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How Lehr's price index is related to Laspeyres' and Paasche's price index

There are good reasons to assume that Lehr's price index (P^{LE}) lies within the bounds of Paasche (P^P) and Laspeyres (P^L), such that $P^P < P^{LE} < P^L$.

Given that Lehr's price index for two adjacent periods, say 0 and 1 is defined as follows $P_{01}^{LE} = \frac{\sum p_{i1}q_{i1}}{\sum p_{i0}q_{i0}} \cdot \frac{\sum q_{i0}\breve{p}_{i,01}}{\sum q_{i1}\breve{p}_{i,01}} = \frac{V_{01}}{Q_{01}^{LE}}$ where $\breve{p}_{i,12} = \frac{p_{i1}q_{i1} + p_{i2}q_{i2}}{q_{i1} + q_{i2}}$ we can relate this index to the other two price indices via the respective quantity indices, that is using the equations $P_{01}^{L} = \frac{V_{01}}{Q_{01}^{P}}$ (Laspeyres) and $P_{01}^{P} = \frac{V_{01}}{Q_{01}^{L}}$ (Paasche). Again the generalized theorem of v. Bortkiewicz on two linear indices[1] can now be applied as follows:[2]

Lehr-Paasche

Using $X_0 = \overline{X} = \frac{\sum x_1 y_0}{\sum x_0 y_0} = \frac{\sum q_1 p_0}{\sum q_{10} p_0} = Q_{01}^{L}$ and $X_1 = \frac{\sum x_1 y_1}{\sum x_0 y_1} = \frac{\sum q_1 \breve{p}_{01}}{\sum q_{10} \breve{p}_{01}} = Q_{01}^{LE}$ we have

$\overline{Y} = \sum \frac{y_1}{y_0} \frac{x_0 y_0}{\sum x_0 y_0} = \frac{\sum \breve{p}_0 q_0}{\sum p_0 q_0}$, hence the covariance is given by

$\operatorname{cov}(LE, PA) = \sum \left(\frac{q_1}{q_0} - \overline{X} \right) \left(\frac{\breve{p}_0}{p_0} - \overline{Y} \right) \frac{p_0 q_0}{\sum p_0 q_0}$, so that $\frac{X_1}{X_0} = \frac{Q_{01}^{LE}}{Q_{01}^{L}} = 1 + \frac{\operatorname{cov}(LE, PA)}{\overline{X} \cdot \overline{Y}}$

$\overline{Y} = \frac{\sum \breve{p}_0 q_0}{\sum p_0 q_0}$ should be seen as price index, because the terms $\frac{\breve{p}_0}{p_0} = \frac{q_0}{q_0 + q_1} + \frac{q_1}{q_0 + q_1} \cdot \frac{p_1}{p_0}$ are simply linear transformations of the price relatives p_{i1}/p_{i0}. This allows concluding: if (transformed) price relatives and quantity relatives are negatively correlated (as usually assumed when the theorem is applied to P^L and P^P), that is cov(LE,PA) < 0, then $Q^{LE} < Q^L$ and consequently $P^{LE} > P^P$.

Lehr-Laspeyres

To demonstrate the corresponding relationship for P^{LE} and P^L requires a tiny modification only. With $y_0 = p_t$ instead of $y_0 = p_0$ we get $X_0 = \overline{X} = Q_{0t}^{P}$ and $\overline{Y} == \frac{\sum p_0 q_0}{\sum \breve{p}_0 q_0}$ so that $\overline{Y}$ may be regarded as reciprocal price index, as $\frac{\breve{p}_0}{p_1} = \frac{q_1}{q_0 + q_1} + \frac{q_0}{q_0 + q_1} \cdot \left(\frac{p_1}{p_0} \right)^{-1}$. The relevant covariance now is

$\operatorname{cov}(LE, LA) = \sum \left(\frac{q_1}{q_0} - \overline{X} \right) \left(\frac{\breve{p}_0}{p_1} - \overline{Y} \right) \frac{p_1 q_0}{\sum p_1 q_0}$, and the general equation

$\frac{X_1}{X_0} = \frac{Q_{01}^{LE}}{Q_{01}^{P}} = 1 + \frac{\operatorname{cov}(LE, LA)}{\overline{X} \cdot \overline{Y}}$ reads as follows: if the (transformed) *reciprocal* price relatives and quantity relatives are positively correlated (or equivalently: if price relatives and quantity relatives are negatively correlated), then $Q^{LE} > Q^P$ and consequently $P^{LE} < P^L$. Note that the weights in the weighted covariance are different now from cov(LE,PA).

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[1] von der Lippe 2007; 196.

[2] For convenience of presentation henceforth the subscripts i = 1,.., n will be omitted. Of course summation takes place over the n commodities.