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Formulas for Chain-linking in QNA
part III of the ECB- presentation (Jan. 2010)

7.2 Steps common to all methods

Average prices (unit values) **(1)** $\bar{p}_{iy} = \frac{\sum_q \sum_i p_{iyq} q_{iyq}}{\sum_q \sum_i q_{iyq}} = \frac{W_{yq}}{\sum \sum q_{iyq}}$

Value (= at current prices) annual: **(2)** $W_y = \sum_q W_{yq} = \sum_q \sum_i p_{iyq} q_{iyq}$

Volume (*quarter specific* prices of the *previous* year y-1) **(3)** $V_{y,y-1,q} = \sum_i p_{i,y-1,q} q_{i,y,q}$

first subscript: period to which quantities refer
second subscript: period to which prices refer
no bar = quarter specific prices, bar = average prices

By contrast: volume (at *average* prices of the *current* year y), their sum is the value for y

(4) $\bar{V}_{y,y,q} = \sum_i \bar{p}_{i,y} q_{iy,q}$, esp. for y = 0 as start volumes for all methods

(4a) $\bar{V}_{0,0,q} = \sum_i \bar{p}_{i,0} q_{i,0,q}$ and **(4a')** $\sum_q \bar{V}_{0,0,q} = W_0$

as opposed to volume (at average prices of the *previous* year)

(5) $\bar{V}_{y,y-1,q} = \sum_i \bar{p}_{i,y-1} q_{i,y,q}$.

The term

(4b) $\bar{V}_{y-1,y-1,q} = \sum_i \bar{p}_{i,y-1,q} q_{i,y-1,q}$

refers to special volumes needed for the OY method and also in the case of q = 4 for the QO method. Note that (4a) is equal to

(6) $\bar{V}_{0,0,q} = \sum_i \bar{p}_{i,0} q_{i,0,q} = W_{0,q} = \sum_i p_{i,0,q} q_{i,0,q}$

System of the formulas

prices	quarter-specific prices	average prices (unit values)
y (same year as quantities)	**(2)** $W_{yq} = V_{y,y,q}$	**(4b)** relevant in OQ* and OY**
y-1 (previous year)	**(3)** hardly rele-vant	**(5)** numerator of almost all links

* for q = 4
** for all quarters

Relevance of the difference between volume/value definitions (figures refer to y = 2006 and q = 4 in the numerical example)

(2) $W_{y,y,q=4} = \sum_i p_{iy,q=4} q_{i,y,q=4}$ =1010 (4) $\bar{V}_{y,y,q=4} = \sum_i \bar{p}_{iy} q_{i,y,q=4}$ = 937.74 and

(5) $\bar{V}_{y,y-1,q=4} = \sum_i \bar{p}_{i,y-1} q_{i,y,q=4}$ = 837.07 see slide **7.3.2 (2)** for the OY method

7.3. Indices for quarters and years (links and updated levels)

7.3.1 AO method (annual overlap, or annual linking)

a) Quarterly indices: links and levels

Quarterly ***link*** AO: **(7)** $L^{AO}_{(y-1)\to y,q} = \dfrac{\overline{V}_{y,y-1,q}}{W_{y-1}/4}$

Quarterly volume ***index*** AO: **(8)** $I^{AO}_{y,q} = I^{AO}_{y-1} L^{AO}_{(y-1)\to y,q}$

starting with **(8a)** $I^{AO}_0 = \frac{1}{4}\sum_q \dfrac{\overline{V}_{0,0,q}}{W_0/4} = 1$ (that is 100%) due to **(4a')**

Equation (eq for short) 8a that is $I^{AO}_0 = 1$ is also used for eq. 10

General principle for the AO quarterly link

A quarter q of year y at *average* prices of the preceding year y-1 $\boxed{\overline{V}_{y,y-1,q} = \sum_i \overline{p}_{i,y-1} q_{i,y,q}}$ is related to the unweighted average of the values (i.e. a fourth of the total value) of the preceding year y-1.

Note that all three methods a link is defined where the numerator is the same viz. $\boxed{\overline{V}_{y,y-1,q} = \sum_i \overline{p}_{i,y-1} q_{i,y,q}}$. The denominators, however, are different in eqs 7, 11, 15

b) Annual indices: links and levels

Annual ***link*** AO: **(9)** $L^{AO}_{(y-1)\to y} = \dfrac{\sum_q \overline{V}_{y,y-1,q}}{\sum_q W_{y-1.q}} = \dfrac{\frac{1}{4}\sum_q \overline{V}_{y,y-1,q}}{\frac{1}{4} W_{y-1}} = \dfrac{\sum_q L^{AO}_{(y-1)\to y,q}}{4}$

Equation 9 shows that aggregated QNA-volumes and direct ANA volumes are consistent (also known as "time consistency" in this context)

Annual volume ***index*** **(10)** $I^{AO}_y = I^{AO}_{y-1} L^{AO}_{(y-1)\to y}$ starting with $I^{AO}_0 = 1$.

The AO quarterly index ***in one formula*** $I^{AO}_{y,q} = \left(\prod_{t=1}^{y-1} \dfrac{\sum \overline{p}_{t-1} q_t}{\sum p_{t-1} q_{t-1}}\right) \dfrac{\sum \overline{p}_{y-1} q_{y,q}}{\sum p_{y-1} q_{y-1}/4}$.

To understand this formula it is useful to assume y = 4 and q= 2. Then we get

$$\left(\prod_{y=1}^{t-1} \frac{\sum_q \overline{V}_{y,y-1,q}}{\sum_q W_{y-1.q}}\right) \frac{\overline{V}_{t,t-1,q}}{W_{t-1.q}} = \left(\frac{\sum_q \sum_i \overline{p}_0 q_{1q}}{\sum_q \sum_i p_{0q} q_{0q}} \frac{\sum_q \sum_i \overline{p}_1 q_{2q}}{\sum_q \sum_i p_{1q} q_{1q}} \frac{\sum_q \sum_i \overline{p}_2 q_{3q}}{\sum_q \sum_i p_{2q} q_{2q}}\right) \frac{\sum_i \overline{p}_3 q_{4,q=2}}{\sum_q \sum_i p_{3q} q_{3q}/4}$$

The first three factors are linking successive years 0 → 3 (see eq. 9) and the fourth factor refers to the specific quarter (q = 2) of the fourth year (which corresponds to eq. 7). This is equivalent to (omitting subscript i for convenience of presentation)

$$\left(\prod_{y=1}^{t-1} \frac{\sum_q \overline{V}_{y,y-1,q}}{\sum_q W_{y-1.q}}\right) \frac{\overline{V}_{t,t-1,q}}{W_{t-1.q}} = \frac{\sum_q \overline{V}_{1,0,q}}{\sum_q W_{0,q}} \frac{\sum_q \overline{V}_{2,1,q}}{\sum_q W_{1,q}} \frac{\sum_q \overline{V}_{3,2,q}}{\sum_q W_{2,q}} \frac{\overline{V}_{4,3,q}}{W_{3,q}}$$. For y = 4 and q = 3 we get

$\left(\frac{\sum_q \sum_i \bar{p}_0 q_{1q}}{\sum_q \sum_i p_{0q} q_{0q}} \frac{\sum_q \sum_i \bar{p}_1 q_{2q}}{\sum_q \sum_i p_{1q} q_{1q}} \frac{\sum_q \sum_i \bar{p}_2 q_{3q}}{\sum_q \sum_i p_{2q} q_{2q}} \right) \frac{\sum_i \bar{p}_{3q} q_{4,q=3}}{\sum_i p_{3q} q_{3q}}$ so that the quarter-to-quarter comparison amounts to $\frac{I^{AO}_{y=4,q=3}}{I^{AO}_{y=4,q=2}} = \frac{\sum_i \bar{p}_{3q} q_{4,q=3}}{\sum_i \bar{p}_{3q} q_{4,q=2}}$, which is "unbiased" and the same as in the case of the QO method. Note the difference between

(3) $V_{y,y-1} = \sum_q V_{y,y-1,q} = \sum_q \sum_i p_{i,y-1,q} q_{iyq}$ and **(5)** $\bar{V}_{y,y-1} = \sum_q \bar{V}_{y,y-1,q} = \sum_q \sum_i \bar{p}_{i,y-1} q_{iyq}$ will

For example t = y = 2006 gives in the numerical example used in the presentation

Table 1

q	(5) $\sum_i \bar{p}_{i,y-1} q_{iyq}$	(3) $\sum_i p_{i,y-1,q} q_{iyq}$	(2) $W_{yq} = \sum_i p_{iyq} q_{iyq}$
	(1)	(2)	(3)
1	596.55	$40 \cdot 10 + 2 \cdot 50 = 500$	600
2	654.30	$45 \cdot 15 + 2 \cdot 65 = 805$	710
3	779.32	$50 \cdot 11 + 3 \cdot 70 = 760$	890
4	837.07	$55 \cdot 10 + 3 \cdot 75 = 775$	1010
Σ	2867.24	2840	3210
Σ/4	716.81	710	802.5

7.3.2 QO method (quarterly overlap [or one-quarter-overlap, or Q4-method])

a) Quarterly indices: links and levels

Quarterly QO volume index ***link***[1]: **(11)** $L^{QO}_{y-1,q=4\to y,q} = \frac{\bar{V}_{y,y-1,q}}{\bar{V}_{y-1,y-1,q=4}}$

In particular we have for y = 1 $L^{QO}_{0,q=4\to 1,q} = \frac{\bar{V}_{1,0,q}}{\bar{V}_{0,0,q=4}}$ and for q = 1

$L^{QO}_{y-1,q=4\to y,q=1} = \frac{\bar{V}_{y,y-1,q=1}}{\bar{V}_{y-1,y-1,q=4}}$ and $L^{QO}_{y,q=4\to y+1,q=1} = \frac{\bar{V}_{y+1,y,q=1}}{\bar{V}_{y,y,q=4}}$

(Other quarters q = 2, 3, analogously; [for q = 4, which then is the basis for a new link).

Quarterly QO volume ***index*** **(12)** $I^{QO}_{y,q=4} = I^{QO}_{y-1,q=4} L^{QO}_{y-1,q=4\to y,q=4}$ starting with

(12a) $I^{QO}_{0,q=4} = \frac{\bar{V}_{0,0,q=4}}{W_0/4}$

According to eq. 12 the quarterly index is formed by multiplying an index *for the fourth quarter* (rather than an annual index as in eq. 8) with a link $L_{y-1,q=4}$

Note that $\bar{V}_{y,y,q=4} \neq V_{y,y,q=4} = W_{y,q=4}$ and that $\bar{V}_{y,y,q=4}$ needs an additional calculation to be made (not necessary in the case of the AO method).

[1] Note that the denominator in eqs. 11, 11a and 11b is not the value $W_{y,q=4}$ of the fourth quarter.

Linking over the years y → y+1 → y+2 etc.

It is important to realize that a quarterly index for q = 4 is chain-linked, however the annual index is not (by contrast to the AO method, see eq. (9)). The year-on-year series for the fourth quarter is generated using

(11a) $L^{QO}_{y,q=4\to y+1,q=4} = \dfrac{\overline{V}_{y+1,y,q=4}}{\overline{V}_{y,y,q=4}}$ and **(12)** starting with $I^{QO}_{0,q=4} = \dfrac{\overline{V}_{0,0,q=4}}{W_0/4}$ (according to **12a**)

General principle for the QO quarterly link

A quarter q of year y at *average* prices of the year y-1 $\boxed{\overline{V}_{y,y-1,q} = \sum_i \overline{p}_{i,y-1} q_{i,y,q}}$ is related to <u>the volume of the forth quarter</u> of the previous year y-1 (that is q=4,y-1) at *average* prices of the previous year y-1

b) Annual indices: links and levels

Annual indices are not derived from a product like eq. **(12)** but from links relating a level referring to a whole year to an index referring to y-1 and q=4 rather than to the whole year y-1. Hence we get a sequence of annual indices, as opposed to a linked index:

The ***link*** generating a QO-type annual index is

(13) $L^{QO}_{y-1,q=4\to y} = \dfrac{\frac{1}{4}\sum_q \overline{V}_{y,y-1,q}}{\overline{V}_{y-1,y-1,q=4}} = \dfrac{\sum_q L^{OY}_{y-1,q=4\to y,q}}{4}$ and the sequence of index levels is

The sequence of *annual* QO volume ***indices*** as products of **(12)** and **(13)** is given by

(14a) $$I^{QO}_1 = I^{QO}_{0,q=4} L^{QO}_{0,q=4\to 1} = \frac{\overline{V}_{0,0,q=4}}{W_0/4} \frac{\frac{1}{4}\sum_q \overline{V}_{1,0,q}}{\overline{V}_{0,0,q=4}}$$

(14b) $$I^{QO}_2 = I^{QO}_{1,q=4} L^{QO}_{1,q=4\to 2} = \frac{\overline{V}_{0,0,q=4}}{W_0/4} \frac{\overline{V}_{1,0,q=4}}{\overline{V}_{0,0,q=4}} \frac{\frac{1}{4}\sum_q \overline{V}_{2,1,q}}{\overline{V}_{1,1,q=4}}$$

(14c) $$I^{QO}_3 = I^{QO}_{2,q=4} L^{QO}_{2,q=4\to 3} = \frac{\overline{V}_{0,0,q=4}}{W_0/4} \frac{\overline{V}_{1,0,q=4}}{\overline{V}_{0,0,q=4}} \frac{\overline{V}_{2,1,q=4}}{\overline{V}_{1,1,q=4}} \frac{\frac{1}{4}\sum_q \overline{V}_{3,2,q}}{\overline{V}_{2,2,q=4}}$$ etc.

Although according to eq. 13 the annual link $L^{QO}_{y-1,q=4\to y} = \dfrac{\sum_q L^{OY}_{y-1,q=4\to y,q}}{4}$ is defined as the arithmetic mean of quarterly links the QO does not deliver time consistent indices (in contrast to the AO method). It will be demonstrated below that eq. 13 and eq. 9 may look similar, but they are in actual fact quite different.

The QO index is also given ***in one formula*** by

$$I^{QO}_{y,q} = \frac{\sum \overline{p}_0 q_{0;4}}{W_0/4} \cdot \left(\prod_{t=1}^{y-1} \frac{\sum \overline{p}_{t-1} q_{t;4}}{\sum \overline{p}_{t-1} q_{t-1;4}} \right) \cdot \frac{\sum \overline{p}_{y-1} q_{y,q}}{\sum \overline{p}_{y-1} q_{y-1;4}}$$

Assume for example y = 4 (= 2009) and q = 2

$$I^{QO}_{4,2} = \frac{557.84}{473.75} \left(\frac{837.07}{557.84} \cdot \frac{1109.37}{937.74} \cdot \frac{2100.90}{1981.57} \right) \frac{3546.16}{3176.31}$$

Table 2*: 100 times the product of the fist 2, 3, … factors of $I^{QO}_{y,q}$

2	3	4	5
$I^{QO}_{06,4}$ = 176.69	$I^{QO}_{07,4}$ = 209.029	$I^{QO}_{08,4}$ = 221.617	$I^{QO}_{09,2}$ = 247.422

* to be compared with the ppt-presentation in part 7.3

The numerical example may also serve for a demonstration of the ***time inconsistency of the QO Method***: Consider now the sum of the in combination with the absolute volumes.

As for quarterly indices holds $I^{QO}_{y,q} = I^{QO}_{y-1,q=4} L^{QO}_{y-1,q=4\to y,q} = I^{QO}_{y-1,q=4} \cdot \frac{\overline{V}_{y,y-1,q}}{\overline{V}_{y-1,y-1,q=4}}$, taking the sum over the four quarters gives

$$\sum_q I^{QO}_{y,q} = I^{QO}_{y-1,q=4} \sum_q L^{QO}_{y-1,q=4\to y,q} = I^{QO}_{y-1,q=4} \cdot \left(\frac{\overline{V}_{y,y-1,1}}{\overline{V}_{y-1,y-1,q=4}} + \ldots + \frac{\overline{V}_{y,y-1,4}}{\overline{V}_{y-1,y-1,q=4}} \right)$$ and therefore

(13a) $$\tfrac{1}{4}\sum_q I^{QO}_{y,q} = I^{QO}_{y-1,q=4} \cdot \frac{\frac{1}{4}\sum_q \overline{V}_{y,y-1,q}}{\overline{V}_{y-1,y-1,q=4}} = I^{QO}_{y-1,q=4} \cdot L^{OQ}_{y-1,q=4\to y} = I^{QO}_{y}$$

This eq. (see also eqs. 13 and 14) might give rise to the misunderstanding that QO volumes might satify time consistency. However, in contrast to eqs 9 and 10 where $\frac{I^{AO}_{y}}{I^{AO}_{y-1}} = L^{AO}_{(y-1)\to y}$ $= \frac{1}{4}\sum_q L^{AO}_{(y-1)\to y,q}$ here annual links are not related to an index level I_{y-1} of the year y-1 but rather to the fourth quarter of that year, that is $I_{y-1,q=4}$.

Table 3 shows in detail how time consistency applies to growth factors $I^{QO}_{y} / I^{QO}_{y-1,q=4}$ but not to $I^{QO}_{y} / I^{QO}_{y-1}$. According to **(13a)** I^{QO}_{y} is an unweighted arithmetic mean of the four indices $I^{QO}_{y,q}$. However, for $I^{QO}_{y} / I^{QO}_{y-1}$ we get a ***weighted*** sum

(13b) $$\frac{I^{QO}_{y}}{I^{QO}_{y-1}} = \frac{I^{QO}_{y,q=1}}{I^{QO}_{y-1,q=1}} \left(\frac{I^{QO}_{y-1,q=1}}{4 \cdot I^{QO}_{y-1}} \right) + \ldots + \frac{I^{QO}_{y,q=4}}{I^{QO}_{y-1,q=4}} \left(\frac{I^{QO}_{y-1,q=4}}{4 \cdot I^{QO}_{y-1}} \right)$$

The weights (terms in brackets) add up to unity.

Table 3: Annual indices derived from QO linking are not time consistent

growth factor	q = 1 (1)	q = 2 (2)	q = 3 (3)	q = 4 (4)	whole year	average of (1) to (4)
(year 1)/ (year 0, quarter 4)	$\frac{125.92}{117.75}$	$\frac{138.11}{117.75}$	$\frac{164.50}{117.75}$	$\frac{176.69}{117.75}$	$\frac{151.30}{117.75}$	$\frac{151.30}{117.75}$
(year 1)/ (year 0)	$\frac{125.92}{77.16} =$ 1.6319	$\frac{138.11}{89.35} =$ 1.5548	$\frac{164.50}{115.74} =$ 1.4213	$\frac{176.69}{117.75} =$ 1.5031	1.5278	$\frac{151.30}{100} =$ 1.5130
(year 2)/ (year 1, quarter 4)	$\frac{203.85}{176.69}$	$\frac{191.55}{176.69}$	$\frac{197.13}{176.69}$	$\frac{209.03}{176.69}$	$\frac{200.49}{176.69}$	$\frac{200.49}{176.69}$
(year 2)/ (year 1)	$\frac{203.85}{125.92} =$ 1.6189	$\frac{191.55}{138.11} =$ 1.3869	$\frac{197.13}{164.50} =$ 1.1984	$\frac{209.03}{176.69} =$ 1.1830	1.3480	$\frac{200.49}{151.30} =$ 1.3245

Eq. 13b reads in this example (last row) as follows

$\frac{200.49}{151.30} = \frac{203.85}{125.92}\left(\frac{125.95}{4 \cdot 151.30}\right) + \ldots + \frac{209.03}{176.69}\left(\frac{176.69}{4 \cdot 151.30}\right)$, that is 1.3245 is a weighted average of the quarterly growth factors 1.6189 etc with the following weights (the terms in brackets) 0.2081, 0.2282, 0.2718, and 0.2920.

7.3.3 OY method (over the year)

a) Quarterly indices: links and levels

Quarterly ***link*** OY (15) $L^{OY}_{y-1,q\to y,q} = \frac{\overline{V}_{y,y-1,q}}{\overline{V}_{y-1,y-1,q}}$

Quarterly volume ***index*** (16) $I^{OY}_{y-1,q\to y,q} = I^{OY}_{y-1,q} L^{OY}_{y-1,q\to y,q}$ starting with $I^{OY}_{0,q} = \frac{\overline{V}_{0,0,q}}{W_0/4}$

Note $I^{OY}_{0,q=4} = I^{QO}_{0,q=4} = \frac{\overline{V}_{0,0,q=4}}{W_0/4}$

General principle for the OY quarterly link

A quarter q of year y at *average* prices of year y-1 $\boxed{\overline{V}_{y,y-1,q} = \sum_i \overline{p}_{i,y-1} q_{i,y,q}}$ is related <u>to the same quarter of the previous year</u> y-1 (that is q,y-1) at *average* prices of that year y-1 (that is quantities of the preceding year and average prices of the preceding year y-1)

b) Annual indices: links and levels

Annual ***link*** OY (17) $L^{OY}_{(y-1)\to y} = \frac{\sum_q \overline{V}_{y,y-1,q}}{\sum_q \overline{V}_{y-1,y-1,q}} = \frac{\sum_q L^{OY}_{(y-1),q\to y,q} \overline{V}_{y-1,y-1,q}}{\sum_q \overline{V}_{y-1,y-1,q}} \neq \frac{\sum_q L^{OY}_{(y-1),q\to y,q}}{4}$

Annual volume ***index*** (18) $I^{OY}_{(y-1,)\to y} = I^{OY}_{y-1} L^{OY}_{(y-1)\to y}$

and $I^{OY}_{y} \neq \frac{1}{4}\sum_q I^{OY}_{y,q} = \frac{1}{4}\sum_q I^{OY}_{y-1,q}\left(\overline{V}_{y,y-1,q} / \overline{V}_{y-1,y-1,q}\right)$

The OY Index for year y and quarter q ***in one formula***

(*) $I^{OY}_{y,q} = \frac{\sum p_0 q_{1;q}}{\sum W_{0q}/4}\left(\prod_{t=1}^{y} \frac{\sum \overline{p}_{t-1} q_{t;q}}{\sum \overline{p}_{t-1} q_{t-1;q}}\right)$ or $I^{OY}_{y,q} = \frac{\sum p_0 q_{1,q}}{W_0/4} \prod_{t=1}^{y} \frac{\overline{V}_{t,t-1,q}}{\overline{V}_{t-1,t-1,q}}$

which is - due to eq. (6) - equivalent to

(**) $I^{OY}_{y,q} = \frac{\sum \overline{p}_0 q_{0;j}}{W_0/4} \prod_{t=2}^{y} \frac{\sum \overline{p}_{t-1} q_{t,q}}{\sum \overline{p}_{t-1} q_{t-1,q}}$

To demonstrate the kind of calculation assume again y = 4 and q = 2. Then formula (*) gives using the figures of our numerical example

$$\frac{423.30}{473.75} \cdot \left(\frac{654.30}{423.30} \cdot \frac{1018.74}{730.42} \cdot \frac{2220.22}{1695.93} \cdot \frac{3546.16}{3361.29}\right) = 2.6605$$

(starting with $\frac{\sum p_0 q_{0;j}}{W_0/4} = \frac{423.30}{473.75}$ and followed by $\frac{\overline{V}_{1,0,q=2}}{\overline{V}_{0,0,q=2}} = \frac{\sum \overline{p}_0 q_{1,q=2}}{\sum \overline{p}_0 q_{0,q=2}} = \frac{654.30}{423.30}$).

For the second formula (**) we get $\frac{\overline{V}_{1,0,q=2}}{W_0/4}\left(\frac{\overline{V}_{2,1,q=2}}{\overline{V}_{1,1,q=2}}\frac{\overline{V}_{3,2,q=2}}{\overline{V}_{2,2,q=2}}\frac{\overline{V}_{4,3,q=2}}{\overline{V}_{3,3,q=2}}\right)$ which means for the example $\frac{654.30}{473.75}\left(\frac{1018.74}{730.42}\frac{2220.22}{1695.93}\frac{3546.16}{3361.29}\right)$.

Table 4: 100 times the product of the fist 1, 2, … factors of formula (*)

1	2	3	4	5
$I^{OY}_{0,2}$ = 89.35	$I^{OY}_{1,2}$ = 138.11	$I^{OY}_{2,2}$ = 192.63	$I^{OY}_{3,2}$ = 252.18	$I^{OY}_{4,2}$ = 266.05

* these are precisely the figures of the ppt-presentation in part **7.3.3**

7.5 Time series and comparisons chain-linked indices

7.5.1 AO method

a) Sequence of annual indices (years 1, 2, …, y)

$$I^{AO}_y = L^{AO}_{0\to1}L^{AO}_{1\to2}L^{AO}_{2\to3}\ldots L^{AO}_{(y-1)\to y} = 1\cdot\frac{\sum\sum\overline{p}_{i0}q_{i1q}}{\sum\sum p_{i0q}q_{i0q}}\cdot\frac{\sum\sum\overline{p}_{i1}q_{i2q}}{\sum\sum p_{i1q}q_{i1q}}\cdot\ldots\cdot\frac{\sum\sum\overline{p}_{i,t-1}q_{i,t,q}}{\sum\sum p_{,t-1,q}q_{i,t-1,q}}$$

(19) $$I^{AO}_y == \frac{\sum\sum\overline{p}_{i0}q_{i1q}}{W_0}\cdot\frac{\sum\sum\overline{p}_{i1}q_{i2q}}{W_1}\cdot\ldots\cdot\frac{\sum\sum\overline{p}_{i,y-1}q_{i,y,q}}{W_{t-1}}$$

or simply $I^{AO}_y = \frac{\overline{V}_{1,0}}{W_0}\cdot\frac{\overline{V}_{2,1}}{W_1}\cdot\ldots\cdot\frac{\overline{V}_{y,y-1}}{W_{y-1}}$

b) Sequence of quarterly indices in year y

$$I^{AO}_{y,q=1} = I^{AO}_{y-1}\frac{\overline{V}_{y,y-1.q=1}}{W_{y-1}/4} = I^{AO}_{y-1}\frac{\sum_i\overline{p}_{i,y-1}q_{i,y,q=1}}{W_{y-1}/4}$$

(20) $I^{AO}_{y,q=1} = I^{AO}_{y-1}\frac{\sum_i\overline{p}_{i,y-1}q_{i,y,q=1}}{W_{y-1}/4}$ analogously $I^{AO}_{y,q=2} = I^{AO}_{y-1}\frac{\sum_i\overline{p}_{i,y-1}q_{i,y,q=2}}{W_{y-1}/4}$ etc.

c) Interpretation of the AO comparisons

<u>D1:</u> y,q and y,q+1 or <u>y,q and y,q-1</u>: successive quarters of the same year only differ from one another with respect to quantities. Using eqs. 5 and 8 we get

(21) $$\frac{I^{AO}_{y,q}}{I^{AO}_{y,q-1}} = \frac{\overline{V}_{y,y-1,q}}{\overline{V}_{y,y-1,q-1}} = \frac{\sum_i\overline{p}_{i,y-1}q_{i,y,q}}{\sum_i\overline{p}_{i,y-1}q_{i,y,q-1}}$$

<u>D2:</u> same quarter, different years, e.g. quarter q in y+1 and q in y

$$I^{AO}_{y+1,q} = I^{AO}_{y-1}\frac{\sum_q\sum_i\overline{p}_{i,y-1}q_{iyq}}{W_{y-1}}\cdot\frac{\sum_i\overline{p}_{i,y}q_{i,y+1,q}}{W_y/4} \text{ and } I^{AO}_{y,q} = I^{AO}_{y-1}\frac{\sum_i\overline{p}_{i,y-1}q_{i,y,q}}{W_{y-1}/4}$$

$$\frac{I^{AO}_{y+1,q}}{I^{AO}_{y,q}} = \frac{\sum_q \sum_i \bar{p}_{i,y-1} q_{iyq} \sum_i \bar{p}_{iy} q_{i,y+1,q}}{W_y \sum_i \bar{p}_{i,y-1} q_{i,y,q}} = \frac{\bar{V}_{y,y-1}}{W_y} \cdot \frac{\bar{V}_{y+1.y,q}}{\bar{V}_{y,y-1,q}} = A \cdot B$$

Or analogously q in y and q in y-1

(22) $$\frac{I^{AO}_{y,q}}{I^{AO}_{y-1,q}} = \frac{\bar{V}_{y.y-1,q}}{\bar{V}_{y-1,y-2,q}} \cdot \frac{\sum_q \bar{V}_{y-1,y-2,q}}{W_{y-1}} = Q^{y,q}_{y-1,q} \cdot A^{y-1,y-2}_{y-1,y-1}$$

The first factor Q (=quarterly) is a quarter specific ratio in which both, (average) prices <u>and</u> quantities are different in numerator and denominator. A (=annual) is a relation between two years (volume [at average prices in y-2] and value in y-1); there only prices are different.

<u>D3:</u> Fourth quarter in y and first quarter in y+1 (y,4 and (y+1),1)

$$I^{AO}_{y,q=4} = I^{AO}_{y-1} \frac{\sum_i \bar{p}_{i,y-1} q_{i,y,q=4}}{W_{y-1}/4} = I^{AO}_{y-1} \frac{\bar{V}_{y,y-1,q=4}}{W_{y-1}/4}$$

$$I^{AO}_{y+1,q=1} = I^{AO}_{y-1} L^{AO}_{(y-1)\to y} \frac{\sum_i \bar{p}_{i,y} q_{i,y+1,q=1}}{W_y/4} = I^{AO}_{y-1} \frac{\sum_q \bar{V}_{y,y-1,q}}{W_{y-1}} \cdot \frac{\bar{V}_{y+1,y,q=1}}{W_y/4}$$

(23) $$\frac{I^{AO}_{y+1,q=1}}{I^{AO}_{y,q=4}} = \frac{\bar{V}_{y+1,y,q=1}}{\bar{V}_{y,y-1,q=4}} \cdot \frac{\sum_q \bar{V}_{y,y-1,q}}{W_y} = Q^{y+1,1}_{y,4} \cdot A^{y,y-1}_{y,y}$$

Eq. 23 can also be interpreted as follows

$$\frac{I^{AO}_{y+1,q=1}}{I^{AO}_{y,q=4}} = \frac{\bar{V}_{y+1,y,q=1}}{\bar{V}_{y,y-1,q=4}} \cdot \frac{\sum_q \bar{V}_{y,y-1,q}}{W_y} = Q^{y+1,1}_{y,4} \div A^{y,y}_{y,y-1}$$

where A may be seen as a kind of price correction (comparing average prices of y and y-1 on the basis of y-quantities). A pure quantity comparison between y+1,q=1 and y,q=4 would be

(23a) $D = \dfrac{\sum_i \bar{p}_{iy} q_{i,y+1,q=1}}{\sum_i \bar{p}_{iy} q_{i,y,q=4}}$ instead of $Q^{y+1,1}_{y,4}$

The "contamination" of the comparison in (23) may be viewed as

(23b) $$\text{cont} = \frac{I^{AO}_{y+1,q=1} / I^{AO}_{y,q=4}}{D} = \frac{\sum_i \bar{p}_{iy} q_{i,y,q=4} / \sum_i \bar{p}_{i,y-1} q_{i,y,q=4}}{\sum_q \sum_i \bar{p}_{iy} q_{i,y,q} / \sum_q \sum_i \bar{p}_{i,y-1} q_{i,y,q}}$$

cont is a relation between two Paasche price indices comparing average prices of y-1 and y, where the structure of the quantities apply to the fourth quarter in the numerator index as opposed to the whole year in the denominator index (see Kirchner)[2]. If the structure of quantities in the fourth quarter of y were not different from structure in the year as a whole cont = 1.

Decomposition of AO volume growth rate

Assume two components, A and B of an aggregate

$$g^{AO}_{y,q+1} = \frac{I^{AO}_{y,q+1} - I^{AO}_{y,q}}{I^{AO}_{y,q}} = \frac{\bar{V}_{y,y-1,q+1} - \bar{V}_{y,y-1,q}}{\bar{V}_{y,y-1,q}} = \frac{\sum_i \bar{p}_{i,y} q_{i,y,q+1} - \sum_i \bar{p}_{i,y} q_{i,y,q}}{\sum_i \bar{p}_{i,y} q_{i,y,q}}$$

[2] Robert Kirchner, Konsequenzen aus der Umstellung der realen Angaben der vierteljährlichen Volkswirtschaftlichen Gesamtrechnung auf Kettenindizes für die aktuelle Analyse der wirtschaftlichen Entwicklung in Deutschland, Vortrag in Rostock 10.6.2005

$$= w_{Ayq}\left(\frac{q_{A,y,q+1}-q_{A,y,q}}{q_{A,y,q}}\right)+w_{Byq}\left(\frac{q_{B,y,q+1}-q_{B,y,q}}{q_{B,y,q}}\right)$$

where $w_{Ayq}=\dfrac{\overline{p}_{A,y-1}q_{A,y,q}}{\sum_{i\subset(A,B)}\overline{p}_{i,y-1}q_{i,y,q}}$. $w_{Byq}=1-w_{Ayq}$.

7.5.2 QO method

a) Sequence of annual indices (years 1, 2, ..., y)

(24) $$I_y^{QO}=\frac{\sum_i\overline{p}_{i0}q_{i,0,q=4}}{\frac{1}{4}\sum_q\sum_i\overline{p}_{i0}q_{i,0,q}}\cdot\frac{\frac{1}{4}\sum_q\sum_i\overline{p}_{i0}q_{i,1,q}}{\sum_i\overline{p}_{i0}q_{i,0,q=4}}\cdot\frac{\frac{1}{4}\sum_q\sum_i\overline{p}_{i1}q_{i,2,q}}{\sum_i\overline{p}_{i1}q_{i,1,q=4}}\cdot\ldots\cdot\frac{\frac{1}{4}\sum_q\sum_i\overline{p}_{i,y-1}q_{i,y,q}}{\sum_i\overline{p}_{i,y-1}q_{i,y-1,q=4}}$$

the first two factors represents I_1^{QO}, the first three I_2^{QO}. Obviously the first two factors give $\dfrac{\sum_q\sum_i\overline{p}_{i0}q_{i,1,q}}{\sum_q\sum_i\overline{p}_{i0}q_{i,0,q}}$ so that $I_y^{QO}=\dfrac{\sum_q\sum_i\overline{p}_{i0}q_{i,1,q}}{\sum_q\sum_i\overline{p}_{i0}q_{i,0,q}}\cdot\dfrac{\frac{1}{4}\sum_q\sum_i\overline{p}_{i1}q_{i,2,q}}{\sum_i\overline{p}_{i1}q_{i,1,q=4}}\cdot\ldots\cdot\dfrac{\frac{1}{4}\sum_q\sum_i\overline{p}_{i,y-1}q_{i,y,q}}{\sum_i\overline{p}_{i,y-1}q_{i,y-1,q=4}}$ or

equivalently $I_y^{QO}=\dfrac{\sum_q\overline{V}_{1,0,q}}{\sum_q\overline{V}_{0,0,1}}\cdot\dfrac{\frac{1}{4}\sum_q\overline{V}_{2,1,q}}{\overline{V}_{1,1,q=4}}\cdot\ldots\cdot\dfrac{\frac{1}{4}\sum_q\overline{V}_{y,y-1,q}}{\overline{V}_{y-1,y-1,q=4}}$

b) Sequence of quarterly indices in year y

(25) $$I_{y,q=1}^{QO}=I_{y-1,q=4}^{QO}\cdot\frac{\sum_i\overline{p}_{i,y-1}q_{i,y,q=1}}{\sum_i\overline{p}_{i,y-1}q_{i,y-1,q=4}},\quad I_{y,q=2}^{QO}=I_{y-1,q=4}^{QO}\cdot\frac{\sum_i\overline{p}_{i,y-1}q_{i,y,q=2}}{\sum_i\overline{p}_{i,y-1}q_{i,y-1,q=4}}\ \text{etc.}$$

c) Interpretation of QO Comparisons

D1: y,q and y,q-1 successive quarters of the same year

(26) $\dfrac{I_{y,q}^{QO}}{I_{y,q-1}^{QO}}=\dfrac{\sum_i\overline{p}_{i,y-1}q_{i,y,q}}{\sum_i\overline{p}_{i,y-1}q_{i,y,q-1}}$ the same formula as eq. **21**

D2: same quarter, different years, e.g. quarter q in y+1 and q in y

(27) $\dfrac{I_{y,q}^{QO}}{I_{y-1,q}^{QO}}=\dfrac{\overline{V}_{y,y-1,q}}{\overline{V}_{y-1,y-2,q}}\cdot\dfrac{\overline{V}_{y-1,y-2,q=4}}{\overline{V}_{y-1,y-1,q=4}}=Q_{y-1,q}^{y,q}\Big/Q_{y-1,y-2,4}^{(*)y-1,y-1,4}$ as opposed to

(22) $\dfrac{I_{y,q}^{AO}}{I_{y-1,q}^{AO}}=\dfrac{\overline{V}_{y.y-1,q}}{\overline{V}_{y-1,y-2,q}}\div\dfrac{W_{y-1}}{\sum_q\overline{V}_{y-1,y-2,q}}\cdot=Q_{y-1,q}^{y,q}\Big/A_{y-1,y-2}^{y-1,y-1}$.

A refers to a whole year (y-1) in numerator and denominator, while A* refers to one quarter (q=4) only, the second factor B is the same

D3: Fourth quarter in y and first quarter in y+1 (y,4 and (y+1),1)

(28) due to (11a) $\dfrac{I_{y+1,q=1}^{QO}}{I_{y,q=4}^{QO}}=L_{y,q=4\to y+1,q=1}^{QO}=\dfrac{\overline{V}_{y+1,y,q=1}}{\overline{V}_{y,y,q=4}}=\dfrac{\sum_i\overline{p}_{iy}q_{i,y+1,q=1}}{\sum_i\overline{p}_{iy}q_{i,y,q=4}}$

Note that this is exactly equal to D according eq. 23a $D = \frac{\sum_i \bar{p}_{iy} q_{i,y+1,q=1}}{\sum_i \bar{p}_{iy} q_{i,y,q=4}}$ so that a comparison between these quarters (y+1,q=1, and y,q=4) is a pure comparison and that (28) differs from (23) $\frac{I^{AO}_{y+1,q=1}}{I^{AO}_{y,q=4}} = \frac{\bar{V}_{y+1,y,q=1}}{\bar{V}_{y,y-1,q=4}} \cdot \frac{\sum_q \bar{V}_{y,y-1,q}}{W_y} = Q^{y+1,1}_{y,4} \cdot A$

as follows: the numerator in (28) is the same as in the Q-factor of (23). However in the denominator of Q in (23) prices refer to y-1. In (28) prices refer to y in both, numerator and denominator.

7.5.3 OY method

a), b) Sequence of quarterly indices over the years (years 1, 2, …, y)

annual indices are simply averages $\frac{1}{4}\sum_q I^{OY}_{yq}$

$$I^{OY}_{yq} = \frac{\sum_i \bar{p}_{i0} q_{i,0,q}}{\frac{1}{4}\sum_q \sum_i \bar{p}_{i0} q_{i,0,q}} \cdot \frac{\sum_i \bar{p}_{i0} q_{i,1,q}}{\sum_i \bar{p}_{i0} q_{i,0,q}} \cdot \frac{\sum_i \bar{p}_{i1} q_{i,2,q}}{\sum_i \bar{p}_{i1} q_{i,1,q=4}} \cdot \ldots \cdot \frac{\sum_i \bar{p}_{i,y-1} q_{i,y,q}}{\sum_i \bar{p}_{i,y-1} q_{i,y-1,q}}$$

the first two factors give $I^{OY}_{1,q}$, the first three $I^{OY}_{2,q}$ etc. Obviously this is equal to

(29) $I^{OY}_{y,q} = \frac{\bar{V}_{1,0,q}}{\frac{1}{4} W_0} \cdot \frac{\bar{V}_{2,1,q}}{\bar{V}_{1,1,q}} \cdot \frac{\bar{V}_{3,2,q}}{\bar{V}_{2,2,q}} \cdot \ldots \cdot \frac{\bar{V}_{y,y-1,q}}{\bar{V}_{y-1,y-1,q}}$. Taking an average over all four quarters will differ from (24) $I^{QO}_y = \frac{\sum_q \bar{V}_{1,0,q}}{\sum_q \bar{V}_{0,0,1}} \cdot \frac{\frac{1}{4}\sum_q \bar{V}_{2,1,q}}{\bar{V}_{1,1,q=4}} \cdot \ldots \cdot \frac{\frac{1}{4}\sum_q \bar{V}_{y,y-1,q}}{\bar{V}_{y-1,y-1,q=4}}$

Sequence of an OY-index for quarter q = 1 over the years y-1 → y

(30) $I^{OY}_{y,q=1} = I^{OY}_{y-1,q=1} \cdot \frac{\sum_i \bar{p}_{i,y-1} q_{i,y,q=1}}{\sum_i \bar{p}_{i,y-1} q_{i,y-1,q=1}} = I^{OY}_{y-1,q=1} \cdot \frac{\bar{V}_{y,y-1,q=1}}{\bar{V}_{y-1,y-1,q=1}} = I^{OY}_{y-1,q=1} \cdot L^{OY}_{y-1,q=1\to y,q=1}$,

correspondingly $I^{OY}_{y,q=2} = I^{OY}_{y-1,q=2} \cdot \frac{\sum_i \bar{p}_{i,y-1} q_{i,y,q=2}}{\sum_i \bar{p}_{i,y-1} q_{i,y-1,q=2}} = I^{OY}_{y-1,q=2} \cdot L_{y,2} =$

c) Interpretation of OY comparisons

<u>D1: y,q and y,q-1</u> successive quarters of the same year

(31) $\frac{I^{OY}_{y,q=2}}{I^{OY}_{y,q=1}} = \frac{I^{OY}_{y-1,q=2} L_{y,2}}{I^{OY}_{y-1,q=1} L_{y,1}}$ the same formula as eq. **21**

<u>D2:</u> same quarter, different years, e.g. quarter q in y+1 and q in y

(32) $\frac{I^{OY}_{y,q}}{I^{OY}_{y-1,q}} = \frac{\bar{V}_{y,y-1,q}}{\bar{V}_{y-1,y-1,q}} = L^{OY}_{y-1,q\to y,q}$ as opposed to (27) $\frac{I^{QO}_{y,q}}{I^{QO}_{y-1,q}} = \frac{\bar{V}_{y,y-1,q}}{\bar{V}_{y-1,y-2,q}} \cdot \frac{\bar{V}_{y-1,y-2,q=4}}{\bar{V}_{y-1,y-1,q=4}}$

<u>D3:</u> Fourth quarter in y and first quarter in y+1 (y,4 and (y+1),1)

(33) $\frac{I^{OY}_{y+1,q=1}}{I^{OY}_{y,q=4}} = \frac{I^{OY}_{y,,q=1}}{I^{OY}_{y,q=4}} L^{OY}_{y,q=1\to y+1,q=1} = \frac{I^{OY}_{y,,q=1}}{I^{OY}_{y,q=4}} \cdot \frac{\bar{V}_{y+1,y,q=1}}{\bar{V}_{y,y,q=1}}$ as opposed to (28) $\frac{I^{QO}_{y+1,q=1}}{I^{QO}_{y,q=4}} = \frac{\bar{V}_{y+1,y,q=1}}{\bar{V}_{y,y,q=4}}$

Appendix

A) Chain indices: Path dependence and oscillating ("bouncing") prices

Text (taken from von der Lippe (2001) sec. 3.4) **referring to part I, sec. 3.5.2 (slides 88 -92)**

"Consider the following situation with periodically recurring amounts of expenditure, called A through D for short:

	q even	q uneven
p even	A = $\sum p_0q_0 = \sum p_2q_0 = \sum p_4q_0 = \ldots$ $\sum p_0q_2 = \sum p_2q_2 = \sum p_4q_2 = \ldots$	B = $\sum p_0q_1 = \sum p_2q_1 = \sum p_4q_1 = \ldots$ $\sum p_0q_3 = \sum p_2q_3 = \sum p_4q_3 = \ldots$
p uneven	C = $\sum p_1q_0 = \sum p_3q_0 = \sum p_5q_0 = \ldots$ $\sum p_1q_2 = \sum p_3q_2 = \sum p_5q_2 = \ldots$	D = $\sum p_1q_1 = \sum p_3q_1 = \sum p_5q_1 = \ldots$ $\sum p_1q_3 = \sum p_3q_3 = \sum p_5q_3 = \ldots$

With $\alpha = \frac{C}{A}$ and $\beta = \frac{D}{B}$ the structure of the matrix **G** (defined in eq. 3.3.8) now is

$$\begin{bmatrix} g_1^0 & g_2^0 & g_3^0 \\ g_1^1 & g_2^1 & g_3^1 \\ g_1^2 & g_2^2 & g_3^2 \\ g_1^3 & g_2^3 & g_3^3 \end{bmatrix} = \begin{bmatrix} \alpha & \alpha^{-1} & \alpha \\ \beta & \beta^{-1} & \beta \\ \alpha & \alpha^{-1} & \alpha \\ \beta & \beta^{-1} & \beta \end{bmatrix}, \ldots.$$

The covariance $\mathrm{Cov}\left(x_{t-1,t}, y_{0,t-1}\right)$ responsible for the change of the drift $D_{0,t-1}^{PL} \rightarrow D_{0t}^{PL}$ can take only two values $\delta = \frac{BC - AD}{C^2} = \frac{B}{C}\left(1 - \frac{\beta}{\alpha}\right)$ and 0 indicating that the drift $D_{0t}^{PL} = \frac{\alpha}{\beta} D_{0,t-1}^{PL}$, or the drift does not change $D_{0t}^{PL} = D_{0,t-1}^{PL}$ respectively. On the basis of this matrix G we get the following results for four indices, two drifts, the covariance (cov), and the LPG (Laspeyres - Paasche gap) of the chain indices[3,4] γ_{0t}^{C} :

t	P_{0t}^{L}	P_{0t}^{P}	$\bar{P}_{0t}^{L}$	D_{0t}^{PL}	cov	$\bar{P}_{0t}^{P}$	D_{0t}^{PP}	$\bar{\gamma}_{0t}$
1	α	β	α	1	δ	β	1	$\alpha-\beta$
2	1	1	(α/β)	α/β	0	β/α	β/α	$(\alpha^2-\beta^2)/\alpha\beta$
3	α	β	$\alpha(\alpha/\beta)$	α/β	δ	$\beta(\beta/\alpha)$	β/α	$(\alpha^3-\beta^3)/\alpha\beta$
4	1	1	$(\alpha/\beta)^2$	$(\alpha/\beta)^2$	0	$(\beta/\alpha)^2$	$(\beta/\alpha)^2$	$(\alpha^4-\beta^4)/(\alpha\beta)^2$
5	α	β	$\alpha(\alpha/\beta)^2$	$(\alpha/\beta)^2$	δ	$\beta(\beta/\alpha)^2$	$(\beta/\alpha)^2$	$(\alpha^5-\beta^5)/(\alpha\beta)^2$
6	1	1	$(\alpha/\beta)^3$	$(\alpha/\beta)^3$	0	$(\beta/\alpha)^3$	$(\beta/\alpha)^3$	$(\alpha^6-\beta^6)/(\alpha\beta)^3$
7	α	β	$\alpha(\alpha/\beta)^3$	$(\alpha/\beta)^3$	δ	$\beta(\beta/\alpha)^3$	$(\beta/\alpha)^3$	$(\alpha^7-\beta^7)/(\alpha\beta)^3$
8	1	1	$(\alpha/\beta)^4$	$(\alpha/\beta)^4$	0	$(\beta/\alpha)^4$	$(\beta/\alpha)^4$	$(\alpha^8-\beta^8)/(\alpha\beta)^4$

[3] The LPG of the *direct* indices is oscillating, taking values $\alpha-\beta$, 0 and so on.
[4] Notation adapted to the ppt-presentation.

Interestingly … there is no drift of the Fisher index, that is DP_{0t}^{F} = 1 (or equivalently $\overline{P}_{0t}^{FC} = P_{0t}^{F}$) and both indices take two alternating values only, $\sqrt{\alpha\beta}$ and 1. Though there definitely exists a LPG the index $\overline{P}_{0t}^{FC}$ seems to enjoy an advantage over both other chain indices, $\overline{P}_{0t}^{LC}$ and $\overline{P}_{0t}^{PC}$, in that some kind of "counteracting" of the drifts D_{0t}^{PL} and D_{0t}^{PP} (or more distinct: an inverse relation between them, irrespective of the sign of the covariance δ, as can easily be seen by comparing the two relevant columns) prevents the drift D_{0t}^{PF} from existing. SNA does not seem to have taken this possibility into account, because SNA *in general* recommends *not* applying chaining (with whichever type of links) when prices are moving cyclically."

We demonstrate two examples

1) positive covariance	2) negative covariance
A = 80, B = 100, C = 120, D = 90	A = 100, B = 80, C = 90, D = 120
$\alpha_1 = 1.5$, $\beta_1 = 0.9$	$\alpha_2 = 0.9$, $\beta_2 = 1.5$
$\delta = (100/120)*(1-0.6) = +1/3$	$\delta = (80/90)*[1-(1/0.6)] = -0.592595$

Obviously in case 2 the roles of Laspeyres and Paasche are simply interchanged: what applies to P_{0t}^{L}, $\overline{P}_{0t}^{L}$, and D_{0t}^{PL} now applies to P_{0t}^{P}, $\overline{P}_{0t}^{P}$, and D_{0t}^{PP}. So it is sufficient to examine case 1. Interestingly the chain indices are drifting away from the direct indices and also the Laspeyres Paasche Gaps (LPG) are not constant. For the LPGs we get

direct indices either $\gamma_2 = -(\alpha_1-\beta_1) = \beta_1-\alpha_1 = \alpha_2-\beta_2 = -0.6$ or 0

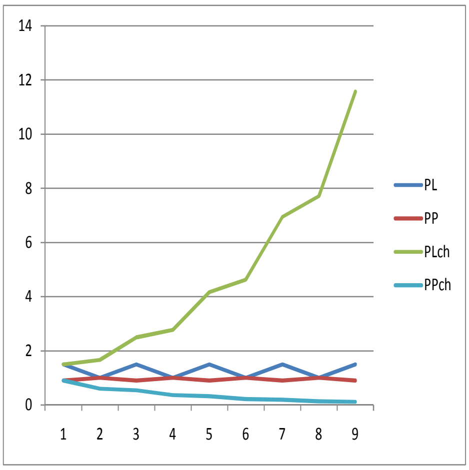
instead of either $\gamma_1 = \alpha_1-\beta_1 = +0.6$ or 0 for case 1. Correspondingly for the gaps between chain indices holds $\overline{\gamma}_2 = -\overline{\gamma}_1$.

P^L	P^P	PLch	D^{PL}	PPch	D^{PP}	g-d	g-ch
1,5	0,9	1,500	1,000	0,900	1,000	0,600	0,600
1,0	1,0	1,667	1,667	0,600	0,600	0,000	1,067
1,5	0,9	2,500	1,667	0,540	0,600	0,600	1,960
1,0	1,0	2,778	2,778	0,360	0,360	0,000	2,418
1,5	0,9	4,167	2,778	0,324	0,360	0,600	3,843
1,0	1,0	4,630	4,630	0,216	0,216	0,000	4,414
1,5	0,9	6,944	4,630	0,194	0,216	0,600	6,750
1,0	1,0	7,716	7,716	0,130	0,130	0,000	7,586
1,5	0,9	11,574	7,716	0,117	0,130	0,600	11,457

The Laspeyres chain index is increasing from 150% up to 1157% while the direct P^L is either 150 or 100. The Paasche chain index is constantly decreasing from 90% to 11.7%. Drifts D^{PL} and D^{PP}, and gaps (g) react correspondingly

Chaining does not automatically reduce the LPG. Fisher's index ($\overline{P}_{0t}^{F} = P_{0t}^{F}$) is given by $\sqrt{\alpha\beta}$ = 1.162 or 1, in both situations, case 1 as well as case 2.

Furthermore: while oscillating prices and quantities may be disadvantageous for chain indices of Laspeyres and Paasche, a Fisher chain index $\overline{P}_{0t}^{F}$ may nonetheless be appropriate.



B) Some additional index formulas

Lowe $$P^{Lo}_{t,0,b} = \sum_i \frac{p_{it}}{p_{i0}} \frac{p_{i0}q_{ib}}{\sum p_{i0}q_{ib}} = \sum_i \frac{p_{it}}{p_{i0}} \cdot s_{i,0b} = \frac{\sum p_{it}q_{ib}}{\sum p_{i0}q_{ib}}$$

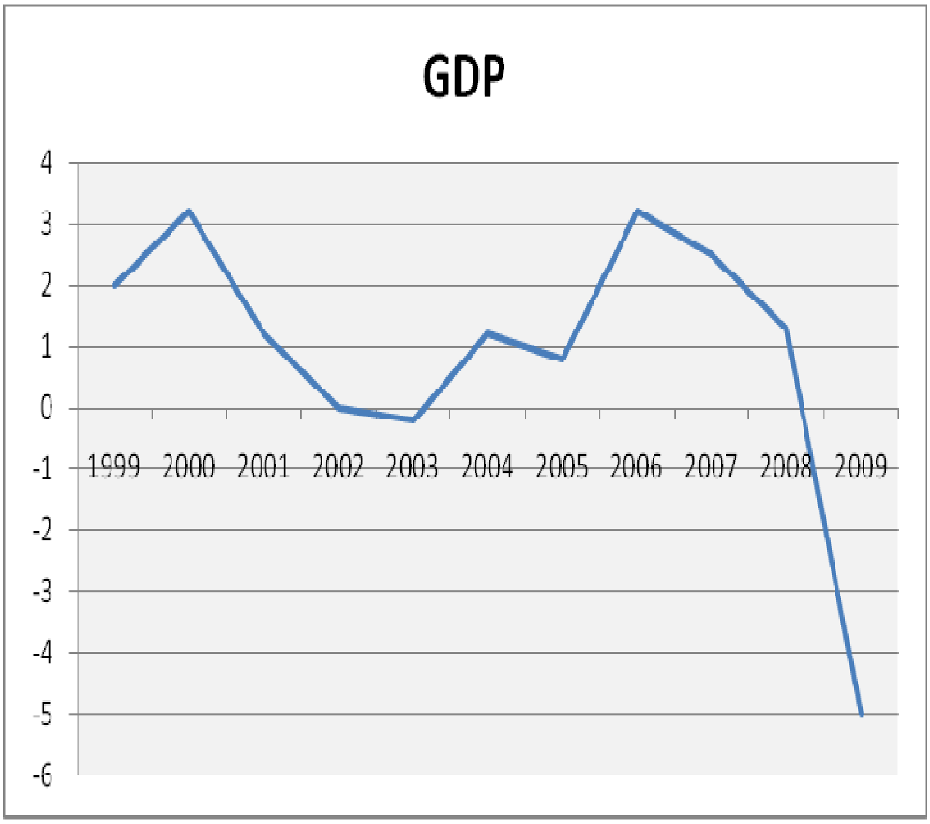
Young $$P^{Y}_{t,b,b} = \sum_i \frac{p_{it}}{p_{i0}} \frac{p_{ib}q_{ib}}{\sum p_{ib}q_{ib}} = \sum_i \frac{p_{it}}{p_{i0}} \cdot s_{i,bb}$$

Paasche $$P^{P}_{0t} = P^{P}_{t,0,t} = \sum_i \frac{p_{it}}{p_{i0}} \frac{p_{i0}q_{it}}{\sum p_{i0}q_{it}} = \sum_i \frac{p_{it}}{p_{i0}} \cdot s_{i,0t} = \frac{\sum p_{it}q_{it}}{\sum p_{i0}q_{it}}$$

Laspeyres $$P^{L}_{0t} = P^{L}_{t,0,0} = \sum_i \frac{p_{it}}{p_{i0}} \frac{p_{i0}q_{i0}}{\sum p_{i0}q_{i0}} = \sum_i \frac{p_{it}}{p_{i0}} \cdot s_{i,00} = \frac{\sum p_{it}q_{i0}}{\sum p_{i0}q_{i0}}$$

C Annual growth rates GDP "chained, volume" DE (formerly slide 8.3.1 (5) of part IV)

Press release: "Bruttoinlandsprodukt (Destatis, Jan 2010), preisbereinigt, verkettet Veränderung gegenüber dem Vorjahr (in Prozent)"



year	GDP
1999	2.0
2000	3.2
2001	1.2
2002	0
2003	-0.2
2004	1.2
2005	0.8
2006	3.2
2007	2.5
2008	1.3
2009	-5.0