p_{i,t+1} \frac{q_{it}}{\sum q_{it} p_{it}} \overline{P}_{0t}^{LC} = \overline{P}_{0,t+1}^{LC} - \overline{P}_{0t}^{LC}$$

is much more complicated than the term to be added to P_{0t}^{L} to get $P_{0,t+1}^{L}$, which is simply

(7.2.15) $$P_{0,t-1}^{L} - P_{0,t}^{L} = \sum_i \Delta p_{it} \left(\frac{q_{i0}}{\sum q_{i0} p_{i0}} \right) = \sum_i \Delta p_{it} w_i .$$

Moreover there is no theory known to explain why the *same* amount of change of prices should be treated quite *differently* as shown in eq. 14 (and just in the way of eq. 14).

Example 7.2.5 (again a modification of **ex. 7.2.2**)

	period 0		period 1		period 2	
i	prices	quantities	prices	quantities	prices	quantities
1	8	6	6	10	4	
2	12	4	16	3	20	

The direct Laspeyres index yields 1.0417 and 1.0833 rising constantly by 4.17 percentage points because $\Delta p_1 \frac{q_{10}}{\sum q_{i0}p_{i0}} + \Delta p_2 \frac{q_{20}}{\sum q_{i0}p_{i0}} = (-2)\frac{6}{96} + (+4)\frac{4}{96} = 0.0417$. The chain index, however, develops as follows: $\overline{P}_{01}^{LC} = \frac{100}{96} = 1.0417$ and $\overline{P}_{02}^{LC} = \frac{100}{96}\cdot\frac{100}{108} = 0.9645$.[57]

f) Inconsistency in aggregation

Whenever more than just one item (commodity) is involved in chaining, or comparing two or more periods the chain-index approach is much more difficult than the direct-index approach. Consider the growth of a stock A by adding successive net increases Z_1, Z_2 etc. The stocks are A, A + Z_1, A + Z_1 + Z_2 etc. and the relatives are $m_{01} = (A + Z_1)/A$, $m_{02} = (A+Z_1 + Z_2)/A$. Obviously $m_{02} = \frac{A+Z_1}{A}\frac{A+Z_1+Z_2}{A+Z_1} = m_{01}m_{12}$. This simple situation[58] becomes much more difficult, however, when stocks and flows are broken down to two sectors (*sub*-aggregates) with stocks $A = A_1 + A_2$, and two flows with $Z_1 = Z_{11} + Z_{12}$ and $Z_2 = Z_{21} + Z_{22}$ respectively:

period	sector 1	sector 2
1	$m_{(1)\,01} = (A_1 + Z_{11})/A_1$	$m_{(2)\,01}$ analog $m_{(1)\,01}$
2	$m_{(1)\,12} = (A_1 + Z_{11} + Z_{21})/(A_1 + Z_{11})$	$m_{(1)\,12} = (A_1 + Z_{11} + Z_{21})/(A_1 + Z_{11})$

Though relatives for each sector (i = 1, 2) obviously remain transitive $m_{(i)\,02} = m_{(i)\,01}\, m_{(i)\,12}$ when aggregated we need, however, changing weights in order to account for the changing structure): $A_i / \sum A_i$ for m_{01}, and $(A_i + Z_{1i}) / \sum A_i + Z_{1i}$ for m_{12} leading to

$$\text{(7.2.16)} \qquad m_{02} = m_{01}m_{12} = \frac{A_1 m_{(1)01} + A_2 m_{(2)01}}{A_1 + A_2} \frac{(A_1 + Z_{11})m_{(1)12} + (A_2 + Z_{12})m_{(2)12}}{(A_1 + Z_{11}) + (A_2 + Z_{12})}.$$

It is difficult to see why this chaining (multiplying) of relatives should be any better than going the direct way using (7.2.16a) $m_{02} = \frac{A_1 m_{(1)02} + A_2 m_{(2)02}}{A_1 + A_2}$.

The successive weights $w_{i\tau}$ to be assigned to sector i in aggregating sectoral links to the total link to arrive at $\overline{P}_{0t}^{LC} = \prod_{\tau=1}^{t}\sum_{i=1}^{n} m_{(i)\tau-1,\tau} w_{i\tau}$ are $\frac{A_i}{\sum A_i}$, $\frac{A_i + Z_{1i}}{\sum A_i + Z_{1i}}$, $\frac{A_i + Z_{1i} + Z_{2i}}{\sum A_i + Z_{1i} + Z_{2i}}$ as shown above. The general term is $A_i + \sum_{\tau=1}^{\tau=t-1} Z_{\tau i} \Big/ \sum_i A_i + \sum_{\tau=1}^{\tau=t-1}\sum_i Z_{\tau i}$ and the chain is given by

$$\text{(7.2.17)} \qquad m_{0t} = \frac{A_1 m_{(1)01} + A_2 m_{(2)01}}{A_1 + A_2} \cdot \ldots \cdot \frac{(A_1 + \sum_{\tau=1}^{\tau=t-1} Z_{\tau 1})m_{(1)t-1,t} + (A_2 + \sum_{\tau=1}^{\tau=t-1} Z_{\tau 2})m_{(2)t-1,t}}{(A_1 + \sum Z_{\tau 1}) + (A_2 + \sum Z_{\tau 2})}$$

as opposed to the simply

$$\text{(7.2.18)} \qquad m_{0t} = \frac{A_1 m_{(1)0t} + A_2 m_{(2)0t}}{A_1 + A_2}$$ in the case of a direct index.

57 We may also well get a sequence of chain indices indicating a decline in prices although the direct index is not changing. This is shown in ex. 7.2.6, not included here.

58 Note that the result above only shows that the stock may be calculated directly *or* indirectly. It is also irrelevant in which and how many subintervals the interval is divided.

To arrive at a relative $m_{0t} = y_t/y_0$ relating to an *aggregate* y composed of n components $y_1, y_2, ..., y_n$ given *links* of the components (sectors) $m_{(i)t-1,t}$ (sectoral links) and their changing weights w_{it} we have in principle *to work out again the complete calculation*, that is to

- aggregate (summate) over the sectoral links using constantly varying weights, and to
- aggregate (multiply) over time using $(\Sigma w_{i0}\, m_{(i)01}) \ldots (\Sigma w_{i,t-1}\, m_{t-1,t})$.

Hence for "users" of price indices it is much more difficult, if not impossible to compile a (chain) price index for his specific (ad hoc) composition of (included and excluded) subaggregates. He has not only got to know the (partial) link for each sub-aggregate and each period under consideration but also the constantly changing weights of each sub-aggregate in each period in order to derive the total links to be multiplied to the chain index of the user's own composition.

g) No structural consistency (of volumes) and proportionality in quantities

No chain index construction is structurally consistent (provides "additive" volumes), not even the *chain* version of Paasche (as opposed to the *direct* Paasche index)[59], let alone the *chain* Fisher price index as deflator. *In addition* to the structural inconsistency (of the direct Fisher as well as chain Fisher deflator) there is at least one more shortcoming of the *chained* Fisher index: it violates the *value index preserving test* (Vogt 1978, Balk 1995) or *proportionality in the quantities*.

axiom violated	direct Fisher	chained Fisher
structural consistency	yes	yes
proportionality in quantities	no	yes

If all quantities change identically by λ such that $\mathbf{q}_t = \lambda\,\mathbf{q}_0$ then it seems to be plain logic that a quantity index should yield $Q(\mathbf{q}_0, \mathbf{p}_0, \lambda\mathbf{q}_0, \mathbf{p}_t) = \lambda$ [60]. However, volumes gained by deflation with $\overline{P}^{FC}_{0t}$ as deflator fails this test:

Example 7.2.7

Assume that prices of two goods, A and B are rising uniformly by 50% from 0 to 3, and quantities remain constant such that $V_{03} = P^L_{03} = P^P_{03} = P^F_{03} = 1.5$. *Direct* Fisher- and Paasche-deflation yields the same result. By contrast, deflation with $\overline{P}^F_{03} = 1.564$, results in

- *volumes* indicating a *decline*, despite the same quantities in 0 and 3, and
- prices *not* rising by 50% but rather by 56.4%.

	period 0		period 1		period 2		period 3	
good	p	q	p	q	p	q	p	q
A	30	5	40	3	50	2	45	5
B	10	15	5	20	10	13	15	15

$\overline{P}^{FC}_{03}$ simplifies to $\sqrt{1.5\frac{\sum p_1q_0 \sum p_2q_1 \sum p_3q_2}{\sum p_0q_1 \sum p_1q_2 \sum p_2q_0}}$, not necessarily amounting to 1.5. The results for the chain indices[61] are $\overline{P}^{LC}_{03} = 665/368 = 1.807$, $\overline{P}^{PC}_{03} = 2277/1682 = 1.354$, and $\overline{P}^{FC}_{03}$

[59] As shown above this is the *only* index function able to fulfill the criterion of structural consistency in volumes.

[60] This also includes the case of no change λ = 1 (identity), in which Q should amount to Q = 1 and $V_{0t} = \mathbf{p}_t'\mathbf{q}_0/\mathbf{p}_0'\mathbf{q}_0 = P^L$. The direct Fisher index will always pass this "value index preserving test", while the chain Fisher index will not as shown above.

[61] Note that in this case the chain indices of Laspeyres and Paasche are *not* closer to the direct Fisher index P^F than their direct counterparts (P^L and P^P). Thus chaining *not* always reduces the "Laspeyres- Paasche-gap".

$= \sqrt{2.4463} = 1.564$. Dividing of $\sum p_3 q_3 = 450$ by $\overline{P}^{FC}_{03}$ gives a chain-index Fisher volume of 287.71, hence a reduction compared with $\sum p_0 q_0 = 300$ by 4.1%. ♦

Moreover in the case of deflation using a direct P^P index (DP-method), the sequence of volumes as well as quantity indices is clearly exclusively reflecting the change of quantities. **Tab. 7.2.4** demonstrates, however, that this is no longer the case whenever a chain index (Paasche [CP] or Fisher [CF])is used for deflation.

The table[62] shows that a change of CF-volumes may well be a *result of a number of influences, not at all of quantities only*. Therefore it is difficult to state what exactly has changed to which extent when a volume of this type indicates a given change.

(7.2.19) $$\overline{Q}^{FC}_{03} = \sqrt{\frac{\sum p_3 q_3}{\sum p_0 q_0}\sqrt{\frac{\sum p_0 q_1 \sum p_1 q_2 \sum p_2 q_3}{\sum p_1 q_0 \sum p_2 q_1 \sum p_3 q_2}}}$$

(7.2.20) $$\overline{Q}^{FC}_{04} = \sqrt{\frac{\sum p_4 q_4}{\sum p_0 q_0}\sqrt{\frac{\sum p_0 q_1 \sum p_1 q_2 \sum p_2 q_3 \sum p_3 q_4}{\sum p_1 q_0 \sum p_2 q_1 \sum p_3 q_2 \sum p_3 q_2}}}$$ and in general

(7.2.21) $$\overline{Q}^{FC}_{0t} = \prod_{k=1}^{k=t}\left(\frac{\sum p_{k-1} q_k}{\sum p_k q_{k-1}} V_{0t}\right)^{1/2}.$$

Table 7.2.4: Comparison of volumes derived from different deflation methods (deflators P_{0t})
DP = direct Paasche; DF = direct Fisher, CP = chain Paasche, CF = chain Fisher

P_{0t}	t = 3 $(\sum p_3 q_3 / P_{03})$	t = 4 $(\sum p_4 q_4 / P_{04})$
DP	$\sum p_0 q_3$	$\sum p_0 q_4$
DF	$\sum p_0 q_3 \left(\frac{\sum p_3 q_3}{\sum p_3 q_0}\frac{\sum p_0 q_0}{\sum p_0 q_3}\right)^{1/2}$	$\sum p_0 q_4 \left(\frac{\sum p_4 q_4}{\sum p_4 q_0}\frac{\sum p_0 q_0}{\sum p_0 q_4}\right)^{1/2}$
CP	$\sum p_0 q_1 \left(\frac{\sum p_1 q_2}{\sum p_1 q_1}\frac{\sum p_2 q_3}{\sum p_2 q_2}\right)$	$\sum p_0 q_1 \left(\frac{\sum p_1 q_2}{\sum p_1 q_1}\frac{\sum p_2 q_3}{\sum p_2 q_2}\frac{\sum p_3 q_4}{\sum p_3 q_3}\right)$
CF	$\left(\sum p_3 q_3 \sum p_2 q_3 \frac{\sum p_0 q_0}{\sum p_1 q_0}\frac{\sum p_0 q_1}{\sum p_2 q_1}\frac{\sum p_1 q_2}{\sum p_3 q_2}\right)^{1/2}$	$\left(\sum p_4 q_4 \sum p_3 q_4 \frac{\sum p_0 q_0}{\sum p_1 q_0}\frac{\sum p_0 q_1}{\sum p_2 q_1}\frac{\sum p_1 q_2}{\sum p_3 q_2}\frac{\sum p_2 q_3}{\sum p_4 q_3}\right)^{1/2}$

Tab. 7.2.4 also reveals a cumulative pattern. Moreover it is not easy to discern a "quantity" in "volume" expressions for CP- and CF-deflation in tab. 7.2.4, or to understand why such a volume is said to be a measure "*at constant prices of t = 0*".

We now examine what determines how a volume changes from say period 2 to 3. In the case of DP-deflation the volume change is depending on *base* period prices only

(7.2.22) $$\frac{Q^L_{03}}{Q^L_{02}} = \frac{\sum q_3 p_0}{\sum q_2 p_0} = \sum \frac{q_3}{q_2}\frac{p_0 q_2}{\sum p_0 q_2} = Q^L_{23(0)}$$ (a Q^L_{03} index re-based on base 2).

The corresponding factor in the case of chain Fisher index deflation (CF deflation) is

(7.2.23) $\sqrt{Q^L_{23} Q^P_{23}} = Q^{FC}_3$ depending on *two* price structures[63] both *other* than p_0.

[62] There is no such table in the SNA in which volumes are presented in terms of all aggregates involved.

[63] The SNA said in the light of this result: "Only in the special case in which time series of fixed base Laspeyres volume indices are used ... it is legitimate to equate ... real GDP with ... GDP 'at constant prices'. When chain indices are used, it is not appropriate to describe real GDP as GDP at constant prices“"(p. 16.71).

h) Determinants of the drift (drift-function, temporal correlation, growth factors)[64]

(7.2.24) $D_{0t}^{PL} = \overline{P}_{0t}^{LC} / P_{0t}^{L}$ is known as *drift-function* for a Laspeyres price index (correspondingly $D_{0t}^{QL} = \overline{Q}_{0t}^{LC} / Q_{0t}^{L}$ is the drift of a Q^L-index). With $g_t^k = \sum \frac{p_{it}}{p_{i,t-1}} \frac{p_{i,t-1} q_{ik}}{\sum p_{i,t-1} q_{ik}} = \frac{\sum p_{it} q_{ik}}{\sum p_{i,t-1} q_{ik}}$ being the growth factor of prices from t-1 to t on the basis of weights (quantities) belonging to period k, we get

(7.2.24a) $D_{02}^{PL} = \frac{\overline{P}_{02}^{LC}}{P_{02}^{L}} = \frac{P_{01}^{L} P_{12}^{L}}{P_{02}^{L}} = \frac{P_{12}^{L}}{P_{02}^{L} / P_{01}^{L}} = \frac{\sum p_2 q_1}{\sum p_1 q_1} \div \frac{\sum p_2 q_0}{\sum p_1 q_0} = \frac{g_2^1}{g_2^0}$, and

(7.2.24b) $D_{03}^{PL} = \frac{\overline{P}_{03}^{LC}}{P_{03}^{L}} = D_{02}^{PL} \frac{g_3^2}{g_3^0} = \frac{g_2^1}{g_2^0} \frac{g_3^2}{g_3^0}$, which clearly displays a *recursive* and *cumulative* pattern: starting with $D_{01}^{PL} = 1$, we get successively $\frac{g_2^1}{g_2^0}, \frac{g_2^1}{g_2^0}\frac{g_3^2}{g_3^0}, \frac{g_2^1}{g_2^0}\frac{g_3^2}{g_3^0} \cdots \frac{g_t^{t-1}}{g_t^0}$, as both parts of D_{0t}^{PL}, viz. $\overline{P}_{0t}^{LC}$ and P_{0t}^{L} can be represented as products of growth factors such that

$\overline{P}_{0t}^{LC} = g_1^0 g_2^1 g_3^2 \cdot \ldots \cdot g_t^{t-1}$ where $g_2^1 = \sum \frac{p_2}{p_1} \frac{p_1 q_1}{\sum p_1 q_1}$, $g_3^2 = \sum \frac{p_3}{p_2} \frac{p_2 q_2}{\sum p_2 q_2}$, … and

$P_{0t}^{PL} = g_1^0 g_2^0 g_3^0 \cdot \ldots \cdot g_t^0$ where $g_2^0 = P_{02}^{L} / P_{01}^{L} = \sum \frac{p_2}{p_1} \frac{p_1 q_0}{\sum p_1 q_0}$, $g_3^0 = \sum \frac{p_3}{p_2} \frac{p_2 q_0}{\sum p_2 q_0}$, …[65].

Both drifts (D) and chain price indices ($\overline{P}$) are chainable (can be multiplied)

(7.2.25) $D_{0t}\overline{P}_{0t} = (D_{0s}\overline{P}_{0s})(D_{st}\overline{P}_{st})$ hence $D_{0t} = D_{0s} D_{st}$ just like $\overline{P}_{0t} = \overline{P}_{0s}\overline{P}_{st}$.[66]

Laspeyres price index drift and Paasche quantity index drift are inversely related to one another according to (7.2.26) $D_{03}^{PL} = 1 / D_{03}^{QP}$.

The drift (D_{02}^{PL} for example) can also be expressed in terms of a covariance (in analogy to the relation of eq. 1.3.12: $C = Q_{0t}^{L}(P_{0t}^{P} - P_{0t}^{L})$ found by L. v. Bortkiewicz) as follows

$D_{02}^{PL} = \frac{Cov(x_{12}, y_{01})}{\overline{x}_{12} \cdot \overline{y}_{01}} + 1$ where Cov(x_{12},y_{01}) is the covariance between[67]

growth ***factors*** of individual prices	quantity ***relatives*** (i.e. ***cumulative*** changes)
$x_{i,12} = \frac{p_{i2}}{p_{i1}}$ likewise $x_{i,23} = \frac{p_{i3}}{p_{i2}}$, ... etc.	$y_{i,01} = \frac{q_{i1}}{q_{i0}}$ correspondingly $y_{i,02} = \frac{q_{i2}}{q_{i0}}$ ...

(7.2.27) $\overline{x}_{12} = \sum \frac{p_2}{p_1} \frac{p_1 q_0}{\sum p_1 q_0} = \frac{\sum p_2 q_0}{\sum p_1 q_0} = P_{02}^{L} / P_{01}^{L} = P_{12(0)}$, and

(7.2.27a) $\overline{y}_{01} = \sum \frac{q_1}{q_0} \frac{p_1 q_0}{\sum p_1 q_0} = \frac{\sum p_1 q_1}{\sum p_1 q_0} = Q_{01}^{P}$ then the covariance can be written as

(7.2.28) $Cov(x_{12}, y_{01}) = \sum (x_{i,12} - \overline{x}_{12})(y_{i,01} - \overline{y}_{01}) \frac{p_1 q_0}{\sum p_1 q_0} = \frac{\sum p_2 q_1}{\sum p_1 q_0} - \overline{x}_{12}\overline{y}_{01}$ or

64 We now turn to some issues which are more or less interesting only from a theoretical point of view.

65 see **sec. 2.5**

66 The drift functions depend (much like the chain index function to which they refer) on the length of the interval (0, t) in question, on how it is subdivided into subintervals, and on the path (pattern of the p's and q's).

67 The subscript i in the following table will be dropped in what follows

$\mathrm{Cov}(x_{12}, y_{01}) = \frac{Q^P_{01}}{P^L_{01}}\left(P^L_{01}P^L_{12} - P^L_{02}\right) = Q^P_{01}\left(P^L_{12} - P^L_{12(0)}\right)$, such that

(7.2.29) $D^{PL}_{02} = \frac{\mathrm{Cov}(x_{12}, y_{01})}{\bar{x}_{12}\cdot\bar{y}_{01}} + 1$. In a similar manner we get

(7.2.29a) $\mathrm{Cov}(x_{23}, y_{02}) = \sum(x_{i,23} - \bar{x}_{23})(y_{i,02} - \bar{y}_{02})\frac{p_2q_0}{\sum p_2q_0} = Q^P_{02}\left(P^L_{23} - P^L_{23(0)}\right)$ etc.

Tab. 7.2.5: Cumulative structure of the drift as a function of the (temporal) covariance

t	drift function D^{PL} and covariance cov(x, y)
t = 2, D^{PL}_{02}	$\frac{\mathrm{Cov}(x_{12}, y_{01})}{\bar{x}_{12}\cdot\bar{y}_{01}} + 1$ (since D^{PL}_{01} = 1)
t = 3, D^{PL}_{03}	$D^{PL}_{02}\left(\frac{\mathrm{Cov}(x_{23}, y_{02})}{\bar{x}_{23}\cdot\bar{y}_{02}} + 1\right) = \left(\frac{\mathrm{Cov}(x_{12}, y_{01})}{\bar{x}_{12}\cdot\bar{y}_{01}} + 1\right)\left(\frac{\mathrm{Cov}(x_{23}, y_{02})}{\bar{x}_{23}\cdot\bar{y}_{02}} + 1\right)$
t = 4, D^{PL}_{04}	$D^{PL}_{03}\left(\frac{\mathrm{Cov}(x_{34}, y_{03})}{\bar{x}_{34}\cdot\bar{y}_{03}} + 1\right) = \left(\frac{\mathrm{Cov}(x_{12}, y_{01})}{\bar{x}_{12}\cdot\bar{y}_{01}} + 1\right)\left(\frac{\mathrm{Cov}(x_{23}, y_{02})}{\bar{x}_{23}\cdot\bar{y}_{02}} + 1\right)\left(\frac{\mathrm{Cov}(x_{34}, y_{03})}{\bar{x}_{34}\cdot\bar{y}_{03}} + 1\right)$

Though the system is becoming apparent, the problem is that it is difficult to draw general conclusions concerning the sign and amount of drift[68].

i) Justification of chainlinking, uniqueness theorem of Funke et al.

The operation of "chaining" (if based on the notion of chainability) is inconsistent with constant adjustment of quantity weights. To link consistently two subintervals

(7.2.30) $P_{0t} = P_{0s}P_{st}$ it is implicitly assumed that $\frac{P_{0t}}{P_{0s}} = \frac{P_{st}}{P_{ss}}$, $P_{ss} = 1$ holds.

This is tantamount to assuming indices at *two* different bases (and mostly with different weights) will change *in proportion*. Likewise in the case of three subintervals we have

(7.2.31) $P_{0t} = P_{0r}P_{rs}P_{st}$ implies $\frac{P_{0t}}{P_{0r}} = \frac{P_{rt}}{P_{rr}}$ ***and*** in addition $\frac{P_{rt}}{P_{rs}} = \frac{P_{st}}{P_{ss}}$.

It was not until the proof of Funke et al. 1979 according to which

> the only index, satisfying the minimal requirements monotonicity, linear homogeneity, identity and commensurability and being able to meet chainability is the so called "**Cobb-Douglas index**" given by $P^{CD}_{0t} = \prod_{i=1}^{n}\left(\frac{p_{it}}{p_{i0}}\right)^{\alpha_i}$ where "weights" $\alpha_1, \alpha_2, \ldots \alpha_n$ are *arbitrary* real constants *not* depending on period 0 nor t or any other period, and $\Sigma\alpha_i = 1$.

that we clearly saw a conflict between the *property of chainability* (violated of course in the case of path dependent chain indices) and the continual adjusting of quantity weights[69].

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68 The same is true for the "spread" (or "gap") between the direct Laspeyres and Paasche index formula (more details cp. my book on chain indices).

69 As I. Fisher already conjectured: there are chainable indices where the weights must be constant and on the other hand there are indices with variable weights necessarily violating chainability.

j) Aspects of official statistics: cost-benefit-considerations, understandability, and a system of chain indices

1. Chain indices will require *more statistical surveys* and thereby cost more, both directly to Statistical Offices and indirectly to respondents facing an increased response burden;
2. for public acceptance it is important that concepts are *understandable** and the Office is seen as *neutral* and impartial; this applies in particular to inflation measurement;
3. a change of methods has far-reaching implications as official statistics has to provide a whole *system* of indices (not only price indices, let alone only consumer price indices, but also indices of production, new orders etc.) which should fit together.

* we find exactly the opposite position on the part of the "chainers" (adherents of chaining): it is difficult to explain to the general public why weights should be kept constant for a couple of years as done in the Laspeyres index

Figure 7.2.2: Comparability and chain indices (part 1. Dimensions of comparability)

Chain indices violate "pure" [a] price comparison in the sense of not only being affected by

prices	**periods 0 and t**
but also by changes in (the structure of) quantities (weights), qualities, types of products, outlets etc.[b]	but also by referring to other periods and depending on the path connecting 0 and t (not only on the endpoints 0 and t)[c]
no elimination of structural change	path dependence (no chainability)

a) "Pure" means that situations to be compared should differ in only *one* aspect in order to avoid difficulties (ambiguities) of interpretation and to *make sure that like is compared with like*.
b) this applies to unit value indices as well for example
c) as the first aspect (i.e. prices) refers to the aggregation over commodities, this (second) notion of "pure" refers to the temporal aggregation (over intervals in time).

Chain indices are "impure" in the sense of reflecting a number of other influences than prices. For things to be meaningfully comparable they must have certain common aspects (CA) and at the same time aspects with respect to which they are different (DA). Only DAs should affect the result of a comparison while the CA are necessary in order to compare like with like. Moreover comparability is treated differently in the *single* (n = 1) commodity case as opposed to the case of a *basket* (case of n ≥ 2 commodities).

Figure 7.2.3: Comparability (part 2): Inconsistencies with respect to the number of commodities)

Comparability on the

Single commodity level	**Composite commodity (basket-) level**
In reporting individual prices much heed is given to ensure comparability by carefully making • adjustments for quality change, or • detailed specifications of products*	on the other hand on the basket level no problem is seen in the continually changing of the • selection of goods, and other price determining characteristics (PCD) • weights attached to the goods

* this applies in particular to goods deemed acceptable and suitable for international comparisons.

Digression:

How complicated a deflated value added becomes when double deflation is made using Fisher chain indices for both, input prices and output prices

$\overline{P}_{0t}^{FC}(O)$ and $\overline{P}_{0t}^{FC}(I)$ respectively are Fisher chain price indices for output and input. The deflated output (DO) is given by dividing $\sum p_2(0)q_2(0)$ by

$\overline{P}_{0t}^{FC}(O) = \left(\frac{\sum p_1(0)q_0(0)}{\sum p_0(0)q_0(0)} \frac{\sum p_2(0)q_1(0)}{\sum p_0(0)q_1(0)} \frac{\sum p_2(0)q_2(0)}{\sum p_1(0)q_2(0)}\right)^{1/2}$, and in exactly the same manner dividing $\sum p_2(I)q_2(I)$ by the corresponding index $\overline{P}_{0t}^{FC}(I)$ yields the deflated input (DI). Finally the implicit deflator price index of value added (Y) is given by

$P_{02}^{imp}(Y) = \frac{\sum p_2(0)q_2(0) - \sum p_2(I)q_2(I)}{DO - DI} = \frac{\text{nominal value added}}{\text{real value added}}$ or in terms of Fisher chain price indices and i_t denoting the input share of output (at current prices) at time t.

(7.2.32) $$P_{02}^{imp}(Y) = \frac{(1-i_2)\overline{P}_{02}^{FC}(O)\overline{P}_{02}^{FC}(I)}{\overline{P}_{02}^{FC}(I) - i_2\overline{P}_{02}^{FC}(O)}$$ or in detail

(7.2.33) $$P_{02}^{imp} = \frac{(1-i_2)\sqrt{\frac{\sum p_2(0)q_2(0)}{\sum p_0(0)q_0(0)}}}{\sqrt{\frac{\sum p_0(0)q_1(0)\sum p_1(0)q_2(0)}{\sum p_1(0)q_0(0)\sum p_2(0)q_1(0)}} - \sqrt{i_0 i_2}\sqrt{\frac{\sum p_0(I)q_1(I)\sum p_1(I)q_2(I)}{\sum p_1(I)q_0(I)\sum p_2(I)q_1(I)}}},$$

With more than two links the result will be all the more complicated. The formula shows that the implicit deflator P^{imp} is reflecting *a number* of influences in addition to prices of a given selection of inputs and outputs at two periods in time.

Chapter 8 Interspatial comparisons of prices and volumes

8.1. Introduction into interspatial comparisons

a) Intertemporal and interspatial comparison	e) Consistent multination. comparisons (transitivity)
b) Uses & limitations: PPP & exchange rates	f) Conditions, axioms and properties
c) Bilateral and multilateral comparison	g) Engel-Gerschenkron effect, P^P-P^L spread
d) Some methods of bilateral comparison	

a) Differences between intertemporal and interspatial comparisons

Figure 8.1.1: Some differences between intertemporal and interspatial comparisons

1	order (defined sequence), no chain index approach, different axioms
2	discrete/continuous variable
3	problems with additional countries (full scale comparisons)
4	different and modifiable size of countries
5	stable relationships less likely, no "economic theory approach"
6	price indices no longer pure numbers
7	"regional deflation"("real" aggregates") purchasing power parities (PPP) vs. exchange rates (ER)
8	consistency as regards comparisons over periods (time) *and* countries (space)