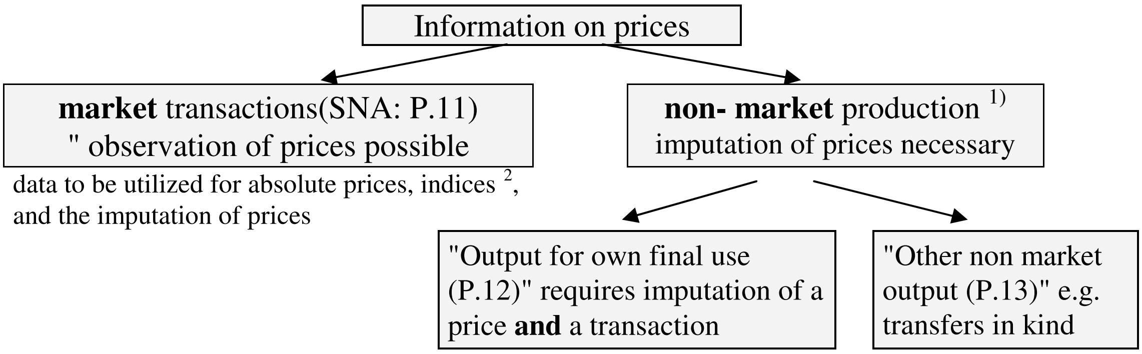
Figure 4.1.1: Scope of price statistics and origin of data on prices



1) Goods and services not actually sold or bartered for other goods or services, but provided free as transfer in kind or produced for own use
2) among them also indices for purposes of deflation

Table 4.1.1: System of price indices in German official statistics

Economic sector [1)]	Purchase prices (**Input**)	Selling (Producer) prices (**Output**)
Agriculture and forestry (NACE-Section A; excluded hunting)	Index of purchase prices of farm supplies	Producer price index of agricultural products, and several Producer price indices of forestry products
Mining [2)], manufacturing, electricity [3)] (C, D, E)	Input price index for production industries [4)]	Index of producer prices for industrial products
Construction (-Section: F)		Price indices for buildings
Wholesale and retail trade (NACE-Section: G)		Index of selling prices in wholesale trade and Retail price index
Hotels and Restaurants (H)		Price index for hotels and restaurants
Transport, storage and communication (NACE-Section: I)		Indices of sea freight rates (NACE-class: 61,10) Price index for postal services (NACE class: 64.11) Price index for telecommunications services (NACE-class: 64.20)
Other service activities [5)]		until 2008 some subindices of the CPI only, now a quarterly index for selected services (more in **fig. 4.1.2a**)
Foreign Trade	Import price index [6)]	Export price index [6)]
Private Households	Consumer price index	

1 Kind of economic activity (Sections in the [EU-] NACE Classification). 2 and quarrying 3 plus gas, water supply; 4 Price index for goods received by production industries. 5 Financial intermediation, real estate; renting and similar business activities etc. NACE: J+K+M+N and parts thereof. 6 Unit value indices in addition.

As to the **source** of information on prices (**who should report prices?**) practical experience shows that the type of information collected by price statistics does not only comprise prices as such but also much of the so called price determining characteristics (PDCs as introduced in **sec. 1.3**). This requires a close cooperation between reporting firms and the NSI, which has to rely on expert knowledge of the firms as to the quality of products and the prevailing conditions on the market. It is therefore often better to collect prices at sellers rather than at buyers not only in the case of indices of selling prices but also in the case of indices of purchase prices (see **fig. 4.1.1** for this distinction).

Table 4.1.2: Some details of selected German official price indices

Name of index	Price collection	Origin of weights
Price indices in agriculture a) Producer Prices b) Input Prices	a) ≈ 1,600 quotations from 300 reporting units for 240 products (monthly) and b) ≈ 6,100 quotations (mostly quarterly) from 650 r.u. [1]	Structures of farmers' sales and purchases provided by the German Federal Ministry of Agriculture
Producer Price Index [2] (only domestic sales) PPI	≈ 13,600 quotations from 6,000 producers monthly. Prices include excise taxes and duties	Turnover and when broken down to smaller units of commodity classification production values [3]
PPI Services	Quarterly index see below fig. 4.1.2a	Turnover of selected firms
Input Prices of Production Industries	No separate price collections [4] ≈ 9,000 domestic and import prices	weights derived from National Accounts estimates
Import and Export - Price Indices	Monthly reports (mail questionnaire) from 2,500 importing and 3,000 exporting firms.[5] In all ≈ 15,000 price quotations.	Foreign Trade Statistics import/ export values for ≈ 10,000 goods in a breakdown by countries and commodities.
- Unit value indices	By-product of foreign trade statistics	
Consumer Price Indices CPIs [6]	≈ 400,000 quotations monthly, price collection mostly decentralised [7], plus ≈ 25,000 rents	(formerly: various types of Family Budget (or "Family Expenditure") Surveys)[9], basket of ≈ 700 items

1 reporting units; 2 Mining and manufacturing industries; 3 Corrected for exports; 4 Prices of PPI, Wholesale- and Import Price Index; 5 Some prices taken from special publications; 6 Until the index system 2000 = 100 different CPIs for different types of households on the basis of so called "Extended Budget Surveys" (sample for *all* types of households in 5 years intervals) and "Continuous Budget Surveys" (a regular survey for 3 specific types of households each month). Meanwhile the coexistence of different types of budget surveys has been given up in Germany. Beginning with the index system 2000 = 100 there is only one single CPI left in Germany. 7 Regional Statistical Offices responsible. 8 Price quotations collected from landlords and tenants; 9 FBS or FES for short

Table 4.1.2a: Some details of the new German producer price index (PPI) of services (with number of items and price quotations [*or models examined*])

NACE*	Name of activity	Items	Prices
6024	Freight transport by road	36	1770
6311	Cargo handling	24	360
7411	Legal activities (Legal consultancy)	*45*/16	*200*/650
74113	Notaries	10	*60*
7413	Market research and public opinion polling	17	400
7412 74121	Accounting, book keeping and auditing etc. (esp. auditing)	*21*/12	*75*/1350
74123	Tax consultancy	21	*75*
7414	Business and management consultancy activities	16	1050
7440	Advertising	6	650**
74701	Industrial cleaning, esp. cleaning of buildings	6	750
74702	Industrial cleaning, esp. chimney sweeping	10	60
	Total	240	7450

* Rev. 1.1 (NACE Nomenclature Générale des activités économiques dans les communautés Européennes, Statistical Classification of Economic Activities in the European Community) The valid revision now is Rev. 2.
** Information taken from a data bank

4.2. Quality adjustment in price statistics

a) Need for quality adjustment	c) Numerical example
b) Classification of methods	d) Hedonic method

a) Need for quality adjustment

The appearance of new products and disappearance of old products (giving rise to looking for a replacement item) shows that the "quality problem" and the "(re-) sampling problem" are related. It is sometimes very difficult to distinguish between

- "pure price changes" that can and should *not* be attributed to changes in the price determining characteristics (PDCs), such as changes in the quality, and
- rises or declines of prices that result from changing PDCs.

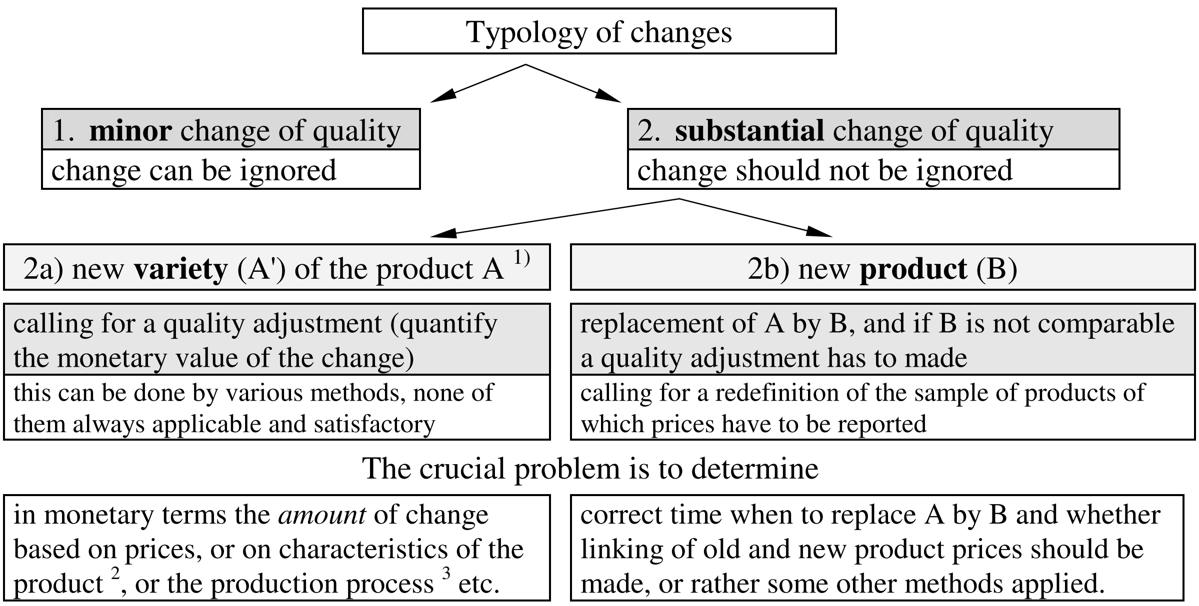
The principle of pure price comparison requires displaying only the first change while neutralizing the second type of change. Thus it may not suffice to compare only prices, just as they are observed. It is rather necessary to *estimate* (and deduct or add) that part of the observed change of prices that can rightly be attributed to changes in the PDCs. The task is to estimate *that* level prices *would have* if the PDCs had in fact been the same. Adjustments are required for the sake of (pure price) comparability.

The task is twofold and requires:

1. to **identify** a quality improvement or deterioration
2. to **evaluate** changes *in monetary terms* (in units of currency), so that an adjustment can be made by reducing (in the case of improvement) or augmenting (in the case of deterioration) the reported price appropriately.

> As a rule "quality" should be defined with regard to the use-value the buyer enjoys, and not (or not primarily) with reference to "experts", or producers; moreover it is *incomparability* of *all* kind that should be taken into account.

Figure 4.2.1: Types of quality change (of good A) and adjustment methods



1 quality improved or diminished
2 e.g. the so called hedonic approach rests on physical and other characteristics of the product A' as compared with A
3. an occasionally used method is, for example, to take production costs as a proxy of the true change in quality

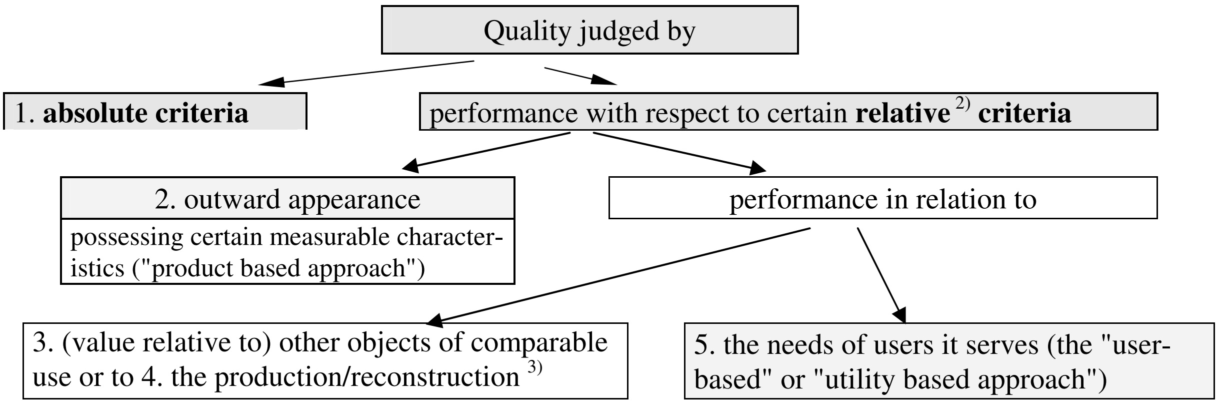
"Quality" is inclusive also of factors not tied to discrete goods and services, such as the variety of supply, time and location of sale, restrictions of competition (reducing the possibilities of free choice), rationing and queuing, the existence of a second (unofficial) market, or maybe also skill in bargaining.

In the case 1 (**fig. 4.2.1**) it may be legitimate to *ignore* the change and to continue with reporting prices as they are observed (a procedure which is the so called "**direct comparison**" or "unadjusted price comparison"). In the cases 2a and 2b change can no longer be ignored; the traditional approach of official statistics in dealing with this situation is the "**matched models method**" (**MM method**, disregarding all non-matching items[22]). Another option besides MM or direct comparison is to try a decomposition of the observed price difference Δp into a quality (thus "justified") difference Δq and the true (pure) price change (or residual component) $\Delta p - \Delta q = \Delta r$.

It should be noticed, however **Fig. 4.2.2** shows that absolute or relative standards may exist, and be relevant. In the case of relative standards the question is: *who* is setting the standards. Another question is whether quality has to be assessed with reference to aspects of its outward appearance, its performance as measured by certain criteria, its use value or its price.

Note also, that there are clearly limits for adjustments to be reasonable and that the correct idea of quality adjustment can easily be misused in a way that eventually all or most of the observed price movement can be denied or "adjusted away".

Figure 4.2.2: The notion of "quality" [1]



1. according to Garvin 1984
2. in comparison with agreed upon standards, the price or the intended use to be made of the product etc.
3. for example less problems with subsequent reconstruction, repair etc.

b) Classification of methods

Unfortunately the terminology concerning the methods differs greatly between statisticians and this unduly complicates the surveying of this subject. In fig. 4.2.3 and 4.2.4 an attempt is made to make classifications and terminological distinctions concerning the adjustment methods clear. These methods are devised in order to decide on the extent to which the observed Δp may be divided into apart representing quality change (Δq) and a residual part Δr respectively.

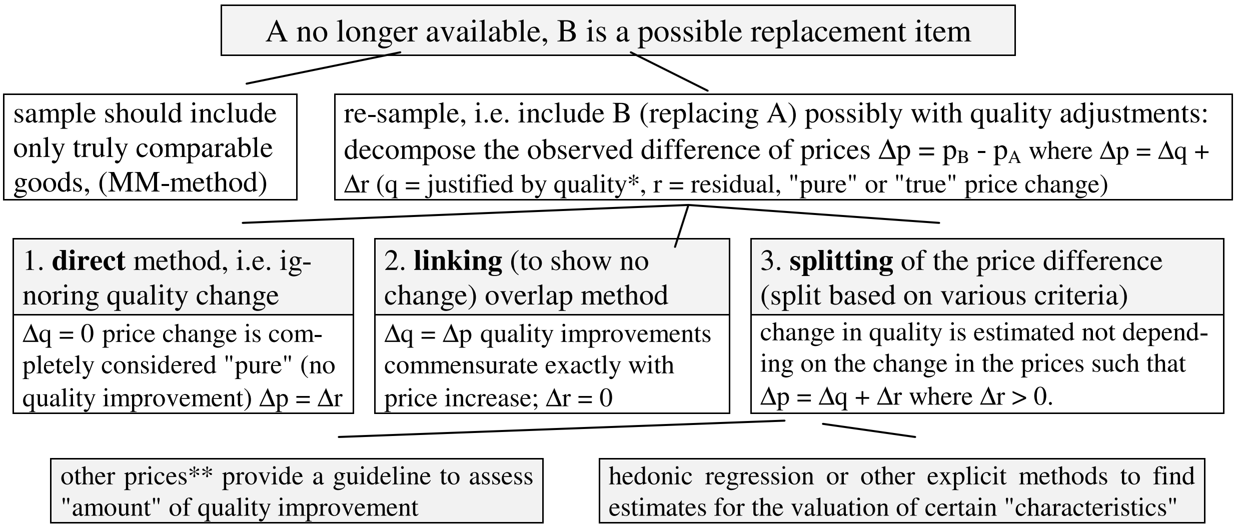
Implicit assumptions and justification of linking (overlapping) methods

Whenever price quotations of (qualitatively different) variants of the same product A, that is A' or B overlap in at least one period it appears reasonable to assume that the difference in price between them is reflective of the consumers' valuation of the quality difference, or that both goods m and n

[22] A method called "[monthly] **chaining and resampling**" requires matched models only for any two *adjacent* periods unlike the strict MM-method requiring comparable items *all along* from 0 to t.

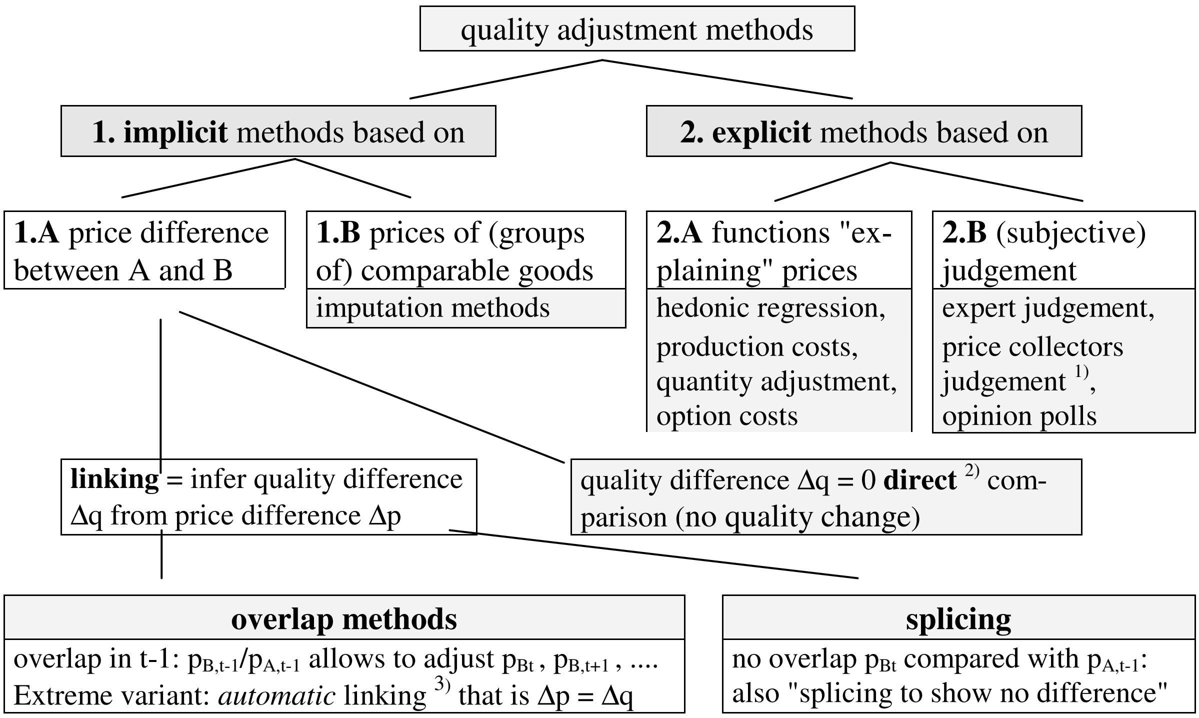
offer the same price/quality ratio PQR. However whether or not these assumptions can be made depends on the specific situation of the market and the nature of the quality change. The overlap method is certainly not applicable if the new good is entirely new without an earlier counterpart, or if the price of A (old), or B (new) is unduly low.

Figure 4.2.3: Components of the price difference and adjustment methods



* Δq should not be mistaken for a difference in quantity rather than quality.
** i.e. prices of similar products or the average price of the group of products to which the product in question belongs.

Figure 4.2.4: System of adjustment methods
A = "old" item, B = (new) replacement item



1) decentralised judgemental adjustment of price collectors (assuming that valuation correctly reflects product features as seen by consumers) as opposed to centralised expert judgements
2) because prices are considered (or assumed to be) **directly** comparable (without adjustment)
3) or *link to show no price difference* banned as unacceptable in the framework of the HICP (see sec. 6.1)

Additional difficulties due to combination such as: cheaper and at the same time better products (case 1), or more expensive and worse (case 2),

	Price of B compared to A	
Quality of B compared to A	higher $p_t^B > p_t^A$ (F < 1)	lower $p_t^A > p_t^B$ (F > 1)
better	$\hat{p}_t^B < p_t^B$ a better quality justifies *reduction*	**?** (1)
worse	**?** (2)	$\hat{p}_t^B > p_t^B$ a worse quality requires *increase* of $\hat{p}_t^B$ relative to p_t^B

or whenever quality has changed but the price remained constant, that is Δp = 0..

c) Numerical example

	actual price of commodity		fictitious [1)] price of
period in time	A (old)	B (new)	B (comparable with A)
0	$p_0^A = 5$	×	$\hat{p}_0^B = p_0^A / F$
...	...	×	...
t-1	$p_{t-1}^A = 10$	×	$\hat{p}_{t-1}^B = p_{t-1}^A / F$ [2)]
t (overlap period)	$p_t^A = 12$	$p_t^B = 15$	$\hat{p}_t^B = p_t^B \cdot F$
t+1	×	$p_{t+1}^B = 18$	$\hat{p}_{t+1}^B = p_{t+1}^B \cdot F$
...	×	...	
t+k	×	$p_{t+k}^B = 23$	$\hat{p}_{t+k}^B = p_{t+k}^B \cdot F$

1 quality adjusted price (as opposed to the observed price) of B so that B is qualitatively comparable to A
2 There is no point in calculating this price, however, because there was no commodity B in period t-1
× means "not available" or "not applicable"

The general principle goes as follows: multiply (after t), or divide (for periods before t) the price by the factor F

(4.2.1) $$\hat{p}_{t+k}^B = p_{t+k}^B \cdot F \text{ (k > 0 after overlap) and } \hat{p}_{t+k}^B = p_{t+k}^A / F \text{ (k < 0 before overlap)}$$

in order to enable a continuation of the series of item A though no longer available. F can take different values depending on which of the following methods is used[23]:

the two extreme solutions		an intermediate solution*
overlap method or: link to show no change	no adjustment or: direct method	the "50%-rule"
$F = \frac{p_t^A}{p_t^B} = \frac{12}{15} = 0.8$	$F = \frac{p_t^B}{p_t^B} = 1$	$F = \frac{\frac{1}{2}(p_t^A + p_t^B)}{p_t^B} = 0.9$

* such that 0.8 < F < 1. The 50% rule is only one example for such a procedure (it is - or was at least - widely in use in Germany).

23 We demonstrate only three cases (out of infinitely many).

Examples for the factor F (eq. 4.2.1) in $p^B_{t+k} \rightarrow \hat{p}^B_{t+k}$

A) IN THE CASE OF THE OVERLAP (OR LINKING) METHOD (no. 2 in fig. 4.2.3):

The method amounts to assuming $\hat{p}^B_t = p^A_t$ because due to $\hat{p}^B_t = p^B_t \cdot F = p^A_t$ the factor F is given by $F = p^A_t / p^B_t$

(4.2.2) $\hat{p}^B_t = p^B_t \cdot F = p^B_t \frac{p^A_t}{p^B_t} = p^A_t \Rightarrow F = \frac{p^A_t}{p^B_t}$, or $\boxed{\hat{p}^B_t = p^A_t + 0 \cdot (p^B_t - p^A_t)}$

attributing none of the price difference to a "genuine" price increase. We may also state the principle as follows (4.2.3) $\frac{p^A_t}{p^A_0} \cdot \frac{p^B_{t+k}}{p^B_t} = \frac{\hat{p}^B_{t+k}}{p^A_0}$, which explains the term "linking" or "chaining" widely used for this method[24]. The numerical example gives F = 0.8 such that:

	0	t-1	t	t+1	t+k
observed price	A: 5	A: 10	A:12, B: 15	B: 18	B: 23
imputed price	$\hat{p}^B_0 = p^A_0 / F =$ 5/0.8 = 6.25	$\hat{p}^B_{t-1} = p^A_{t-1} / F =$ 10/0.8 = 12.5	$\hat{p}^B_t = p^A_t = 12$	$\hat{p}^B_{t+1} = p^B_{t+1} F =$ $18 \cdot 0.8 = 14.4$	$\hat{p}^B_{t+k} = p^B_{t+k} F$ = 18.4
price relatives	1	2 = 10/5	2.4 = 12/5 = 15/6.25	2.88 = 18/6.25 = 14.4/5	3.68 = 23/6.25 = 18.4/5

The price relatives are given by $\hat{p}^B_{t+k} / p^A_0 = p^B_{t+k} F / p^A_0$, or equivalently $p^B_{t+k} / \hat{p}^B_0 = p^B_{t+k} / (p^A_0 / F)$

It is in practice generally more convient to adjust one single base period price $p^A_0 \rightarrow \hat{p}^B_0$ and to compare[25] subsequently each *observed* price p^B_{t+1}, p^B_{t+2}, ... with this adjusted (fictitious) price $\hat{p}^B_0$ rather than to continually adjust each current period price $p^B_{t+1} \rightarrow \hat{p}^B_{t+1}$, $p^B_{t+2} \rightarrow \hat{p}^B_{t+2}$ ... and to compare them with an unadjusted base period price p^A_0.

To summarize the method: only the adjusted (imputed) prices $\hat{p}^B_t$, $\hat{p}^B_{t+1}$, ... are reflecting the true (genuine) increases of the price of B (as compared with A)

	observed price increase of commodity B	true increase in price (rise of the quality adjusted price of B)	extent to which increase in price of B is owed to improved quality
period	p^B_{t+k}	$\hat{p}^B_{t+k} = p^B_{t+k} F$ (F = 0.8)	$p^B_{t+k} - \hat{p}^B_{t+k} = p^B_{t+k}(1-F)$
t	15	12.0	3.0
t + 1	18	14.4	3.6
t + k	23	18.4	4.6

B) IN THE "NO-ADJUSTMENT-METHOD" (= "direct comparison" No. 1 in fig.4.2.3):

As F = 1 there is no quality adjusted price $\hat{p}^B_{t+k}$ which is lower than the observed price p^B_{t+k}

(4.2.5) $\boxed{\hat{p}^B_t = p^A_t + 1 \cdot (p^B_t - p^A_t) = p^B_t}$, or regarding all of the difference $p^B_t - p^A_t$ as a true (not quality induced) price increase.

24 The change over an interval from 0 to t+k (denoted by [0, t+k]) is derived from linking two subintervals [0, t] and [t, t+k].

25 that means to divide each readily available *observed* price of B in t+1, t+2 etc. by a fictitious base period price.

	0	t	t+1	t+k
observed price	A: 5	A:12, B: 15	B: 18	B: 23
adjusted price	$\hat{p}_0^B = p_0^A = 5$	$\hat{p}_t^B = p_t^B = 15$	$\hat{p}_{t+1}^B = p_{t+1}^B = 18$	$\hat{p}_{t+k}^B = p_{t+k}^B = 23$
price relatives*	1 (1)	15/5 = 3 (2.4)	18/5 = 3.6 (2.88)	23/5 = 4.6 (3.68)

* in brackets the case of linking (F = 0.8)

C) IN THE CASE OF THE 50% RULE: we assign half of the difference between the prices that is $\left(p_t^B - p_t^A\right)/2$ to the quality component and half to a real price increase such that

(4.2.6) $$\hat{p}_t^B = p_t^A + \frac{p_t^B - p_t^A}{2}$$, or equivalently $F = \frac{\frac{1}{2}\left(p_t^A + p_t^B\right)}{p_t^B} = 0.9$ (instead of 0.8, or 1)

yielding results of $\hat{p}_t^B$ exactly in between the two methods above:

	0	t	t+1	t+k
A	5	12		
B		15	18	23
overlap	6.25	12	14.4	18.4
direct	5	15	18	23
50%-rule	5.55	13.5	16.2	20.7

To derive the price relatives it again proves more convenient to divide the (sequence of) observed prices of B by a constant fictitious base period price such that the price relatives are given by

$\frac{p_{t+1}^B}{\hat{p}_0^B}, \frac{p_{t+2}^B}{\hat{p}_0^B}, \ldots$ instead of calculating $\frac{\hat{p}_{t+1}^B}{p_0^A}, \frac{\hat{p}_{t+2}^B}{p_0^A}, \ldots$.

d) Hedonic Method

If "quality" can be decomposed into certain quantitative "characteristics" and if sufficient price information on goods representing the different levels (or variants) of these features is available the hedonic method[26] of quality adjustment is applicable. The price p_i (dependent variable) of an item i is a function of k characteristics (regressors) x_{1i}, …, x_{ki} And since prices are known of goods with different values of the variables $X_1, \ldots, X_k$ it is possible to run the regression of prices on the set of regressors $x_1, \ldots, x_k$ possibly including also dummy variables for qualitative aspects (e.g. brand name) or time periods. The hedonic method consists in imputing a price

- justified for a bundle of characteristics (measurable traits) of a good, or
- prevailing in a period t (using time dummies D_t) and assuming all characteristics being constant

on the basis of a regression analysis.

The (possibly linear) regression

(4.2.1) $\hat{p}_i = b_0 + b_1 x_{1i} + b_2 x_{2i} + \ldots + b_k x_{ki}$ or $\hat{p}_i = b_0 + \sum_j b_j x_{ji}$, (i = 1,..., n and j = 1, ..., k)

of n prices (p_i) on k "characteristics" is known as the "hedonic price function". It is assumed that the regression coefficients (partial derivatives or gradients) $b_j = \partial \hat{p}_i / \partial x_{ji}$ measure the buyers' marginal willingness to pay for a small amount more of the respective characteristics x_k. The price $\hat{p}_i$ represents the "quality adjusted" price as opposed to the actually observed price p_i of good i. The Formula of the *indirect* approach is given by

[26] or "regression-", "characteristic method" etc.

(4.2.2) $\hat{p}_i = b_0 + \sum_j b_j x_{ji} + c_1 D_1 + ... + c_T D_T$, (i = 1, ... , n j = 1, ..., k time dummies D_1, …, D_T).

Figure 4.2.6: Variants of the hedonic method

a) Two methods depending on what regressors the regression function contains

data from a single reference period in order to calculate the implicit prices of each of the characteristics	data are covering all periods, time dummies are included; regression is re-run each time the index is compiled
indirect -, or individual coefficient method	**direct** [a)]-, or **time dummy** variables method
can be used to estimate a price for an item not on the market or to adjust post-hoc the observed price of a replacement item (method can be combined with other methods of quality adjustment, more appropriate for official statistics)	difference between the time dummies is taken to represent the genuine price change [b)] (quality change excluded); price movements over a number of years are estimated in one regression. Less observations needed than in indirect method
relies much on the stability of the function;	usually backwards looking studies

With both methods frequent updates (systematic methods of timing are vital!) necessary; hedonics in a COLI (rather than COGI) context requires to take more regressors into account (e.g. convenience, state of competition, availability of alternatives etc.

a) because price index is calculated directly from the regression
b) those price changes that cannot be associated with a change in the characteristics

b) List of some major problems with hedonics

1	selection and measurement of "characteristics"; expensive data collection [a)]
2	functional form [b)], collinearity, heteroscedasticity, estimation problems (stepwise regression)
3	weighting (e.g. with sales values; the choice of weights is far from clear [c)])
4	interpretation of the regression coefficients (theory!!); problems with entirely new goods

a) therefore priority setting (due to limited resources) is an important issue
b) in general semi-log or double-log regression functions often provided a better fit than linear functions
c) expenditure weights for example automatically give larger weights to expensive models

c) Various methods depending on which prices are observed/imputed

sets	D (disappeared)	M (matched)	N (new)
prices in t = 0	p_{D0}	p_{M0}	p_{N0}
prices in t=1	p_{D1}	p_{M1}	p_{N1}

Prices p_{D1} and p_{N0} are necessarily imputed; other prices of the universe $D \cup M = U^0$ can be observed in t = 0 and prices of the universe $M \cup N = U^1$ can be observed in t = 1:

imputation concept	weights $s_i^0/2$; prices of U^0		weights $s_i^1/2$; prices of U^1	
	observed	imputed	observed	imputed
SI single	p_{M0} p_{M1} p_{D0}	p_{D1}	p_{M0} p_{M1} p_{N1}	p_{N0}
DI double	p_{M0} p_{M1}	p_{D1} p_{D0}	p_{M0} p_{M1}	p_{N0} p_{N1}
FI full*		$p_{D1}p_{D0}$ p_{M0} p_{M1}		$p_{N0}p_{M0}$ p_{M1} p_{N1}

* equivalent to "exact" hedonic imputation of Feenstra

Hence in the order SI $\Rightarrow$ DI $\Rightarrow$ FI more and more use is made of imputation.

According to the Schultze Panel "the best candidates for hedonic analysis are categories of goods for which quality change is frequent but incremental and for which the characteristics changes are easy to measure"[27]

The German price statistics of buildings or constructions (or in general in pricing "unique" products) follows in principle the same ideas as the hedonic approach, with the exception that "weights" in a weighted sum differ from regression coefficients.

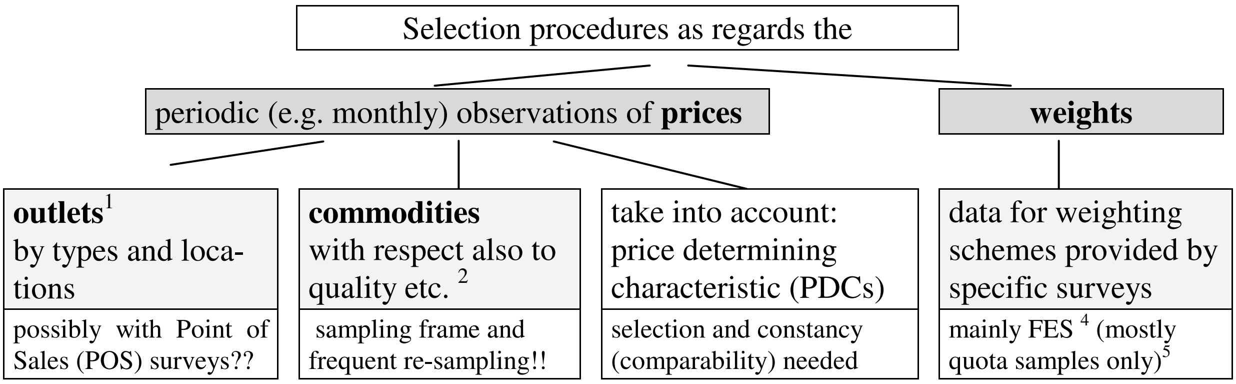
4.3. Sampling in price statistics[28]

A price index combines data on prices and weights, both of which likely to be subject to a sampling error and there is little known about the sampling aspects of the *combination* of price relatives *and* weights. Moreover it is often difficult to define the *unit of observation*, and thus the universe, and more often than not impossible to dispose of a *"sampling frame"* (from which a sample can be drawn).

Given the need to observe such "side conditions" the sampling problem is in practice much more complicated than it might be supposed on the face of it. Furthermore whenever new products emerge, or in general the PDCs change *we should draw a new (costly) sample*.

The *definition of the target population* does not only comprise goods and reporting units, it also requires observing all aspects (PDCs) of sales and purchases, in particular a constant monitoring of the (continually changing) representative PDCs. Moreover we should be able to dispose of a *"sampling frame"* (from which a sample can be drawn), however, as a rule a complete and up-to-date list of all units to be sampled is not only lacking in the case of items (goods and services) but in most countries also in the case of outlets.

Figure 4.3.1: Stages in index calculation where selections are involved



1) reporting firms (where constantly reliable and representative price quotations can be made)
2) e.g. regular availability (presently but also in future)
3) in price statistics it is crucial to have all sorts of PDCs correctly represented and kept constant as time goes on
4) family expenditure surveys (or family budget surveys, FBS)
5) due to nonresponse problems with voluntary surveys

The use of scanner data

Problems of re-selection of commodities

Two extreme strategies of dealing with dynamic changes in the universe of commodities

1. the **MM-method** (matched models) = a *fixed sample of items* is followed over time (pure price comparison at the expense of representativity), as opposed to

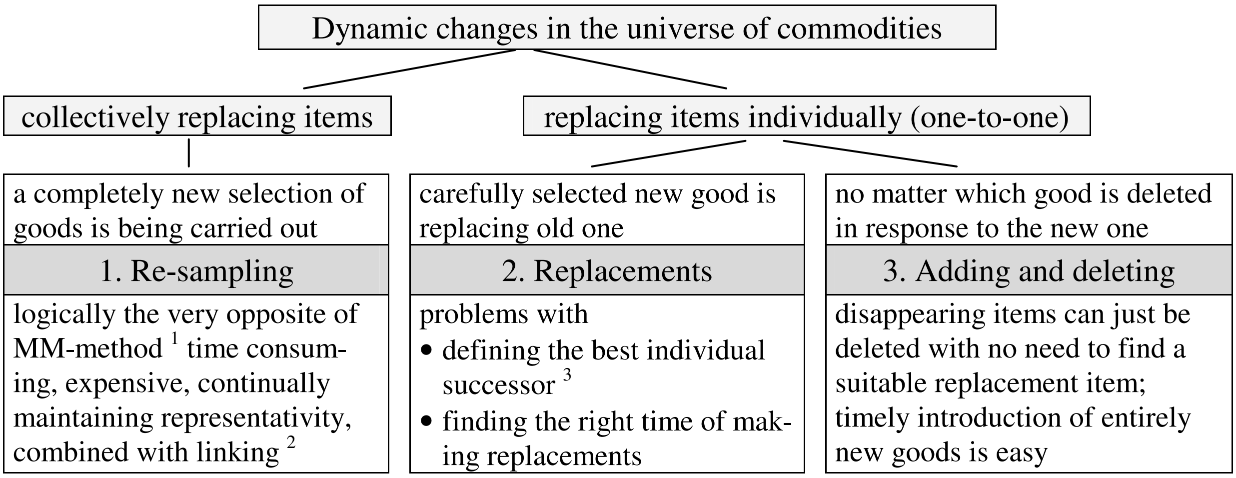
[27] Why frequent change? Because there should exist a sufficient number n of models ("varieties") varying significantly with respect to the characteristics.

[28] We address here also the re-sampling problem already mentioned in sec. 4.2.

2 continuous (e.g. monthly) **re-sampling** of items in order to maintain the representativity.

Definition and incorporation of "new goods"

Figure 4.3.1: Dealing with appearances and disappearances of goods



1 matched models (see sec. 4.2 part c2) 2 or "overlapping" (see sec. 4.2 part e1 and e2)
3 "most sold" (more representativity) or "most like" (then the least problems with quality adjustment) good? Unlike the Adding-and-deleting method (no 3) the Replacements method cannot adequately reflect completely new developments as each newly introduced good should have some relation or other to a replaced one.

De Haan's Generalised Ideal (Fisher) Index (P^{GF}) (also called "matched" Fisher index)

Denote $\mathbf{q}_0^D$ and $\mathbf{p}_0^D$ the vectors of quantities and prices respectively of goods available in 0 but not in t (= disappearing). Likewise $\mathbf{q}_0^C$, and $\mathbf{p}_0^C$ are vectors related to "ongoing" continually observable in both periods 0 and t. New goods (in t but not yet available earlier) are labelled N (vectors $\mathbf{p}_t^N$ and $\mathbf{q}_t^N$) As $\mathbf{q}_t^D = \mathbf{q}_0^N = \mathbf{0}$ the vectors $\mathbf{p}_t^D$ and $\mathbf{p}_0^N$ have to be estimated The conventional Paasche index excludes goods which disappeared such that

$$P_{0t}^P = (\mathbf{p}_t^{C'}\mathbf{q}_t^C + \mathbf{p}_t^{N'}\mathbf{q}_t^N)/(\mathbf{p}_0^{C'}\mathbf{q}_t^C + \mathbf{p}_0^{N'}\mathbf{q}_t^N) = (A_1 + A_2)/(B_1 + B_2),$$

much like P_{0t}^L does not reflect prices of new goods[29]. P^{GF} is given by $P_{0t}^{GF} = \sqrt{P_{0t}^L P_{0t}^P}$, and is related to the Fisher index using ongoing (C) goods only (i.e. confined to the "intersection universe" $P_{0t}^{F,int} = \left[(\mathbf{p}_t^{C'}\mathbf{q}_t^C)(\mathbf{p}_t^{C'}\mathbf{q}_0^C)/(\mathbf{p}_0^{C'}\mathbf{q}_t^C)(\mathbf{p}_0^{C'}\mathbf{q}_0^C)\right]^{1/2} = \sqrt{A_1C_1/B_1D_1}$ such that

$$P_{0t}^{GF} = P_{0t}^{F,int} \cdot \sqrt{\frac{(A_1 + A_2)/A_1}{(D_1 + D_2)/D_1}} \cdot \sqrt{\frac{(C_1 + C_2)/C_1}{(B_1 + B_2)/B_1}} = P_{0t}^{F,int} \cdot \mu \cdot \lambda \text{ where } \lambda = \sqrt{\frac{1 + C_2/C_1}{1 + B_2/B_1}}$$

According to de Haan μ "re-scales" $P^{F,int}$ "for the fact that the expenditures of new and disappearing goods, have not been taken into account"[30]. The factor λ "contains imputed prices" (backward imputed for N - goods in the B_2 aggregate, and forward imputed prices in C_2) and "is needed to handle new and disappearing goods in the correct way". Ratios such as C_2/C_1 are comparing prices of disappearing goods with ongoing ones on the basis of base period quantities just like B_2/B_1 is comparing prices of new goods with ongoing ones on the basis of current period quantities.

29 In P^P certain base period prices (of N-goods) are fictitious, and so are current period prices of D-goods in P^L.

30 That is A_2 for new and D_2 for disappeared goods respectively, which means that values such as A_2 and D_2 are needed to correct a current period term (A - term) in the numerator and a base period term (D - term) in the denominator. Note that the second factor (μ) is composed of observed values only, unlike the third factor (λ).

Chapter 5 Deflation, structural consistency

5.1. Introduction into deflation methods

a) Objectives and types of deflation	d) Deflating National Accounts aggregates
b) Direct and indirect volume measurement	e) Deflation in real terms
c) Inflation measurement and deflation	f) Terms of trade effect

a) Objectives and types of deflation

Figure 5.1.1: Notion of "deflation"

a) ***Types of "revaluation"***

Revaluation of aggregates for

intertemporal comparisons (= **deflation**)	interspatial (e.g. international) comparisons
calculation of $\Sigma q_{it}p_{i0}$ with current (t) quantities q_{it} and base period (0) prices p_{i0}	calculation of $\Sigma q_{iA}p_{iB}$ with reference country quantities q_{iA} and base country (B) prices p_{iB}

b) Types of deflation

Deflation of aggregates

in volume terms	**in real terms**
"at constant prices"	at constant purchasing power of money
Volume - approach (quantity interpretation of the result intended): isolation of the quantity component	Real - income- approach (adjusting income for inflation): the aim is to estimate (adjust for) the effect of inflation
Applicable to value changes only, that can be decomposed into price- and quantity-changes, i.e. to commodity flows (CFs)	Applicable also to values that do not have price and quantity dimensions on their own, i.e. to non-commodity flows (NCFs)
Deflate aggregates $A_1, A_2, ..., A_m$ (representing CFs) by using m price indices P_i (i = 1,...,m) for the m aggregates (P_1 refers to the commodities in A_1 and P_2 to A_2 and so on)	Deflate aggregates $A_1, A_2, ..., A_m$ (all or many of them NCFs) using *one single* deflator that accounts for the general development of the purchasing power

the typical problem in applying the method is

double deflation (i.e. deflating C = A - B by deflating A and B separately)	**choice of *the*** general (universal) **deflator**, that has to be applied to *all* aggregates

Abbreviations: CF = commodity flow, NCF = non-commodity flow

b) Direct and indirect volume measurement, new products

Figure 5.1.2: Alternative methods to estimate volumes or volume (quantity) indices

method

- **indirect** method
  - method 1: using prices p_{i0} instead of p_{it}, i.e. by individual re-pricing of each aggregate
  - method 2: divide the value by an appropriate price index (deflator) (global re-pricing)
- **direct** method
  - method 3: extrapolation of quantities, or using an appropriate quantity index

Assume good 2 is new such that $p_{20} = q_{20} = 0$. To deflate $\left(p_{1t}q_{1t} + p_{2t}q_{2t}\right)/p_{10}q_{10}$ using p_{1t}/p_{10} as deflator is equivalent to making the assumption $\hat{p}_{20} = p_{10}\left(p_{2t}/p_{1t}\right)$, since

(5.1.1) $$\frac{p_{10}\left(p_{1t}q_{1t} + p_{2t}q_{2t}\right)}{p_{1t}p_{10}q_{10}} = \frac{p_{10}q_{1t} + \left(p_{10}\dfrac{p_{2t}}{p_{1t}}\right)q_{2t}}{p_{10}q_{10}}$$ (given $p_{20} = q_{20} = 0$), or to assuming

$\frac{p_{1t}}{p_{10}} = \frac{p_{2t}}{\hat{p}_{20}}$, such that both prices vary in proportion, and $\frac{p_{10}q_{1t} + \hat{p}_{20}q_{2t}}{p_{10}q_{10} + \hat{p}_{20}\cdot 0}$.

c) Inflation measurement and deflation (volume measurement)

Figure 5.1.3: Alternative views concerning uses of price indices

Price level (inflation) measurement and deflation are

one task (SNA recommendations)	two different tasks (traditional position)
the same price index serving both purposes; deflator of GDP, and also used to measure inflation: SNA prescribes a chain index of Fisher type	requiring different methods; **two price indices**, for example P^L (CPI) for inflation measurement, and P^P for deflation
factor reversal test gains importance (same formula for CPI and deflator)	product test sufficient, allowing for different formulas, Laspeyres and Paasche

d) Deflating National Accounts aggregates

Figure 5.1.4: Production, expenditure and income approach in National accounts (NA)

NA breakdown into aggregates, predominantly

- commodity flows (CFs) and balancing items
- non-commodity flows (NCFs)

Production approach: supply side	**Expenditure approach: uses side →GDP = F - M**	**Income approach: distribution of incomes**
Total supply (S)	Final expenditure (F), GDP	National income
S = Y + M + T;	F = S –Z = C + G +K + X	NDP + PI

Definitions	
C	Final consumption of households (Private cons.)
D	Consumption of fixed capital (K.1)
F	Final expenditure
G	Final consumption of government (Collective cons.)
GDP	Gross domestic product
K	(gross) Capital formation (P.5)*
M	Imports (P.7)
PI	net Primary incomes from the rest of the world
T	Taxes less subsidies on products (D.21-D.31)
X	Export
Z	Intermediate consumption (P.2)
S	Total supply
VA	value added (prominent example of an NCF)
Y	Output (P.1)

Identities
Final consumption = FC = C + G
Final domestic expenditure FD = FC + K
F = FD + X
GDP = F – M
NDP = net domestic product = GDP – D
VA value added (= Y – Z, a balancing item)

* of which gross fixed capital formation (= investment, P.51), Changes in inventories (P.52) and Acquisition less disposal of valuables (P.53)

e) Deflation in real terms and the choice of "the" deflator (the "general price level")

(5.1.2) $$\text{real income} = y_r = \frac{\text{nominal income}}{\text{consumer price index}} = \frac{y_n}{CPI}.$$

(5.1.2a) $$\frac{\sum p_{kt}q_{kt}}{P_{0t}^L} = \frac{\sum p_{kt}q_{kt}}{\sum p_{it}q_{i0}} \sum p_{i0}q_{i0} = S_1 \neq S_2 = \sum p_{k0}q_{kt}$$ (5.1.2b) $$Q_{0t}^P = \frac{V_{0t}}{P_{0t}^L}.$$

index	compares numerator (N)	with denominator (D)
P_{0t}^L pure price movement	**fictitious** aggregate $\sum p_t q_0$: basket q_0 with actual prices p_t (**constant quantities**) (N differs from D only with respect to prices)	**actual** budget $\sum p_0 q_0$ prices p_0 and quantities q_0 at base period 0
Q_{0t}^P index of real income	**actual** cost of living $\sum p_t q_t$ with actual (present) prices p_t and quantities q_t	**fictitious** $\sum p_t q_0$ **:** cost of living at **constant prices**

Alternatives (discussed in literature): GDP deflator, deflator of domestic final demand, CPI etc. See tab. 5.1.1. for more detail.

Table 5.1.1: Comments on possible general deflators

price index (PI)	coverage	interpretation as "the" deflator
CPI, Consumer Price Index	Only individual consumption (definition differs from to the PCI).	Unlike PCI a P_{0t}^L index (Laspeyres)
SPI, for Total Supply	Covers unlike YPI in addition to domestic produced imports too.	Useful for Input-Output-Tables rather than real income deflation.
FPI, for **F**inal Uses (or: Final Expenditure) = F	Comprises final consumption, capital formation (= final domestic expenditure) *and*[1] export.	In the framework of the so-called "asset inflation" it is desirable to include capital formation as in FPI.
FDPI, Final Domestic Demand	Final Domestic Demand is FD = F – X, hence also comprising investment	This index shows to which extent inflation is "homemade".
GDP deflator	Gross domestic product differs from FPI by the exclusion of Imports.	This price index is resulting from double deflation.
PCI, Private consumption [2]	Differs from CPI as regards some taxes, payments for services etc.	Like value added gained by double deflation.
ECI, Employment Cost Index	Compensation of employees [3] by occupations.	Measures the price of labour services only, no other factors.

1 unlike Final **domestic** expenditure (which is exclusive of exports)
2 more precisely "Final consumption of households"
3 comprising wages and salaries and employer's social contributions

f) Terms of trade effect

(5.1.3) $$N = X - M$$

(5.1.4) $$\frac{N}{P_N} = \frac{X}{P_X} - \frac{M}{P_M} = X_V - M_V = N_V$$ (V = volume).

(5.1.5) $$N_R = \frac{N}{P} = \frac{X - M}{P}.$$

(5.1.6) $G = N_R - N_V$; $G > 0$ represents a gain, $G < 0$ a loss from external trade.

(5.1.7) $$T_N = \frac{P_X}{P_M}$$ (net barter terms of trade), and

(5.1.8) $T_G = \frac{X_V}{M_V}$, the gross (barter) terms of trade. $T_I = \frac{P_X X_V}{P_M} = T_N X_V = T_N T_G M_V =$ income t.o.t. It is often said that an improvement in the terms of trade ($T_N > 1$) affects the trade balance favourable by which is meant an increasing surplus (N > 0) or a decreasing deficit (N < 0). It turns out that more complication is involved in saying "favourable". The difficulty is that it depends on whether a surplus or a deficit has been assumed as starting point.

(5.1.9) $N = M_V P_M (T_N T_G - 1) = M(T_N T_G - 1)$.

situation	surplus	deficit
1: $P_X > 1$, $P_M < 1$	nominal surplus becomes greater	deficit becomes smaller or changes into a surplus
2: $P_X > P_M > 1$	greater surplus	change either way
3: $P_M < P_X < 1$	change either way possible, surplus greater or smaller	improvement, i.e. diminishing deficit or surplus

5.2. Deflation in volume terms, aggregation and double deflation

a) Interpretation of volumes, axioms	e) Equality test (ET)
b) Double (indirect) deflation	f) Structural consistency of volumes
c) Aggregative properties of index functions	g) New results concerning aggregative consistency.
d) Consistency in aggregation	

a) Interpretation of "volumes", axiomatic considerations

Prices	Quantities	
	(1) same* rate ω	(2) different rates
(1) same* rate λ	case 11	case 12
(2) different rates	case 21	case 22

* the case of constant prices/quantities is the special case of $\lambda = 1$, or $\omega = 1$ respectively

Cases 11 and 21 (Volumes should be proportional in the quantities)

if $q_{it} = \lambda q_{i0}$ $\forall i$ then the volume V_t should be λ–fold: $V_t = \lambda V_0$ (as usual identity is a special case $\lambda = 1$)

It can easily be shown (see below sec. 7.2) that notably a deflation with a ***chain*** Fisher price index (in contrast to a ***direct*** Fisher price index) as deflator as recommended in the SNA can well violate this requirement of proportionality (given that there are at least three links multiplied to a chain).

Case 12 (Reflection of differential quantity movement [RQM] if prices change uniformly)

RQM: when all prices change at the same rate, the change of volumes should equal the change of quantities. Under such conditions (i.e. when prices change uniformly) dynamics and structure of volumes should be equal to dynamics and structure of quantities.

Example 5.2.1

	p_{i0}	q_{i0}
A_1	30	70
A_2	50	30
B_1	90	20
B_2	120	30
Σ		150

To study the cases 12 and 22 we make the following assumptions

	prices/quantities case 12		prices/quantities case 22		price-/quantity relat. case 22	
A1	45	84	30	84	1.0	1.2
A2	75	48	40	48	0.8	1.6
B1	135	36	81	36	0.9	1.8
B2	180	24	168	24	1.4	0.8
prices	uniform change		non-uniform change			
quant.	non-uniform change		non-uniform change			

In the case of a uniform change in prices by $\lambda = 1.5$ (case 12) and quantities changing at different rates (from 0.8 to 1.8) price indices P^L and P^P and consequently also P^F are necessarily equal ($P^L = P^P = P^F = 1.5$). Hence in case 12 deflations by P^P and P^F will yield the same result:

	value	V_{0t}	$\Sigma p_t q_0$	**volume** $\Sigma p_0 q_t$	volume structure
A	7380	2.05	5400	4920	0.446
B	9180	1.70	8100	6120	0.554
sum	16560	1.84	13500	11040	1

Comparison of volumes and quantities reveals that the rise (from base period 0 to t) of volume by 22.67% is no longer equal to the rise of quantity which was 28%.

	volumes			quantities		
	structure (time 0)	structure (time t)	Q^L_{0t}	structure (time 0)	structure (time t)	M_{0t}
A	0.4	0.446	1.3667 = 2.05/1.5	0.667	0.6875	1.32
B	0.6	0.554	1.1333 = 1.7/1.5	0.333	0.3125	1.20
			1.2267 = 1.84/1.5			1.28

The group of commodities for which this rise of quantities was above average (that is group A) also experienced an increase in volume above average and therefore a higher share with respect to volumes (0.446 > 0.4) as well as quantities. This is in line with the requirement **RQM**. ♦

(5.2.1) $Q^L_{0t} = \Sigma x_i w_i$ where $x_i = q_{it}/q_{i0}$ and $w_i = p_{i0}q_{i0}/\Sigma p_{i0}q_{i0}$

(5.2.2) $M_{0t} = \Sigma x_i v_i$ where $v_i = q_{i0}/\Sigma q_{i0}$.

The difference between volumes and quantities is a result of the structure of base period prices. The same increase in quantity will be weighted higher if a high priced good is involved and lower in the case of a low priced good (prices of the base period).[31]

Case 22 (Pure quantity comparison [PQC] when prices change differently)

PQC (pure quantity comparison) requires the movement of volumes to be reflective of changes in quantities irrespective of how prices changed (uniform or non-uniform). This is equivalent to **linearity (additivity) in quantities*** q_{it}, or to the criterion of **structural consistency of volumes** (**SCV**), often called "additivity"** too.

* of the volumes and the quantity index resulting from deflation

** or **A2** in **fig. 5.2.2**

This rather restrictive requirement can be met by using the *same* single set of prices (say p_{i0} as in the case of the traditional deflation with P^P_{0t}).

[31] Prices p_{i0} (acting as constant weights) will also play a part when we try to justify that a "volume" can reasonably be taken as proxy for "total quantity".

It appears justified to regard volume as a measure of the aggregated "quantity" just *because* the sums $\Sigma q_{it}p_{it}$ and $\Sigma q_{it}p_{i0}$ are different only due to different prices, p_{it} and p_{i0} respectively and each element in the sequence $\Sigma q_{i0}p_{i0}$, $\Sigma q_{i1}p_{i0}$, $\Sigma q_{i2}p_{i0}$, ... differs from the other ones only by the quantities.

Example 5.2.2 (= ex. 5.2.1 ctd)

	value	V_{0t}	P^P	volume P^P	volume P^{F*}
A	4400	1.233	0.9024	4920	4881.7
B	6948	1.287	1.1353	6120	5871.7
sum	11388	1.265	1.0315	11040	10658.6**

* (deflation with Fisher index) for purpose of comparison only (see ex. 5.51 for more details)
** obtained by deflating 11388 with the overall Fisher price index, but 4881.7 + 5871.7 = 10753.4 ≠ 10658.6.

It should be noticed that with respect to the *quantity* movement this case (22) does not differ from the preceding case 12. Therefore, the results of deflation using Paasche price indices remain the same (indicated by shadows). But this is not true for deflation with a Fisher price index. Moreover the example shows that Fisher deflation violates **structural consistency** (**SCV**) requiring the sum of (separately) deflated sub-aggregates to equal deflated total-aggregate. ♦

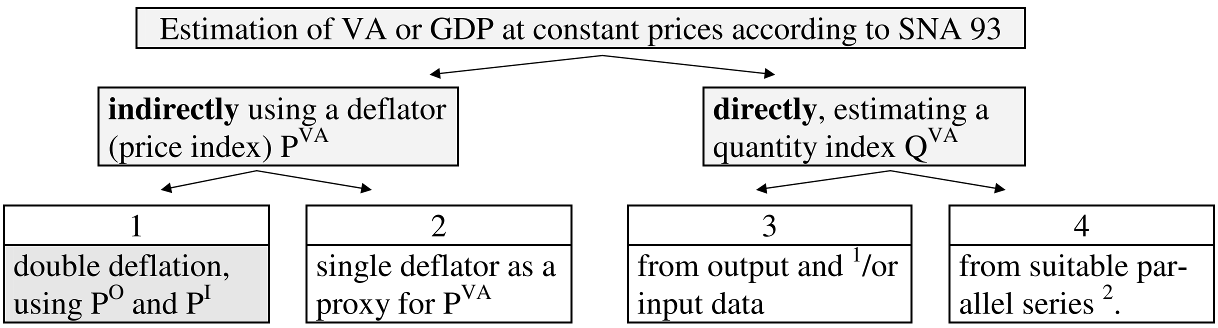
(5.2.3) $$Q^F_{0t} = Q^L_{0t}\sqrt{1+r_{ab}V_aV_b}$$

Table 5.2.1: Three deflators in the light of criteria for good deflation (in volume terms)

Criterion	P^P_{0t}	P^F_{0t}	$\overline{P}^{FC}_{0t}$
quantities change uniformly → volume are **proportional in the quantities** quantities change differently, prices uniformly → **volumes reflect quantity movement** [**RQM**]	yes	yes	no
quantities and prices change differently → volumes **linear in** the **quantities** q_{it} (PQC = SVC)	yes	no	no

b) Double (indirect) deflation method

Figure 5.2.1: Alternative estimations of value added (VA) or GDP at constant prices as recommended in the rev. SNA 1993



1 the case of a double indicator method
2 like wages, consumption of raw material and energy etc

Let the symbol O(s,k) denote output with quantities relating to time s and prices related to time k. Input I and value added Y will receive equivalent symbols. Then we have by definition

(5.2.4) $Y(t,t) = O(t,t) - I(t,t)$

and we define value added at constant prices Y(t,0) as follows

(5.2.4a) $Y(t,0) = O(t,0) - I(t,0)$. (5.2.5) $P_{0t}^{impl}(Y) = \frac{Y(t,t)}{Y(t,0)}$.

(5.2.6) $Y(t,0) = \frac{O(t,t)}{P_{0t}^{P}(O)} - \frac{I(t,t)}{P_{0t}^{P}(I)} = O(t,0) - I(t,0)$.

(5.2.7) $P_{0t}^{imp}(Y) = \frac{Y(t,t)}{Y(t,0)} = \frac{P_{0t}^{P}(O) \cdot P_{0t}^{P}(I) \cdot (1-i)}{P_{0t}^{P}(I) - i \cdot P_{0t}^{P}(O)}$, where $i = \frac{I(t,t)}{0(t,t)}$.

(5.2.8) $\frac{1}{P_{0t}^{P}(O)} = i\frac{1}{P_{0t}^{P}(I)} + (1-i)\frac{1}{P_{0t}^{imp}(Y)}$. Hence

> The output - deflator $P^P(O)$ can be regarded as a weighted harmonic mean of the input deflator $P^P(I)$ [both indices P(O) and P(I) of Paasche type], and the implicit value added deflator $P^{imp}(Y)$, the weights being the quotas i and (1-i) respectively.

$P_{0t}^{imp(F)}(Y)$ and $\frac{1}{P_{0t}^{P}(O)} = i\frac{1}{P_{0t}^{P}(I)}R_1 + (1-i)\frac{1}{P_{0t}^{imp(F)}(Y)}R_2$

where $R_2^2 = P_{0t}^{P}(O) / P_{0t}^{L}(O)$ and $R_1^2 = R_2^2\left[P_{0t}^{P}(I) / P_{0t}^{L}(I)\right]$,

c) Aggregative properties of index functions and deflation methods

Figure 5.2.1: The usage of the terms "additivity" in (price)[1)] index theory

the term "additivity" is used in connection with questions like

- how the index reacts to changes of individual prices at time (period) 0 and/or t (as function of prices)
  - **A1***: additivity (linearity) of the function [2)]
- how a global index can be decomposed (partitioned) into sector indices [3)], or the sector indices can be aggregated to a global index
  - **A1**: aggregative consistency of the index function, **ACF**
- whether the index used to deflate a value aggregate and value subaggregates will provide volumes that can be summed up like values
  - **A2**: structural consistency of volumes (in deflation), **SCV**

if linearity (additivity **A1***) is given → then also aggregative consistency **A1** (**ACF**)[*]

*) the converse is **not** true: index formulas, satisfying A1* will also satisfy A1 but not vice versa: see tab. 5.2.1

Table 5.2.2: Additivity and aggregative consistency[32]

additivity (**A1***)	aggregative consistency (**A1**) yes	aggregative consistency (**A1**) no
yes	Laspeyres index, Paasche, Marshall-Edgeworth index and many other indices	not possible
no	logarithmic Laspeyres index, quadratic mean index*)	Fisher's ideal index, formula of Drobisch

*) Examples for aggregative consistent index functions without being linear, but the converse is not possible.

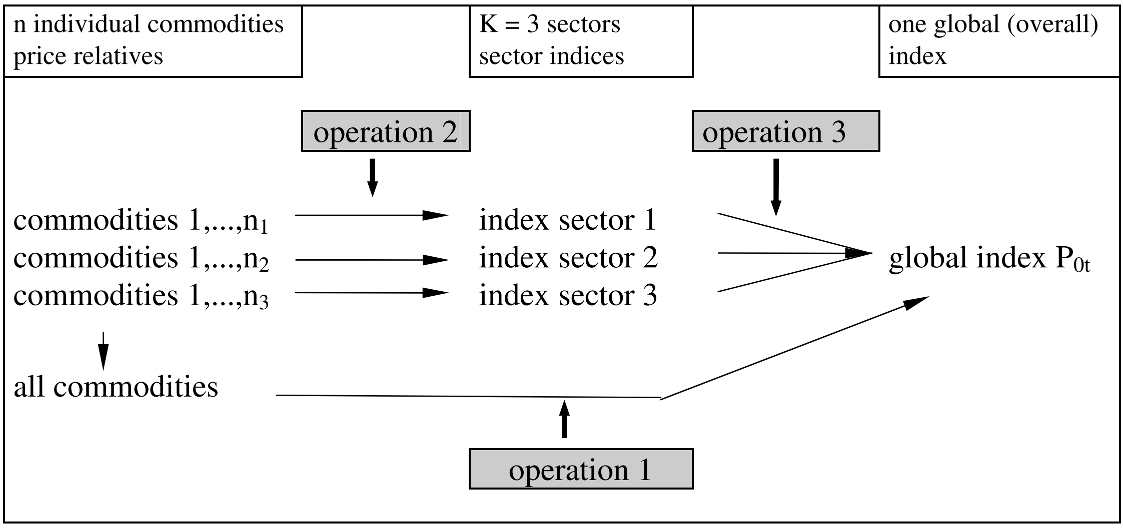
32 Whilst additivity (linearity, A1*) can be regarded as more restrictive a condition than aggregative consistency (ACF) the so called "equality test" (see part e of this section) is a weak version of ACF (A1).

d) Consistency in aggregation (aggregative consistency of the index function) (A1)

To introduce the idea of consistency in aggregation we consider n commodities that can be classified into K non-overlapping (distinct) and exhaustive (all n goods comprising, such that $n = n_1 + n_2 + ... + n_K$) groups or "sectors" (k = 1,...,K).:

> The function P_{0t} is called **aggregative consistent** (or consistent in aggregation) if the same formula (function f) can be used to derive
> 1. the overall (global) price index P_{0t} as a function of n price relatives $a^i_{0t} = p_{it}/p_{i0}$ (i = 1, ..., n), that is $P_{0t} = f(a^i_{0t})$ for all n commodities
> 2. the K sector indices $P^1_{0t},...,P^K_{0t}$ for the K subaggregates (sectors), calculated separately as functions f of $n_1,...,n_K$ price relatives (commodities) where $n = n_1 + ... + n_K$
> 3. the global price index P_{0t} as a function f of the K sector indices $P^1_{0t},...,P^K_{0t}$, such that $P_{0t} = f(P^k_{0t})$, k = 1,...,K.

Figure 5.2.2: The concept of aggregative consistency (K = 3 sectors)



(5.2.9) $$P^F_{0t} = \sqrt{\left(g_1 a^1_{0t} + g_2 a^2_{0t} + ... + g_n a^n_{0t}\right)\left(w_1 a^1_{0t} + w_2 a^2_{0t} + ... + w_n a^n_{0t}\right)},$$

(5.2.9a) $$\sqrt{\left(g_1 P^{F1}_{0t} + g_2 P^{F2}_{0t} + ... + g_K P^{FK}_{0t}\right)\left(w_1 P^{F1}_{0t} + w_2 P^{F2}_{0t} + ... + w_K P^{FK}_{0t}\right)}$$

with "sector Fisher indices" defined by $P^{Fk}_{0t} = \sqrt{P^{Lk}_{0t} P^{Pk}_{0t}}$, but rather by

(5.2.10) $$P^F_{0t} = \sqrt{\left(g_1 P^{L1}_{0t} + g_2 P^{L2}_{0t} + ... + g_K P^{LK}_{0t}\right)\left(w_1 P^{P1}_{0t} + w_2 P^{P2}_{0t} + ... + w_K P^{PK}_{0t}\right)}$$

where $P^{Fk}_{0t} = \sqrt{P^{Lk}_{0t} P^{Pk}_{0t}}$. Thus Fisher's index is *not* aggregative consistent (it also fails the even weaker "equality test"). This will be demonstrated in **ex 5.2.3**.

e) Equality test (ET)

The function P satisfies the equality test if

(5.2.14) $$P_{0t} = f(P^1_{0t}, P^2_{0t}, ..., P^K_{0t}) = f(\lambda, \lambda, ..., \lambda) = \lambda \quad \text{(equality test)}$$

Given all sector indices P^k_{0t} (k = 1,2, ... ,K) equal λ, so should the global index yield $P_{0t} = \lambda$. This seems to be a rather weak condition, such that

if aggregative consistency is given ⟶ then is also the equality test met

Example 5.2.4

Consider two commodities and weights $g_1 = 0.6$, thus $g_2 = 0.4$ to calculate P_{0t}^{L} using sectoral indices, and correspondingly weights $w_1 = 0.4$, $w_2 = 0.6$ to calculate P_{0t}^{P}. Then assume $P_{0t}^{L1} = 1.25$ and $P_{0t}^{P1} = 1.2$, and likewise $P_{0t}^{L2} = 2$ and $P_{0t}^{P2} = 0.75$ such that $P_{0t}^{F1} =$. $P_{0t}^{F2} = \sqrt{1.5}$ But the global index P_{0t}^{F} is not $\sqrt{1.5}$, required by the equality test but rather $\sqrt{1.55 \cdot 0.93} = \sqrt{1.4415}$, because the global indices are $P_{0t}^{L} = 1.55$ and $P_{0t}^{P} = 0.93$. Note that we obtain the global indices P_{0t}^{L}, P_{0t}^{P}, and thus also P_{0t}^{F} from the sectoral ones using a *weighted arithmetic* mean, whilst the sectoral indices P_{0t}^{F} are gained from the sectoral Laspeyres and Paasche indices by taking an *unweighted geometric* mean. This is the reason why P_{0t}^{F} is unable to meet the equality test.

f) Structural consistency of volumes (A2), a case for using Paasche indices in deflation

Let V_1, V_2, ... ,V_K denote *values* (aggregates at current prices) referring to *sub*-aggregate 1 to K, and V_T to the *total* (T) aggregate respectively, such that by definition

(5.2.15) $V_1 + V_2 + ... + V_K = \sum V_k = V_T$ (k = 1, 2, ... , K).

Each volume is defined by dividing a value by its corresponding price index (deflator), P_1, P_2, ... , P_K. To satisfy SCV the following equation has to hold for P_T, the "total deflator"

(5.2.16) $$\frac{V_1}{P_1} + ... + \frac{V_K}{P_K} = \frac{V_T}{P_T}.$$

Next consider value shares (or "weights") w_k to describe the fact that total value V_T is broken down into K subaggregates' values

(5.2.17) $$\frac{w_1 V_T}{P_1} + ... + \frac{w_K V_T}{P_K} = \frac{V_T}{P_T}, \text{ where } w_K = \frac{V_k}{V_T}$$

and after division of both sides of the equation by V_T we get.

(5.2.18) $$w_1 \frac{1}{P_1} + ... + w_K \frac{1}{P_K} = \frac{1}{P_T}$$

which simply means that P_T has to be a *weighted harmonic* mean of sectoral indices (deflators) with weights being value shares ($p_t q_t / \sum p_t q_t$) that is $w_k = V_k/V_T$.

The only deflator price index capable of producing structurally consistent volumes SCV at all levels of aggregation is the Paasche index due to the fact that this index is based on a harmonic mean (of price relatives or sub-indices [sectoral deflators] respectively). P^P is *characterized* by SCV.

If price indices P_1, P_2, P_T are replaced by sectoral *Fisher* deflators- $P_K^F = \sqrt{P_K^L P_K^P}$ we get

(5.2.19) $$\frac{V_1}{P_1^P}\sqrt{\frac{P_1^P}{P_1^L}} + ... + \frac{V_K}{P_K^P}\sqrt{\frac{P_K^P}{P_K^L}} \neq \frac{V_T}{P_T^P}\sqrt{\frac{P_T^P}{P_T^L}},$$

with the result that the left hand side (LHS) will usually differ from the right hand side (RHS). Interestingly other properties called "reflection of quantity movement" (**RQM**, in the case of uniform price movement) and "pure quantity comparison" (**PQC**, in the case that prices change non-uniformly) are most closely related to structural consistency.

Given some base period values $V_k^B = \sum p_{k0} q_{k0}$ for any k = 1,...,K, as for example private consumption (V_1^B) or investment (V_2^B) at period 0 as a starting point it might be desirable to "update" these aggregates using suitable quantity indices Q_k, such that

(5.2.20) $$V_1^B Q_1 + ... + V_K^B Q_K = (V_1^B + V_K^B) Q_T.$$

The only total-aggregate (Q_T) quantity index permitting this type of consistent "updating" of base period (sub-aggregate) volumes $\sum p_0 q_0$ to current period volumes $\sum p_0 q_t$ needs to be an *arithmetic* mean of Q_1, Q_2, ... with weights g_k, hence a *Laspeyres quantity index* Q_{0t}^L. Thus

the harmonic mean in P corresponds to an arithmetic mean in Q, such that we get the pair P^P, Q^L. In our view it seems much more reasonable to seek a pair of indices, P and Q such that P is aggregative consistent (SCV), and Q is linear in the quantities (PQC) than to require factor reversibility.

Note: The resulting (from deflation by P_{0t}^F) index Q_{0t}^F is at least *proportional* in the quantities, while $\overline{Q}_{0t}^{FC}$ is unable to meet even this criterion. $P_{0t}^P\, Q_{0t}^L$ (see below sec. 7.2).

The sequence of (direct) Paasche and (direct) Fisher deflation volumes is given by

t	Paasche	Fisher
1	$\sum p_0 q_1 = Q_{01}^L \cdot \sum p_0 q_0$	$\sum p_0 q_1 \left[\left(\frac{\sum p_1 q_1}{\sum p_1 q_0} \frac{\sum p_0 q_0}{\sum p_0 q_1} \right)^{1/2} \right] = \sum p_0 q_2 \sqrt{\frac{Q_{01}^P}{Q_{01}^L}}$
2	$\sum p_0 q_2 = Q_{02}^L \cdot \sum p_0 q_0$	$\sum p_0 q_2 \left[\left(\frac{\sum p_2 q_2}{\sum p_2 q_0} \frac{\sum p_0 q_0}{\sum p_0 q_2} \right)^{1/2} \right] = \sum p_0 q_2 \sqrt{\frac{Q_{02}^P}{Q_{02}^L}}$

g) Some new results concerning aggregative consistency (ACF)

The advantage of aggregative consistent index functions (notion **A1** in **fig. 5.2.1**) is that the results of index compilations are independent of which aggregation levels are being distinguished and in which order aggregation is being carried out. Invariance with respect to the kind and sequence of aggregation is most attractive for analytical purposes in particular.

Example 5.2.5

i		p_{i0}	q_{i0}	p_{it}	q_{it}
1	Non durable goods	100	130	165	40
2	Durable goods	100	95	85	160
3	Services	100	40	60	120

Commodities 1 and 2 are combined in the sub-aggregate "goods" while commodity 3 constitutes a sub-aggregate of its own. It can easily be verified that the price index of Fisher amounts to $P_{0t}^{F(1)}$ = 1.016 calculated in one single stage using the three individual price relatives, whereas the same formula applied in two stages (price relatives → sub indices, and thereafter sub indices→ all item index) yields $P_{0t}^{F(2)}$ = 0.995.

The example is particularly striking as the results differ not only in amount but also in the sign of the growth rate, viz. + 1.6% as opposed to - 0.5%. ♦

Aggregative consistency (ACF) requires the one-stage (or direct) compilation to equal the two-stages (or indirect) compilation of an index number. This property is clearly violated in the case of PF. It has been found (von Auer 2004) for ACF to hold an index function to be expressible in terms of price relatives (r_i), and satisfy commensurability. Furthermore it turned out that most index functions are *able to be expressed in at least three different ways* in terms of price relatives $r_i = p_{it}/p_{io}$ and value (expenditure) shares $v_i^{st}/V^{st} = p_i^s q_i^t / \sum p_i^s q_i^t$, that is

(5.2.22) $P_{0t} = \tilde{P}(r, v^{00}, v^{0t})$ for example $P_{0t}^{L} = \sum \frac{p_t}{p_0} \frac{p_0 q_0}{\sum p_0 q_0} = \sum r_i \left(v^{00} / V^{00} \right)$

(5.2.23) $P_{0t} = \hat{P}(r, v^{tt}, v^{t0})$ or $\left(P_{0t}^{L}\right)^{-1} = \sum \frac{p_0}{p_t} \frac{p_t q_0}{\sum p_t q_0} = \sum \frac{1}{r_i} \frac{v^{t0}}{V^{t0}}$, and finally

(5.2.24) $P_{0t} = \overline{P}(r, v^{00}, v^{tt})$ in the case of P_{0t}^{L} identical to eq. 22.

Note that the Marshall-Edgeworth index P_{0t}^{ME} has the following r-v-representations

$\tilde{P}(r, v^{00}, v^{0t}) = \frac{\sum r_i (v_i^{00} + v_i^{0t})}{\sum (v_i^{00} + v_i^{0t})}$; $\hat{P}(r, v^{tt}, v^{t0}) = \frac{\sum (v_i^{tt} + v_i^{t0})}{\sum \frac{1}{r_i} (v_i^{tt} + v_i^{t0})}$, and $\overline{P}(r, v^{00}, v^{tt}) =$

$\sum (r_i v_i^{00} + v_i^{tt}) / \sum (v_i^{00} + v_i^{tt} / r_i)$. Though evidently all three representations are perfectly equivalent and one can easily be transformed into the other, this - most surprisingly - does not mean that an index function able to satisfy aggregative consistency in one specific r-v-representation will do so in another r-v-representation as well.

Table 5.2.4: Index formulas and their aggregative properties

Criterion (type of consistency)	equality of one- and multistage compilation of the ... form	Formulas satisfying the criterion
Strict consistency	all three forms	Laspeyres, Paasche
Specific consistency	only 5.2.24 ($\overline{P}$ form)	Stuvel, Vartia I, Walsh Vartia
Weak consistency	one or two forms* but not all three forms	Marshall-Edgeworth (here only eq. 22 [$\tilde{P}$] applies)
No consistency whatsoever	none of the formulas 5.2.22 - 5.2.24 applies	Banerjee (BA2), Drobisch, Fisher ,Törnquist, Theil, Vartia II, Walsh (or Walsh 1), Walsh 2

Note: Conspicuously precisely those formulas which are generally worshipped as "superlative" perform remarkably bad. This applies in particular to the formulas of Fisher, Törnquist and Walsh

5.3. Harmonization of deflation methodology in Europe

In general *"indirect" methods using "deflator" price* indices are preferred to "direct" methods of extrapolating quantities (see **fig. 5.1.2**) as price indices are more readily available and subject to less variance than (direct) quantity indicators. However, in quite a few instances (esp. non-market services in education, administration and health) the "direct" method of (directly) observing "quantites" was preferred. Another feature of the procedures laid down to harmonize volume measurement in Europe is the distinction made between three types of methods

A methods:	most appropriate methods
B methods:	methods which can be used when A methods cannot be applied, and
C methods:	methods which shall not be used.

For a method to be eligible as an A method the following conditions should be met:

- coverage should be complete with regard to the aggregate in question,
- quality change should be taken into account,
- consistency with definitions and equations of national accounts has to be ensured, and finally especially in the case of output-deflation
- producers' prices (= basic prices) rather than purchasers' prices or input cost should be used.

Decisions on some of the more complicated problems of deflation methodology such as deflation of certain non-market outputs were initially postponed until results of a special *research programme* will be available.

Table 5.3.1: Selected problems of output deflation by product

Type of output	Major problems to be solved for an appropriate deflation	Recommended method of deflation in the HVM (Handbook Vol. Measurem. (EU)
Agriculture fishing, forestry	Production process spread over more than one accounting period; seasonality and complexity of subsidies	Suitable price indices (PPIs) are often available, quality has to be taken into account
Mining and quarrying	Less problems with insufficient availability of data and heterogeneity	Pure quantity data (direct approach), but also unit value indices acceptable
Manufacturing	Diffic. definition of quality for products like fashion goods or high-tech goods. STS-Regulation [1] requires detailed monthly statistics on production and prices. Only few data on capital goods [2]	Quantity indicators if available or price indices (sometimes difficulties with large equipment goods; model or specification pricing)[3]. Problems with rapid productivity growth (if input indicators are used)
Computers and related equipment	Quality adjustment is no small task, due to the rapid progress in technology in this field in particular	Hedonic method recommended, judgemental approach possible but considered much less appropriate
Energy, gas, water etc.	Price discrimination (e.g. bulk users pay less than private customers), division between producer and distributor	Deflation should treat production and distribution (service) differently; there are many vertically integrated firms
Construction	Long duration, uniqueness of projects; more input than output data available	Input price indices should be abandoned in favour of true output price indices
Wholesale and retail trade services	Trade margin is considered the equivalent of the value of trade services. Value and volume of services is often assumed to correlate with sales [4]. Price differences between outlets cannot be taken as indication of differences in the quality of service provided.	Double deflation of sales and purchases in constant prices if two separate indices are available. changes in the quality of services should be taken into account (explicit adjustments still are rare to date); problems with appearance of new trade channels (e-commerce)
Hotels and restaurants	Wide coverage of extremely different standards: discounts for block booking and many small producers	Price indicators as part of the CPI available yet efforts should be made to directly measure output [5]
Transport, communication	Complicated system of prices (one-off-, season-, group tickets) giving different rights [6] under varying conditions. Discounts + subsidies common [7]. Volume indicators more available than price ind.	Output as number of tickets of a specified type multiplied by price of each ticket in the base period. Appropriate PPIs are A-method; volume indicators are B-method.
Postal and telecommunication services	Genuine PPIs available in many countries only recently. Different prices according to quality differences + different packages of services ("bundles" incompar.)	CPI adjusted to basic prices or PPIs A-method (provided that quality adjustment is made). Volume indicators comprising the full range of services are B-methods
Financial intermediation	Management of accounts, loans, savings etc. directly charged, or indir. measured [8] central bank output best measured by inputs	These services are not well covered by PPIs. Suitability of various volume indicators currently explored.
Insurances and pension funds	Direct volume measurement and deflator of gross premiums (part of CPI) inappropriate. Bundling of products (various insurances) is very common	No A-method available, output indicators such as the number of acquisitions, policies etc. is considered a B-method

Real estate, renting and busines service	If separate prices exist for charged for services, deflation by a PPI is A- meth. [9], often fees are related to property prices. Mostly knowledge-based services. For letting of residential buildings CPI information. For owner occupied dwelling [10] rental equivalent approach acceptable	While output price indices for real estate services are rarely available prices of the corresponding assets will be eligible as B-methods Acquisition price of new dwellings is explicitly downgraded to be a C-method, though often regarded as a solution of the owner occupation problem (!)
Computer and related services	Bundling computer software with hardware and consultancy common. For standard (non-customised) and package software PPI data exist. Other software and consultancy service is mostly unique [11].	Hedonic methods are widely used in this field. Model pricing of well defined services could be A-method. Avoid simply making use of foreign (US) indices relating to comparable products.
Research and development	Output by nature unique. Model pricing not applicable [12], sometimes charge-out rates + hourly fees available (B-meth.)	Output data such as number of patents, publications, diplomas, citations and the like (surprisingly) are deemed acceptable
Other business services[13]	"Representative pricing" applied to a list of products (contracts, tax returns) would be A-method. Problems with ensuring constant quality (standards). A most heterogeneous group of services with most different possible indicators of output.	Crude volume measures such as the number of tax returns, or projects (in the case of technical consultancy), or the amount of advertising space as well as rates, fees etc. are unsatisfactory, due to the rapid productivity changes in this field
Public administration, education health and social work	Weighted (based on costs) sum of output indicators (relatives). Due to increasing interest in the efficiency of government more and more indicators of output, quality and productivity will be available	Output indicators (if quality is accounted for and closely related to the kind of administrative work) are considered A-method Input methods are B only (even if broken down into sufficient details)
Recreational, cultural and sporting services	Comprising widely different activities such as services provided to an audience, libraries, gambling, films, radio and TV, news agencies and the like	In many cases number of tickets sold by type of event is a good output indicator. Some activities are encompassed in CPI, e.g. personal services (hairdressing etc.)

1) Short Term Statistics in the European Union, Commission Regulation 1165/98.
2) like airplanes, ships etc. Explicitly excluded in the STS Regulation.
3) See **sec. 6.3** for problems with unique goods where making pure price comparisons calls for these above-mentioned methods of "pricing".
4) A most widely used method implies that the volume of margins follows the volume of sales and assumes that no quality change in the services takes place.
5) Crude volume data (e.g. number of clients) and input data for deflation are considered C methods however.
6) e.g. to travel in a defined area or within a limited time period only.
7) In air traffic e.g. depending on flexibility, or time before the date of travel, time of travel (peak, peak-off), destinations etc. Such factors are regarded as indicating different quality.
8) This is the case called FISIM= Financial Intermediation Service Indirectly Measured, an item in the SNA which causes considerable problems as regards volume measurement. Value is assumed to correspond to the difference between loans' and deposits' interest rate.
9) This applies also to services auxiliary to financial intermediation.
10) See **sec. 6.2** for more details concerning OOD. Classification of dwellings should be sufficiently detailed (floor area, number of rooms, location etc.).
11) See **sec. 6.3** for a discussion of unique products.
12) There is of course no price for achieving an already known research result again. On the other hand fuitile R&D activity without a useful result still should be regarded as a kind of "output".
13) Such as legal services, (advices etc.), accounting, tax consultancy, management, architectural, engineering and related technical consultancy, labor recruitment, advertising, cleaning, security services etc.. Some services are basically of a routine nature (for example certain book-keeping work) other services are tailor-made and therefore by nature unique. In the case of management services there is no "market" for this kind of work and thus a deflation of inputs acceptable.

5.4. Fisher's ideal index far from being ideal

It is difficult to disentangle the complex of influences that make a Fisher index rise or decline. It is not only a change of price (quantities) to which P^F (or Q^F) will react, and in the case of *chain* Fisher indices situations will be even more arcane. On the other hand, monetary policy as well as economic analysis in general, aims at interpretations in terms of "goals" and "controls". Thus there is a need for empirical figures that can be viewed as effects of some instruments or used as instruments themselves[33]. It is also desirable for an index to be understandable and accepted by the general public.

Figure 5.4.1: Fisher's ideal index far from being ideal

Some shortcomings of Fisher's ideal index

interpretation	use for deflation	other properties
No weighted mean of relatives -, nor a ratio of expenditures [1)] interpretation	no structural consistency (equations for values will not hold for volumes)[2)]	not consistent in aggregation, not even weak equality test is met [3)]
only "opaque" interpretation of being a geometric mean of P^L and P^P	resulting quantity index Q^F is not linear (additive) in the quantities	more difficulties in compiling (compared with fixed base indices) [4)]

1) changing cost of a certain "budget"
2) aggregation of volumes follows the same rules as aggregation of values.
3) no simple function exists by which sectoral indices of P^F-type can be aggregated to a total P^F-index.
4) applies to situations only where hitherto Paasche- or Fisher-indices were uncommon (e.g. as CPIs)

According to Pfouts 1966, an index should answer a specific question, but "the purpose of the ideal index is at best opaque". To interpret P^F there is not much left apart from saying, it is the geometric mean of P^L and P^P. No weighted mean of price (quantity) relatives interpretation can be provided. There is no possibility to interpret P^F as expenditure for a certain (not necessarily constant) given basket of commodities either. Nor is P^F additive (in all connotations distinguished in **sec 5.2**).

According to Stuvel 1989, p. 49 ff. Fisher's index is also unable to pass the (compared to ET) even weaker "Withdrawal-and-entry test" (WT). In general a test becomes "weaker" as additional conditions are introduced in its description. We arrive from ET to WT assuming a sub-aggregate is consisting of only one single commodity. Once *all* subaggregates consist of one commodity only we get proportionality (P), making the assumption of identical price relativesλ. And as $\lambda = 1$ an even more special condition is set: identity (I). Hence[34]

ET $\rightarrow$ WT $\rightarrow$ P $\rightarrow$ I	from left to right tests are becoming weaker

Fisher's index satisfies only the last (i.e. rightmost) two criteria, P and I, Paasche and Laspeyres indices *all* of them, and chain indices *none* (!!).

There is a fundamentally different philosophy underlying this SNA-recommendation as compared with ours: The emphasis of SNA was *not* put *on* structural consistency of volumes nor on permitting a consistent update of volumes, *but rather on* factor reversibility, or the desire to avoid constant prices p_{i0} of the past in defining "volumes". In our view, however, criteria like

- comparability, consistency requirements, interpretation of volumes in terms of "quantities", and isolation of a pure "quantity", or pure "price" component should come first, whereas
- criteria like reversibility, symmetry, updating of weights should come second, however,

the SNA has decided to take precisely the opposite order of preferences.

33 as for instance the volume of money, as measured by M_1, M_2, M_3 and so on.

34 It is possible to conclude in this order: an index to which WT applies will also meet P and I. The converse order of conclusion is not permitted.

A sequence of Paasche deflation volumes is given by

(5.5.3) $\sum p_0q_2$, $\sum p_0q_3$,... in contrast to the sequence of Fisher deflation volumes

(5.5.4) $$\sum p_0q_2\left[\left(\frac{\sum p_2q_2}{\sum p_2q_0}\frac{\sum p_0q_0}{\sum p_0q_2}\right)^{1/2}\right], \sum p_0q_3\left[\left(\frac{\sum p_3q_3}{\sum p_3q_0}\frac{\sum p_0q_0}{\sum p_0q_3}\right)^{1/2}\right], \ldots, \text{ or}$$

(5.5.4a) $$\sum p_0q_2\sqrt{Q^P_{02}/Q^L_{02}},\ \sum p_0q_3\sqrt{Q^P_{03}/Q^L_{03}}$$

which does not appear to be easy to understand.

Chapter 6 Price indices and unit value indices in official statistics

6.1. The Consumer Price Index (CPI) and the Harmonized Index of Consumer Prices (HICP) in Europe

The European Union (EU) and the European Monetary Union (EMU) gave rise to

- a harmonization of the Member Countries' CPIs, and
- to establish a Harmonized Index of Consumer Prices (HICP) aggregated over a group of countries such as the EU or the EMU Member Countries .

Such indices are necessary in order to provide a constant monitoring of social and economic performance across the member states and to assess the convergence progress (with respect to possible future EMU Member States[35]), and to effectively combat inflation by the European Central Bank (ECB). Methods should be harmonized while institutional conditions and practices in economic relations of different countries will in general remain different to some extent, as for example:

- supply and "purchase" of financial services, especially with respect to insurances,
- organization of health and educational services, which in some countries is dominated by private institutions, in other countries it is a domain of governmental activities,
- practices of paying for governmental services by fees, taxes and the like and provision of local facilities and infrastructure elements, such as transport, energy, communication
- private or semi-governmental social and welfare institutions

As by the end of 1991 the so called "Maastricht Treaty" laid down certain "convergence criteria", among them also the inflation rate, 9t became necessary to make national CPIs more comparable. A Working Party (WP) of delegates of NSIs of all Member Countries and observers of European institutions was established (first meeting in June 1993) to promote work on CPI harmonization. Proposals of the WP were to be approved by the Statistical Programme Committee (SPC) of the Directors of EU Member Countries' NSIs. The WP established Task Forces to deal with details of CPI methodology.

The Consumption concept of the HICP is known as the **household final monetary consumption expenditure (HFMCE)**. It covers consumption expenditure

- by private and institutional households irrespective of their nationality
- in monetary transaction on the economic territory of the Member State
- on goods and services used for the direct satisfaction of individual needs
- in one or both of the time periods being compared, and
- unlike the National Accounts deflator the HFMCE covers only that part of the consumption that is considered relevant to inflation measurement (imputed cost of owner occupied housing therefore still is excluded).

[35] The aggregation over the EMU countries of the Hcalled MUICP (see below).