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Unit Value Indices as Proxies for Price Indices

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Abstract

The unit value index (UVI) as compiled in Germany for exports and imports is compared with two other indices, viz. an index of Drobisch which unfortunately is likewise known as "unit value index" and the "normal" Laspeyres price index (PI) of exports and imports. The UVI may be viewed as a Paasche index compiled in two stages where unit values instead of prices are used in the low level aggregation stage. Unit values are average prices referring to an aggregate of (more or less homogeneous) commodities. The focus of the paper is on the decomposition of the discrepancy between UVIs and PIs (the "unit value bias") into a (well known) Laspeyres (or substitution) effect or "L-effect" and a structural component or "S-effect" due to substituting unit values for prices. It is shown that amount and sign of S depends on the correlation between the change of quantities of those goods that are included in the aggregate and their respective base period prices. By contrast to L the correlation between quantity and price movement is not relevant for S.
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JEL: C43, C80, E01, F10

1. Introduction

Only few countries (among which Germany and Japan) are able to provide on a monthly basis both, a unit value index (UVI) and a true price index (PI) for measuring the price development in export and import. This offers the opportunity to study empirically the impact of the methodological differences between these two indices (Silver (2007), Silver (2008), von der Lippe (2007b)).[1] These differences and in particular some considerable shortcomings of UVIs gave rise to concerns as they are internationally much more common and can be viewed only as an unsatisfactory surrogate of PIs.

The problem with UVIs is, however that the term is used for quite different indices. On the one hand there are indices actually compiled in official statistics as for example the German export and import[2] UVIs where unit values as a sort of average prices (for a *group* of goods) take the part prices of individual goods have in the case of a price index (which thus uses data on a much more disaggregated level). On the other hand the term UVI is also in use for an index that should preferably be called "Drobisch's index", and which is of theoretical interest only[3] because this index requires the calculation of a total unit value of all goods (and maybe also services) at two points in time, 0 (base period) and 1 (present period). Most of the literature to be found under the key word "unit value index" is dealing with the UVI in the sense of Drobisch's index. This applies for example to Balk 1994, 1998, 2005 and Diewert 1995, 2004.

Sec. 2 of the paper aims at making clear some properties of unit values and the difference between the above mentioned indices. In sec. 3 a decomposition of the "discrepancy" between a Paasche UVI and the "normal" Laspeyres PI is derived. It introduced two components of the discrepancy, a "Laspeyres" or substitution effect (henceforth "L-effect") and a "structural" or "S-effect" respectively. While the former is already well known and sufficiently understood it was a challenge to give in sec. 3 and 4 an interpretation to the S-effect which is apparently closely related to the heterogeneity of the aggregate underlying the calculation of unit values. In sec. 4 a covariance is found as a determinant of the S-effect. Sec. 5 concludes. In the annex we give some information concerning the German official statistics as well as our empirical study.

2. Unit value index and Drobisch's index

2.1. Definition and some properties of unit values

It is important to realize that unit values are defined only for several goods grouped together in a sub collection of goods defined by a classification of products (e.g. of commodities for production or for foreign trade statistics). The relevant unit of the classification is called "commodity number" (CN) and the unit value is a kind of average price of the n_k goods in the k^{th} CN (k = 1, …, K)

(1) $$\tilde{p}_{kt} = \frac{\sum_j p_{kjt} q_{kjt}}{\sum_j q_{kjt}} = \sum_{j=1}^{n_k} p_{kjt} \frac{q_{kjt}}{Q_{kt}} = \sum p_{kjt} m_{kjt} \quad \text{in periods } t = 0, 1$$

where the summation takes place over the j = 1, …, n_k ($n_k < n$) goods of a CN and refers to periods 0 (base period), or 1 (reference period) respectively. In general only in the case of a

[1] Some of the hypotheses examined in this research as well as conceptual and empirical differences between customs-based UVIs as opposed to survey-based price indices (PIs) are described in the annex.

[2] The method of a UVI is also quite common in the case of indices of wages or prices for certain services (air transport for example).

[3] Both indices are also quite different as regards their axiomatic performance.

commodity number (CN), like the k-th CN sums $Q_{k0} = \sum_{j=1}^{n_k} q_{kj0}$ or $Q_{kt} = \sum q_{kjt}$ of quantities have a meaningful interpretation. As a consequence of the definition a number of observations concerning unit values can be made:

1. If all n_k prices in t are equal $p_{kjt} = \bar{p}_{kt}$ $(\forall j = 1,\dots,n_k)$ the unit value coincides with the unweighted arithmetic mean irrespective of the quantities

   (1a) $\tilde{p}_{kt} = \bar{p}_{kt}$.

2. If all of quantities are equal $q_{kjt} = q_{kt}$ eq. 1a holds and also

   (2) $Q_{kt} = n_k q_{kt}$.

3. From eq. 1 follows that unit values violate proportionality. If all n_k individual prices change λ–fold ($p_{kj1} = \lambda p_{kj0}\ \forall j$) the unit value as a rule does not change λ–fold provided the quantity-structure coefficients m change

   (3) $\tilde{p}_{k1} = \sum \lambda p_{kj0} m_{kj1} = \lambda \sum p_{kj0} m_{kj1} \neq \lambda \tilde{p}_{k0} = \lambda \sum p_{kj0} m_{kj0}$ and due to

   (3a) $$\frac{\tilde{p}_{k1}}{\tilde{p}_{k0}} = \frac{Q_{k1}}{Q_{k1}} \sum_j \frac{p_{kj1}}{p_{kj0}} \left(\frac{p_{kj0} q_{kj1}}{\sum_j p_{kj0} q_{kj0}} \right) = \sum_j \frac{p_{kj1}}{p_{kj0}} \left(\frac{p_{kj0} m_{kj1}}{\sum_j p_{kj0} m_{kj0}} \right)$$

   the situation $p_{kj1} = \lambda p_{kj0}$ results in $\frac{\tilde{p}_{k1}}{\tilde{p}_{k0}} = \sum_j \lambda \left(\frac{p_{kj0} m_{kj1}}{\sum_j p_{kj0} m_{kj0}} \right) \neq \lambda$ because the weights (in brackets) do not add up to unity (unless $m_{kj1} = m_{kj0}$ for all k and j).[4]

4. From eq. 3a follows that the ratio of unit values $\tilde{p}_{k1} / \tilde{p}_{k0}$ is *not* a mean value of price relatives[5] p_{kj1}/p_{kj0} as the weights are $p_{kj0} m_{kj1} = p_{kj0} q_{kj1} / \tilde{p}_{k0} Q_{k1}$ and summing up to

   (4) $$\sum_j \frac{p_{kj0} q_{kj1}}{\tilde{p}_{k0} Q_{k1}} = \frac{Q_{k0}}{Q_{k1}} \cdot Q_{01}^{L(k)} = \frac{Q_{01}^{L(k)}}{\tilde{Q}_{01}^{k}} = S_{01}^{k}.$$

   where $Q_{0t}^{L(k)}$ is the Laspeyres quantity index of the k[th] CN. When no price changes within each CN we get

   (4a) $\tilde{p}_{k1} / \tilde{p}_{k0} = Q_{01}^{L(k)} / \tilde{Q}_{01}^{k} = S_{01}^{k}$ for each k instead of the general formula

   (4b) $\tilde{p}_{k1} / \tilde{p}_{k0} = V_{01}^{k} / \tilde{Q}_{01}^{k}$

   where $V_{01}^{k} = \sum_j p_{kj1} q_{kj1} / \sum_j p_{kj0} q_{kj0}$ the value ratio (index) of the k[th] CN

5. In a similar vein we conclude: if the quantity structure (m-coefficients) within each CN remains constant we get[6]

   (5) $$Q_{01}^{L(k)} = \frac{Q_{k1} \sum_j p_{kj0} m_{kj1}}{Q_{k0} \sum_j p_{kj0} m_{kj0}}$$ (using $m_{kj1} = m_{kj0}$)

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[4] As violation of proportionality implies identity (the special case of λ = 1) this means that unit values may indicate rising or declining prices although all prices remain constant.
[5] It therefore may also violate the mean value property. This also applies to Drobisch's index.
[6] Equation 5 is equivalent to the absence of the so called S-effect and will gain importance in section 3.3.

$$= \frac{Q_{k1}}{Q_{k0}} = \tilde{Q}_{01}^{k}.$$

6. Unit values violate commensurability which is due to the fact that Q_{kt} is affected from changes in the quantity units to which the price quotations refer. It can easily be seen what happens when the quantity to which prices of a good in the k-th CN, say i refer changes. Assume prices refer to pounds (in both periods 0 and 1) rather than to kilogram, then

   (6) $$\tilde{Q}_{01}^{*k} = \frac{Q_{k1}^{(i)} + 2q_{ki1}}{Q_{k0}^{(i)} + 2q_{ki0}} \neq \tilde{Q}_{01}^{k} = \frac{Q_{k1}^{(i)} + q_{ki1}}{Q_{k0}^{(i)} + q_{ki0}}$$

   where $Q_{kt}^{(i)}$ denotes the sum over the quantities of all goods in the CN except for i. Hence the $\tilde{p}_{k1}/\tilde{p}_{k0}$ does not remain unchanged due to the denominator $\tilde{Q}_{01}^{k}$ in eq. 4b.

2.2. Drobisch's index

The index defined by

(7) $$P_{01}^{UD} = \frac{\sum_k \sum_j p_{kj1} q_{kj1} \Big/ \sum_k \sum_j q_{kj1}}{\sum_k \sum_j p_{kj0} q_{kj0} \Big/ \sum_k \sum_j q_{kj0}} = \frac{Q_0}{Q_t} \frac{\sum_k \sum_j p_{kj1} q_{kj1}}{\sum_k \sum_j p_{kj0} q_{kj0}} = \frac{V_{01}}{Q_1/Q_0} = \frac{\tilde{p}_1}{\tilde{p}_0}$$

is unfortunately more often than not called "unit value index"[7] although it is quite different from an index defined by eq. 8 (the index PU^P instead of P^{UD}) which is also called "unit value index". To avoid confusion and this ambiguity the index P^{UD} should better be called "Drobisch's index" as it was being proposed by Drobisch (1871).[8]

It should be noted, however, that it is in general not possible - let alone meaningful - to summate over the quantities of all n = Σn_k commodities, as required in the compilation of "Drobisch's" index. Hence unlike the K terms O_{kt} the term $Q_t = \sum_k \sum_j q_{kjt} = \sum_k Q_{kt}$ that is Q_0 or Q_1 respectively is in general not defined. Drobisch's index therefore is interesting only from a theoretical point of view. It is *not* compiled in the practice of official statistics.[9]

Moreover the index P^{UD} can *not* be viewed as being aggregated over "low level" unit value ratios $\tilde{p}_{k1}/\tilde{p}_{k0}$ because

(7a) $$P_{01}^{UD} = \sum_k \frac{\tilde{p}_{k1}}{\tilde{p}_{k0}} \left(\frac{\tilde{p}_{k0} M_{k1}}{\sum_k \tilde{p}_{k0} M_{k0}} \right) \text{ where } M_{kt} = Q_{kt} \Big/ \sum_k Q_{kt} = Q_{kt}/Q_t$$

shows that Drobisch's index P^{UD} (unlike the unit value index PU^P introduced in sec. 2.3) is not a mean value of unit value ratios in the same way as the value index is not a mean of price relatives because the weights (in brackets) do not add up to unity (unless for all k holds M_{k1} =

[7] See also the contribution of Ludwig von Auer in this journal. It is a pity that due to this terminology the purely theoretic P^{UD} may easily be confounded with the "unit value index" PU^P as it is in actual fact compiled in practice and will be introduced shortly in sec. 2.3. Silver (2007, 2008) presents empirical findings concerning "unit value indices" which can only be PU^P indices and at the same time the formula of eq.7 (that is P^{UD}) as definition of the "unit value index".

[8] The label "Drobisch's index" is, however, uncommon which is possibly due to the fact that it is already in use for another index also advocated by Moritz Wilhelm Drobisch (1802 – 1894), viz. the *arithmetic* mean of a Laspeyres and a Paasche price index. For more details concerning his index P^{UD} (eq. 7) see also the contribution of von Auer who, however, does not mention the "unit value index" of official statistics, that eq. 8.

[9] The same applies to what might be called the corresponding "unit value" (or Drobisch's) *quantity* index defined by $\Sigma_i q_{i1}/\Sigma_i q_{i0}$ mentioned for example in the contribution of Diewert.

M_{k0}). Hence Drobisch's index not only reflects changes within CNs (via $\tilde{p}_{k1}/\tilde{p}_{k0}$) but also between CNs.

2.3. Unit value indices (UVI) and price indices (PIs) in official statistics

The "unit value" index as in actual fact calculated in official statistics of some countries differs from eq. 7 in that unit values are established only for CNs. There are no "total" or all-items unit values $\tilde{p}_1$ and $\tilde{p}_0$ involved i UVIs (as opposed to Drobisch's index).

UVIs are *necessarily* compiled *in two steps*, in the first unit values $\tilde{p}_{k1}$ and $\tilde{p}_{k0}$ (instead of prices) are calculated and in the second they - or ratios of them that is $\tilde{p}_{k1} / \tilde{p}_{k0}$ - are incorporated in the Paasche price index formula

(8) $$PU_{01}^{P} = \sum_k \frac{\tilde{p}_{k1}}{\tilde{p}_{k0}} \frac{\tilde{p}_{k0} Q_{k1}}{\sum_k \tilde{p}_{k0} Q_{k1}} = \frac{\sum_k \tilde{p}_{k1} Q_{k1}}{\sum_k \tilde{p}_{k0} Q_{k1}}.$$

In contrast to Drobisch's index, this index is evidently a weighted arithmetic average of unit value ratios $\tilde{p}_{k1}/\tilde{p}_{k0}$. There is of course no obvious reason why the Paasche formula should be preferred to the Laspeyres formula $PU_{01}^{L} = \sum \tilde{p}_{k1} Q_{k0} \big/ \sum \tilde{p}_{k0} Q_{k0}$ which would be equally useful.

In a unit value index for the measurement of *prices*, that is in PU indices quantities act as weights. It is also possible to measure the dynamics of *quantities* on the basis of sums of quantities Q_{kt} which then gives QU-indices and where unit values consequently take the part of weights. So for example

(8a) $$QU_{01}^{L} = \frac{\sum Q_{k1} \tilde{p}_{k0}}{\sum Q_{k0} \tilde{p}_{k0}}$$

is a unit value quantity index of the Laspeyres type.[10] Of the many possible variants of PU and QU indices respectively, in what follows we focus on two indices only, viz. PU_{01}^{P} and QU_{01}^{L}.

Unit value indices of the type PU may be viewed as two-stage or two-level index compilations where in the first (low) level use is made of unit values rather than prices. There are, however, some differences to the usual notion of "low level" aggregation which applies to situations in which no information about quantities is available, and therefore no weights can be established (unlike the upper level for which the introduction of weights is characteristic), Moreover in low-level aggregation prices usually are referring to the same commodity in different outlets. Here (and also in the case of using scanner data for the purposes of price statistics) quantities are known and unit values refer to different commodities grouped together by a classification.

In order to make unit value indices (UVIs) and the corresponding "true" price indices (PIs) comparable we make in what follows the assumption - unrealistic though[11] - that a price index is comprising all K CNs with all $n = \Sigma n_k$ commodities. We then get

[10] Note that this differs from the "unit value quantity index" (better Drobisch's quantity index) $\Sigma_k Q_{k1}/\Sigma_k Q_{k0}$ as mentioned in the preceding footnote.

[11] Strictly speaking the assumption is not justified, however, because price indices are based on a sample survey whereas unit value indices are resulting from a comprehensive customs statistics. This inaccuracy may be acceptable because our focus is on the formal aspects of the differences between the two types of indices. In addition to the coverage there are many more conceptual and methodological differences between UVIs and PIs for

(9) $$P_{0t}^{L} = \frac{\sum_{i=1}^{n} p_{it}q_{i0}}{\sum_{i=1}^{n} p_{i0}q_{i0}} = \frac{\sum_{k=1}^{K}\sum_{j=1}^{n_k} p_{kjt}q_{k0}}{\sum_{k=1}^{K}\sum_{j=1}^{n_k} p_{kj0}q_{kj0}} = \frac{\sum_{k}\sum_{j} p_{kjt}q_{kj0}}{\sum_{k} \tilde{p}_{k0}Q_{k0}}$$

for the Laspeyres *price* index.

2.4. Unit value index and Drobisch's index

From the observations concerning properties of (ratios of) unit values in sec. 2.1 and eq. 9 it easily follows that indices $U_{01} = U(\ldots)$ that is Drobisch's index P_{01}^{UD} and the unit value index (in our terminology) PU_{0t}^{P} (or PU_{0t}^{L}) have the following axiomatic properties in common:

a) axioms not satisfied

Proportionality (and identity by implication) $U(\mathbf{p}_0, \lambda\mathbf{p}_0, \mathbf{q}_0, \mathbf{q}_1) = \lambda$

Commensurability $U(\mathbf{\Lambda p}_0, \mathbf{\Lambda p}_1, \Lambda^{-1}\mathbf{q}_0, \Lambda^{-1}\mathbf{q}_1) = U(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1)$

Mean value property $\min(p_{i1}/p_{i0}) \leq U_{01} \leq \max(p_{i1}/p_{i0})$

b) axioms satisfied

Linear homogeneity $U(\mathbf{p}_0, \lambda\mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1) = \lambda\, U(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1)$

Additivity (in current period prices)
$U(\mathbf{p}_0, \mathbf{p}_1^{*}, \mathbf{q}_0, \mathbf{q}_1) = U(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1) + U(\mathbf{p}_0, \mathbf{p}_1^{+}, \mathbf{q}_0, \mathbf{q}_1)$ for $\mathbf{p}_1^{*} = \mathbf{p}_1 + \mathbf{p}_1^{+}$,

Additivity (in base period prices)
$[U(\mathbf{p}_0^{*}, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1)]^{-1} = [U(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1)]^{-1} + [U(\mathbf{p}_0^{+}, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1)]^{-1}$ for $\mathbf{p}_0^{*} = \mathbf{p}_0 + \mathbf{p}_0^{+}$

On the other hand there are also some significant differences.

1. while Drobisch's index meets transitivity the unit value index does not
2. as $P_{01}^{UP} \cdot \tilde{Q}_{01} = V_{01}$ (product test, or weak factor reversibility, see eq. 8) the factor (index) $\tilde{Q}_{01} = Q_1/Q_0 = \sum_k Q_{k1} \big/ \sum_k Q_{k0}$ is sometimes called "unit value quantity index" (better: Drobisch's quantity index)[12]; the corresponding relation concerning the PU and QU indices is given in the quite important eq. 10 below.
3. As to the time reversal test the product of $P_{01}^{UD} = \tilde{p}_1/\tilde{p}_0$ and $P_{10}^{UD} = \tilde{p}_0/\tilde{p}_1$ is unity and we have (similar to the "normal" price and quantity indices) $PU_{01}^{L}PU_{10}^{P} = PU_{01}^{P}PU_{10}^{L} = 1$.
4. As will be seen later it is possible that although $P_{01}^{L} = P_{01}^{P} = PU_{10}^{P} = 1$ holds[13] Drobisch's index P_{01}^{UD} may differ from unity because P_{01}^{UD} is affected from changes in the M_{kt} terms (eq. 7a) between (rather than within) CNs.

Such differences in the axiomatic properties reinforce once more the need of making a clear distinction between the two types of indices, Drobisch's index and the unit value index (e.g. of Paasche).

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example in German foreign trade statistics. In the annex we try to give an account of the differences and the empirical findings as regards their consequences.

[12] See footnote 9.

[13] That is a situation where no price changed and the *structure* of quantities *within* the K CNs remain constant (that is where both, L and S effect – introduces later – are absent) and y

3. Unit value index and price index

3.1. Decomposition of the discrepancy between unit value index and price index

The basis of the following decomposition is

(10) $$V_{01} = PU_{01}^{L} QU_{01}^{P} = PU_{01}^{P} QU_{01}^{L} = \sum p_1 q_1 \Big/ \sum p_0 q_0 \ ,$$

a relationship patterned after the well known identity

(10a) $$V_{01} = P_{01}^{L} Q_{01}^{P} = P_{01}^{P} Q_{01}^{L} \, .$$

In combination with the formula of Ladislaus von Bortkiewicz[14] for the covariance between price and quantity relatives weighted with expenditure shares $p_0q_0/\Sigma p_0q_0$

(11) $C = Q_{01}^{L}\left(P_{01}^{P} - P_{01}^{L}\right)$, due to the fact that

(11a) $$C = \sum_i \left(\frac{p_{i1}}{p_{i0}} - P_{01}^{L}\right)\left(\frac{q_{i1}}{q_{i0}} - Q_{01}^{L}\right)\frac{p_{i0}q_{i0}}{\sum p_{i0}q_{i0}} = V_{01} - Q_{01}^{L}P_{01}^{L} = Q_{01}^{L}P_{01}^{P} - Q_{01}^{L}P_{01}^{L}$$

using eq. 11 leads to the following multiplicative decomposition of the discrepancy D

(12) $$D = \frac{PU_{01}^{P}}{P_{01}^{L}} = \left(\frac{C}{Q_{01}^{L}P_{01}^{L}} + 1\right)\left(\frac{Q_{01}^{L}}{QU_{01}^{L}}\right) = \frac{P_{01}^{P}}{P_{01}^{L}} \cdot \frac{PU_{01}^{P}}{P_{01}^{P}} = L \cdot S \, .$$

D has two components or distinct "effects" which may work in the same or in opposite direction, so that they may be positively or negatively correlated.

The term L is referred to as Laspeyres- or simply ***L-effect*** reflecting the fact that $P^P \neq P^L$. The theorem of L. von Bortkiewicz in eq. 11a states in essence that it is the covariance C that determines sign and amount of the L-effect. A negative covariance ($P^P < P^L$) may arise from rational substitution among goods in response to price changes on a given (negatively sloped) demand curve. The less frequent case of a positive covariance is supposed to take place when the demand curve is shifting away from the origin (due to an increase of income for example).

L is since long a well known and well understood effect, much in contrast to the second component of the discrepancy which will henceforth be called structural component (or ***S-effect*** for short). It refers to changing quantities within a group of goods k = 1,…,K (for which unit values are established). S is related to the composition ("structure") of the CNs.

Both effects, L and S can be expressed in terms of quantity indices as well as in terms of price indices

(12a) $$L = \frac{C}{Q_{01}^{L}P_{01}^{L}} + 1 = \frac{Q_{01}^{P}}{Q_{01}^{L}} = \frac{P_{01}^{P}}{P_{01}^{L}}$$

(12b) $$S = \frac{Q_{01}^{L}}{QU_{01}^{L}} = \frac{PU_{01}^{P}}{P_{01}^{P}}$$

The distinction between L and S springs from the fact that it is difficult to compare P^L to PU^P directly. It is useful to divide the comparison into two parts: we compare P^L to P^P on the basis of L, and P^P to PU^P on the basis of S. In general both effects, S and L respectively, will coexist. It is also possible that either or both effects vanish (the latter situation is L = S = 1 and $PU^P = P^P = P^L$).

Table 1 displays various inequalities which can easily be inferred from a closer inspection of eqs. 12a and 12b. In quadrants I and III the effects S and L are working in the same direction

[14] This is a special case of the more general theorem of Bortkiewicz we are going to refer to in sec. 4.1.

(in which case we can combine two inequalities), generating thereby D > 1, or D < 1. By contrast in quadrants II and IV they take the opposite direction so that the sign of D-1 is indeterminate.

Table 1

	L < 1 (C < 0)	**L = 1** (C = 0)	**L > 1** (C > 0)
S > 1	**II:** D is indefinite	$PU^P > P^L = P^P$	**I:** $PU^P > P^P > P^L \Rightarrow$ **D > 1**
S = 1	$PU^P = P^P < P^L$	$PU^P = P^P = P^L$	$PU^P = P^P > P^L$
S < 1	**III:** $PU^P < P^P < P^L \Rightarrow$ **D < 1**	$PU^P < P^L = P^P$	**IV:** D is indefinite

Our empirical study revealed that the most frequently observed case is quadrant III where both effects are negative and reinforce each other to yield $PU^P < P^P < P^L$ (or equivalently $Q^P < Q^L < QU^L$).

3.2. How individual commodities contribute to the L-effect

It is useful to study the covariance (as the decisive term in L) broken down to the level of individual commodities i = 1, …, n. The formula

(13) $$L = \sum_{i=1}^{n} L_i = \sum_{i=1}^{n} \left(\frac{p_{i1}/p_{i0}}{P_{01}^L}\right)\left(\frac{q_{i1}/q_{i0}}{Q_{01}^L}\right)\frac{p_{i0}q_{i0}}{\sum p_{i0}q_{i0}} = 1 + \frac{C}{P_{01}^L Q_{01}^L}$$

where $C/P_{01}^L Q_{01}^L$ is a sort of a "centred" covariance (divided by the respective means), relates individual price and quantity relatives to L and thus shows how a single good contributes to a the L-effect.[15]

(13a) $$L_i = \left(\frac{p_{i1}/p_{i0}}{P_{01}^L}\,\frac{q_{i1}/q_{i0}}{Q_{01}^L}\right)\frac{p_{i0}q_{i0}}{\sum p_{i0}q_{i0}} = \left(\frac{p_{i1}}{P_{01}^L}\,\frac{q_{i1}}{Q_{01}^L}\right)\frac{1}{\sum p_{i0}q_{i0}}$$

follows that below average price relatives $p_{i1}/p_{i0} < P_{01}^L$ and/or below average quantity relatives $q_{i1}/q_{i0} < Q_{01}^L$ contribute to a "negative L-effect" (that is L - 1 < 0).[16] Moreover eq. 13 also shows that the L-effect will disappear (L = 1) when one or more of the following conditions apply:

- all price relatives are equal $p_{i1}/p_{i0} = P_{01}^L$ or unity (no price changes) $p_{i1}/p_{i0} = P_{01}^L = 1$ (in which case C = 0 because $P_{01}^L = P_{01}^P$)
- the same applies mutatis mutandis to quantity relatives (C also vanishes when $Q_{01}^L = Q_{01}^P$)
- the covariance C between price and quantity relatives disappears.[17]

We start our attempts to derive formulas for S in the next section by showing in quite the same manner under which conditions the S-effect will vanish (or equivalently S = 1).

3.2. How individual commodities contribute to the S-effect

A formula useful to explain the contribution of the k-th CN (not the i-th commodity) to S is

[15] In sec. 4.1 we try to find a similar equation in order to explain the S-effect.
[16] The "negative" effect is empirically more frequently observed.
[17] The theorem of Bortkiewicz shows that for the L-effect to exist it is essential that price and quantity relatives are correlated.

(14) $$S=\frac{Q_{01}^{L}}{QU_{01}^{L}}=\sum_k \frac{Q_{01}^{L(k)}}{\tilde{Q}_{01}^{k}}\cdot\frac{\tilde{Q}_{01}^{k}s_{k0}}{\sum_k \tilde{Q}_{01}^{k}s_{k0}}=\sum_k S_{01}^{k}\cdot\frac{\tilde{p}_{k0}Q_{k1}}{\sum_k \tilde{p}_{k0}Q_{k1}}$$

with expenditure shares $s_{k0}=Q_{k0}\tilde{p}_{k0}\big/\sum_k Q_{k0}\tilde{p}_{k0}=\sum_j p_{kj0}q_{kj0}\big/\sum_k\sum_j p_{kj0}q_{kj0}$ because

(14a) $$QU_{01}^{L}=\sum_k \tilde{Q}_{0}^{k}s_{k0}$$ and

(14b) $$Q_{01}^{L}=\sum_k Q_{01}^{L(k)}s_{k0}.$$

Weights equivalent to $\tilde{p}_{k0}Q_{k1}\big/\sum_k \tilde{p}_{k0}Q_{k1}$ in (14) are $\tilde{Q}_{01}^{k}v_0^k\big/\sum_k \tilde{Q}_{01}^{k}v_0^k$ where $v_0^k=\sum_j p_{kj0}q_{kj0}$

Our aim therefore will be to explain the the ratios $Q_{01}^{L(k)}\big/\tilde{Q}_{01}^{k}$ we encountered already in eq. 4, and which are reflecting the contributions of the K CNs to S. This will be done in sec. 4.1.

It should be noted, however, right at the outset that the structural effect owes its existence to the two-stage compilation of the unit-value index (UVI). If summation would take in one stage over the individual commodities (not grouped into CNs) the S-effect would disappear.[18] An equivalent condition is (for all k) $n_k = 1$ (or perfectly homogenous CNs), or $p_{kjt}=p_{kt}=\tilde{p}_{kt}$ $q_{kjt} = q_{kt} = Q_{kt}$, $m_{kj1} = m_{kj0} = 1$ yielding $PU_{01}^{P}=\frac{\sum_k \tilde{p}_{k1}Q_{k1}}{\sum_k \tilde{p}_{k0}Q_{k1}}=\frac{\sum_k p_{k1}q_{k1}}{\sum_k p_{k0}q_{k1}}=P_{01}^{P}$ using eq. 8.

The S-effect will also vanish (S = 1) if one or more of the following conditions is given

1. for all j = 1, …, n_k holds $m_{kj1} = m_{kj0}$ (no structural change within a CN), or
2. all n_k base period prices of a CN k are equal $p_{kj0}=\tilde{p}_{k0}\ \ \forall j=1,\dots,n_k$
3. all quantities change at the same rate λ so that $q_{kj1}/q_{kj0}=Q_{01}^{L(k)}=\tilde{Q}_{01}^{k}=\lambda$ for all j and k, or more specific, they remain constant (λ = 1).

Furthermore given 1 and 3, and constant prices, that is a situation without both, L and S effect and therefore $P_{01}^{L}=P_{01}^{P}=PU_{10}^{P}=1$ it is still possible that $P_{01}^{UD}\neq 1$ as above mentioned already.

Statement 1 follows from $S=PU_{01}^{P}\big/P_{01}^{P}$ (eq. 12b) and

(14c) $$PU_{01}^{P}=\sum_k P_{01}^{P(k)}\frac{Q_{k1}\sum_j p_{kj0}m_{kj1}}{\sum_k Q_{k1}\sum_j p_{kj0}m_{kj0}}$$ as compared to

$$P_{01}^{P}=\sum_k P_{01}^{P(k)}\frac{Q_{k1}\sum_j p_{kj0}m_{kj1}}{\sum_k Q_{k1}\sum_j p_{kj0}m_{kj1}}$$

so that assuming $m_{kj1} = m_{kj0}$ for all j and k gives $P_{0t}^{P}=PU_{0t}^{P}$ and thus S = 1. Likewise statement 1 can also be derived from (14) and (5) and from $S=Q_{0t}^{L}\big/QU_{0t}^{L}$ amounting to

[18] Unlike the L-effect the S effect only exists when commodities are grouped together in CNs. There can be no S-effect when there is no heterogeneity and/or structural change within the CNs. It appears therefore sensible to study the S-effect by examining the situation *within* the CNs.

(15) $$S = \frac{Q^L_{01}}{QU^L_{01}} = \frac{\sum_k Q_{k1} \sum_j m_{kj1} p_{kj0}}{\sum_k Q_{k1} \sum_j m_{kj0} p_{kj0}} = \frac{A}{B}$$

($Q^L_{01} = A/C$ and $QU^L_{01} = B/C$ have the same denominator $C = \sum_k Q_{k0} \sum_j m_{kj0} p_{kj0}$ and different numerators A and B respectively) such that $m_{kj1} = m_{kj0}$ entails A = B.

Statement 2 follows from the definitions of the terms $Q^{L(k)}_{01}$ and $\tilde{Q}^k_{01}$ used in eq. 14 and from

(16) $$\tilde{Q}^k_{01} = \sum_j \frac{q_{kj1}}{q_{kj0}} \frac{q_{kj0}}{\sum_j q_{kj0}} = \sum_j \frac{q_{kj1}}{q_{kj0}} \cdot m_{kj0} \text{ and}$$

(16a) $$Q^{L(k)}_{01} = \sum_j \frac{q_{kj1}}{q_{kj0}} \frac{q_{kj0} p_{kj0}}{\sum_j q_{kj0} p_{kj0}} = \sum_j \frac{q_{kj1}}{q_{kj0}} \cdot s_{kj0} .$$

where $s_{kj0} = q_{kj0} p_{kj0} / \sum q_{kj0} p_{kj0}$ and $m_{kj0} = q_{kj0} / \sum q_{kj0}$.

Equal prices in 0 lead to equality of quantity (m) and expenditure (s) weights $m_{kj0} = s_{kj0}$, or equivalently $Q^{L(k)}_{01} = \tilde{Q}^k_{01}$.

Comparing (16) and (16a) also shows that, what matters is the base period price structure. As

(17) $$\frac{s_{kj0}}{m_{kj0}} = \frac{p_{kj0}}{\tilde{p}_{k0}},$$

holds by definition a commodity j with an above average price $p_{kj0} > \tilde{p}_{k0}$ tends to contribute positively to the S-effect (or in other words, to $S = Q^L_{0t} / QU^L_{0t} > 1$), and correspondingly a below average price contributes negatively to the S-effect (or to S < 1).

Statement 3 is obvious as in this case $Q^{L(k)}_{01} = \tilde{Q}^k_{01} = \lambda$ so that S = 1. Using $S = PU^P_{0t} / P^P_{0t} = 1$, (7) and (10a) we see that under such restricted conditions the unit value index coincides with Drobisch's index $PU^P_{0t} = P^{UD}_{0t} = P^P_{0t}$.

Table 2 summarizes some special conditions under which no S effect or no L effect will arise.

Table 2

	L - effect	S - effect
perfectly homogeneous CNs (or $n_k = 1$)	not affected	vanishes: S = 1
all quantities within the CNs change at the same rate λ (also no quantity changes $\lambda = 1$ for all j and k)	vanishes: L = 1	S = 1
all prices change at the same rate ω (also no price changes $\omega = 1$ for all j and k)	L = 1	not affected $S = PU^P/P^P$ (if $\omega = 1$ $S = PU^P$)
constant structure of quantities within each CN ($m_{kjt} = m_{kj0}$)	not affected	S = 1
equal prices in 0 (all n_k prices p_{kj0} are equal)	not affected	S = 1

In a situation in which the L-effect is vanishing, for example when all prices rise at the same rate ω (or in particular $\omega = 1$) S specializes to $S = PU^P_{01} / \omega$, (or $S = PU^P_{01}$ respectively). Unlike the L-effect the S-effect is possible even though no price is changing.[19] The reason is that ac-

[19] Therefore in the example of sec. 4.3 $\omega = 1$ (L = 1) is assumed to demonstrate the S-effect taken in isolation.

cording to our observation 4 in sec. 2.1 $\tilde{p}_{k1}/\tilde{p}_{k0}$ may well differ from 1 although all individual prices remain constant.

On the other hand, when S vanishes, for example because all prices of a CN in 0 are equal, $L = Q^P_{01}/Q^L_{01}$ does not vanish but only reduces to $L = Q^P_{01}/\tilde{Q}_{01}$ (since in this case $Q^L_{01} = \tilde{Q}_{01}$).

4. Interpretation to the S-effect

4.1. A covariance expression for the S-effect

We now try to explain the K terms $S^k_{01} = Q^{L(k)}_{01}/\tilde{Q}^k_{01}$ in eq. 14, introducing K covariances between (the structure of) base period prices and quantity relatives. The "within-CN" indices $Q^{L(k)}_{01}$ and $\tilde{Q}^k_{01}$ are not only two different ways of measuring the development of quantities in the k^{th} CN, they are also *linear* quantity indices. We therefore can again make use of Bortkiewicz's reasoning. According to the *generalized* theorem of Bortkiewicz for two linear indices[20] the ratio X_1/X_0 of two linear indices

(18) $$X_1 = \frac{\sum x_1 y_1}{\sum x_0 y_1} \quad \text{and} \quad (18a) \quad X_0 = \frac{\sum x_1 y_0}{\sum x_0 y_0}$$

is given by $\frac{X_1}{X_0} = 1 + \frac{c_{xy}}{\overline{X} \cdot \overline{Y}}$ with the covariance

(19) $$c_{xy} = \sum \left(\frac{x_t}{x_0} - \overline{X} \right) \left(\frac{y_1}{y_0} - \overline{Y} \right) w_0 = \frac{\sum x_1 y_1}{\sum x_0 y_0} - \overline{X} \cdot \overline{Y}$$

and weights $w_0 = x_0 y_0 / \sum x_0 y_0$. The mean of the x_1/x_0 terms is with these weights $\overline{X} = X_0$, however, $\overline{Y} = \frac{\sum y_1 x_0}{\sum y_0 x_0} \neq X_1$.

Note that the theorem does not allow comparing any two indices for example

$X_1 = PU^P_{01} = \frac{\sum_k \tilde{p}_{k1} Q_{k1}}{\sum_k \tilde{p}_{k0} Q_{k1}}$ where $\tilde{p}_{k1} = y_1, \tilde{p}_{k0} = y_0, Q_{k1} = x_1$ cannot be compared with

$$P^{UD}_{01} = \frac{\sum_k \tilde{p}_{k1} \left(\frac{Q_{k1}}{Q_1} \right)}{\sum_k \tilde{p}_{k0} \left(\frac{Q_{k0}}{Q_0} \right)} = \frac{\sum_k \tilde{p}_{k1} M_{k1}}{\sum_k \tilde{p}_{k0} M_{k0}},$$

because this ratio cannot be written as a X_0 - term (according to eq. 18a) corresponding to X_1 as defined above.

To compare, however, the terms $X_1 = Q^{L(k)}_{01}$ and $X_0 = \tilde{Q}^k_{01}$ in $S^k_{01} = Q^{L(k)}_{01}/\tilde{Q}^k_{01}$ requires to make the assumptions $x_0 = q_0$, $x_1 = q_1$, $y_0 = 1$, $y_1 = p_0 w_0 = q_0/\Sigma q_0$ leading to $X_1 = Q^{L(k)}_{01}$, $\overline{X} = X_1 = X_0 = \tilde{Q}^k_{01}$ and $\overline{Y} = \tilde{p}_{k0}$. The resulting covariance then is

(20) $$c_k = \sum \left(\frac{q_{kj1}}{q_{kj0}} - \tilde{Q}^k_{01} \right) (p_{kj0} - \tilde{p}_{k0}) \frac{q_{kj0}}{\sum q_{kj0}} = \sum \left(\frac{q_{kj1}}{q_{kj0}} - \tilde{Q}^k_{01} \right) (p_{kj0} - \tilde{p}_{k0}) m_{kj0}$$

[20] See von der Lippe (2007), pp. 194 – 196. Eq. 11a is only the special case of $X_0 = P^L$ and $X_1 = P^P$.

$$= \frac{\sum_j q_{kj1} p_{kj0}}{\sum_j q_{kj0}} - \tilde{p}_{k0}\tilde{Q}^k_{01} = \tilde{p}_{k0}\left(Q^{L(k)}_{01} - \tilde{Q}^k_{01}\right) \qquad \text{(using (1)).}$$

It can easily be verified that in fact $\frac{X_1}{X_0} = 1 + \frac{c_k}{\overline{X} \cdot \overline{Y}} = 1 + \frac{\tilde{p}_{k0}\left(Q^{L(k)}_{01} - \tilde{Q}^k_{01}\right)}{\tilde{p}_{k0}\tilde{Q}^k_{0t}} = \frac{Q^{L(k)}_{01}}{\tilde{Q}^k_{01}} = S^k_{01}$.[21]

Eq. 20 tells us, that a commodity j tends to raise (lower) $S = Q^L_{01} / QU^L_{01}$ as a weighted sum of $S^k_{01} = Q^{L(k)}_{01} / \tilde{Q}^k_{01}$ whenever the covariance is positive (negative) and the commodity j has a non-negligible weight given by the share $m_{kj0} = q_{kj0}/\Sigma q_{kj0}$ of the total quantity at the base period. If quantities of goods with above average prices ($p_{kj0} > \tilde{p}_{k0}$) in the base period tend to rise to an extent below average ($q_{kjt} / q_{kj0} < \tilde{Q}^k_{0t}$) the covariance will be negative and S tends to be less than unity (in short: $c_k < 0 \rightarrow S < 1$). A negative covariance $c_k < 0$ also ensues from an above average rise of quantities of those goods where base period prices were below average. Correspondingly one may infer: $c_k > 0 \rightarrow S^k_{01} > 1 \rightarrow S > 1$.

Due to eq. 14 S^k_{01} and thereby the covariance c_k will contribute more or less to S depending on the somewhat hybrid weights $\tilde{p}_{k0} Q_{k1} / \sum_k \tilde{p}_{k0} Q_{k1}$.
Another way of defining X_t and X_0 ($x_0 = q_0$, $x_t = q_1$, $y_0 = p_0$, $y_t = 1$ $w_0 = p_0q_0/\Sigma p_0q_0$) leads to

(21) $$c^*_k = \sum \left(\frac{q_{kj1}}{q_{kj0}} - Q^{L(k)}_{01}\right)\left(\frac{1}{p_{kj0}} - \frac{1}{\tilde{p}_{k0}}\right)\frac{p_{kj0} q_{kj0}}{\sum p_{kj0} q_{kj0}}$$

$$= \frac{\sum_j q_{kjt}}{\sum_j p_{kj0} q_{kj0}} - Q^{L(k)}_{01} \cdot \frac{1}{\tilde{p}_{k0}} = \left(\tilde{p}_{k0}\right)^{-1}\left(\tilde{Q}^k_{01} - Q^{L(k)}_{01}\right)$$

using weights $s_{kj0} = p_{kj0}q_{kj0}/\Sigma\, p_{kj0}q_{kj0}$ rather than $m_{kj0} = q_{kj0}/\Sigma q_{kj0}$. However, c^*_k explains $1/S^k_{01}$ $= Q^{L(k)}_{01} / \tilde{Q}^k_{01}$ the k-th CN contribution to $S^{-1} = QU^L_{01} / Q^L_{01}$ instead of S, since

$$\frac{X_1}{X_0} = 1 + \frac{c^*_k}{\overline{X} \cdot \overline{Y}} = 1 + \frac{\left(\tilde{p}_{k0}\right)^{-1}\left(\tilde{Q}^k_{01} - Q^{L(k)}_{01}\right)}{\left(\tilde{p}_{k0}\right)^{-1} Q^{L(k)}_{01}} = \frac{\tilde{Q}^k_{01}}{Q^{L(k)}_{01}}.$$

S^{-1} is a weighted sum of these terms with weights given by $Q^{L(k)}_{0t} s_{k0} / \sum_k Q^{L(k)}_{0t} s_{k0}$ instead of $\tilde{p}_{k0} Q_{k1} / \sum_k \tilde{p}_{k0} Q_{k1} = \tilde{Q}^k_{01} s_{k0} / \sum_k \tilde{Q}^k_{01} s_{k0}$.

Both covariances have their specific merits and demerits. From eq. 20 and 21 follows

(22) $$\left(\tilde{p}_{k0}\right)^2 c^*_k = -c_k .$$

Thus the covariances necessarily have different signs. The covariance c_k is useful because it relates to S rather than S^{-1}, however, on the other hand c^*_k can more readily be compared to

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[21] It was only when I presented this paper at the Meeting of the Ottawa Group in Neuchâtel that I became aware of the fact that G. Párniczky (1974) had already mentioned c_k in his (largely unknown) paper dating back to 1974. Moreover, he did so with explicit reference to Bortkiewicz. However, he tried to explain Drobisch's index P^{UD} rather than PU^P. Also the combination of S and L-effect was not his concern. Unlike our exposition his was in need of making a distinction between "within-group" and "between-group" covariances. Finally we do not agree with his main result "that disaggregation in general is not likely to improve the accuracy of the unit value index" (he also used in the sense of Drobisch's index). This is clearly at odds with the conventional wisdom that splitting CNs into smaller (and thus more homogeneous) CNs will in general tend to reduce the S-effect.

the covariance C responsible for the L-effect, in which according to eq. 11a also use is made of weights s_{kj0} rather than m_{kj0}.

Table 3 provides a synopsis of all 2^3 = 8 possible situations concerning L and S and the covariances C (eq. 11a) and c_k^* (eq. 21)[22] depending on whether

1. quantity relatives are above ($q_{kjt}/q_{kj0} > Q_{0t}^{L(k)}$, labelled **QR +**), or below average (**QR -**)
2. price relatives are above ($p_{kjt}/p_{kj0} > P_{0t}^{L(k)}$,**PR +**), or below average (**PR -**)
3. base period prices are above ($p_{kj0} > \tilde{p}_{k0}$,**P_0 +**), or below average (**P_0 -**).

Our empirical study[23] reached the conclusion that L < 1 and S < 1 seems to be the most frequent combination. Situations in which this takes place are highlighted in table 3.

Table 3

L- effect and S- effect depending on two covariances

	price relatives **PR +**		price relatives **PR -**	
	P_0 +	**P_0 -**	**P_0 +**	**P_0 -**
QR +	$C > 0 \rightarrow$ **L > 1**		$C < 0 \rightarrow$ **L < 1**	$C < 0 \rightarrow$ **L < 1**
	$c_k^* < 0 \rightarrow$ **S > 1**	$c_k^* > 0 \rightarrow$ **S < 1**	$c_k^* < 0 \rightarrow$ **S > 1**	$c_k^* > 0 \rightarrow$ **S < 1**
QR -	$C < 0 \rightarrow$ **L < 1**	$C < 0 \rightarrow$ **L < 1**	$C > 0 \rightarrow$ **L > 1**	
	$c_k^* > 0 \rightarrow$ **S < 1**	$c_k^* < 0 \rightarrow$ **S > 1**	$c_k^* > 0 \rightarrow$ **S < 1**	$c_k^* < 0 \rightarrow$ **S > 1**

4.2. A simplified situation to study the determinants of S

Assume only two commodities in one CN only (so n_j = 1, K = 1, and we therefore simply drop the subscript k in what follows), equal quantity shares $m_{10} = m_{20}$ = ½ in the base period, p_{10} = p, and $p_{20} = \lambda p$. Further $\mu = m_{21}/m_{20} = m_{21}/0.5 = 2m_{21}$ measures the change in the quantity share of commodity 2 ($m_{11} = 1-\mu/2$ because $m_{21} = \mu/2$, and $0 \le \mu \le 2$).

In order to bring L ≠ 1 into the play prices have got to change and they should change at a different rate. So denote the price relative of good 1 by $\pi = p_{11}/p_{10}$ and let $p_{21}/p_{20} = \eta(p_{11}/p_{10}) = \eta\pi$ be the price relative of the second good. In order to study the special situation with no L-effect (L = 1) where prices remain constant simply assume $\eta = \pi = 1$.

In the general (L ≠ 1) as well as the special (L = 1) case we get $QU_{01}^P = QU_{01}^L = \tilde{Q}_{01}$. This result and $PU_{01}^P = PU_{01}^L$ is due to the fact that we assumed K = 1 (only one CN) in which case also Drobisch's index P_{01}^{UD} is equal to PU_{01}^P.

Table 4 (part a) summarizes the results.

Part b of table 4 shows that economically rational behaviour, that is the situations II and IV in which the unit value is declining ($\Delta < 0$ although prices remained constant) will also lead to a negative S-effect (S < 1). Once changing prices are considered L < 1 will also ensue from this kind of behaviour. It is therefore not surprising that empirical evidence seems to support the expectation that most frequently both "effects", L and S operate in the same direction and will predominantly be negative. It should be borne in mind, however, that the conclusions are de-

[22] For conditions concerning c_k (of eq. 20) you simply have to change > to < and vice versa.
[23] For details see the Annex.

rived only under most restrictive assumptions (no price changes, only one CN), and they may well be no longer tenable under more general conditions.

Table 4 (part a)

	general case	special case (no change of prices $\eta = \pi = 1$)
$\tilde{p}_0$	$(p/2)(1+\lambda)$	
$\tilde{p}_1$	$(p/2)\pi(2-\mu+\mu\eta\lambda)$	$(p/2)(2-\mu+\mu\lambda)$
$\tilde{p}_1 - \tilde{p}_0$	$=\Delta^* = (p/2)[\pi(2-\mu(1-\eta\lambda))-(1+\lambda))]$	$=\Delta = (p/2)(1-\lambda)(1-\mu)$ a)
P^L_{01}	$\pi(1+\eta\lambda)/(1+\lambda)$	1
P^P_{01}	$\pi(2-\mu+\eta\lambda\mu)/(2-\mu+\lambda\mu)$	1
$L = P^P_{01}/P^L_{01}$	$\frac{2-\mu+\eta\lambda\mu}{1+\eta\lambda}\cdot\frac{1+\lambda}{2-\mu+\lambda\mu}$ $= (2-\mu+\eta\lambda\mu)/(1+\eta\lambda)S$	1
C (covariance)b)	$2\tilde{Q}_{0t}\lambda\pi(1-\mu)(1-\eta)/(1+\lambda)^2$	0 (as $\eta = 1$)
V_{01}	$\pi\tilde{Q}_{01}(2-\mu+\mu\eta\lambda)/(1+\lambda)$	$= Q^L_{01} = \tilde{Q}_{01}PU^P_{01} = \tilde{Q}_{01}S$
$D = LS$	$\frac{PU^P_{0t}}{P^L_{0t}} = \frac{2-\mu(1-\eta\lambda)}{1+\eta\lambda}$	$\frac{2-\mu+\mu\lambda}{1+\lambda} = S$
$PU^P_{01} = PU^L_{01}$	$\pi(2-\mu+\eta\lambda\mu)/(1+\lambda) = S\cdot P^P_{0t}$	$(2-\mu+\mu\lambda)/(1+\lambda)$ = S (as P^P_{01}=1)
Q^P_{01}	$\tilde{Q}_{01}(2-\mu+\eta\mu\lambda)/(1+\eta\lambda)$	$\tilde{Q}_{01}(2-\mu+\mu\lambda)/(1+\lambda) = Q^L_{01}$
Q^L_{01}	$\tilde{Q}_{01}(2-\mu+\mu\lambda)/(1+\lambda)$	
$QU^P_{01} = QU^L_{01}$	$QU^P_{01} = QU^L_{01} = \tilde{Q}_{01}$	
covariance c_k c)	$\sum_j\left(\frac{q_{j1}}{q_{j0}}-\tilde{Q}_{01}\right)(p_{j0}-\tilde{p}_0)\frac{q_{j0}}{\sum q_{j0}} = \tilde{Q}_{01}\frac{p}{2}[(1-\lambda)(1-\mu)] = \tilde{Q}_{01}\Delta$	
c_k^*	$\tilde{Q}_{01}(\lambda-1)(1-\mu)/p(1+\lambda)^2/2 = -\Delta/(\tilde{p}_0)^2$	
$S = Q^L_{01}/\tilde{Q}_{01}$	$\frac{2-\mu+\mu\lambda}{1+\lambda} = 1+\frac{(1-\lambda)(1-\mu)}{1+\lambda} = 1+\frac{2\Delta}{p(1+\lambda)} = 1+\frac{\Delta}{\tilde{p}_0}$	

a) note $\Delta \neq 0$, although no price changed ($\eta = \pi = 1$) if only $\lambda \neq 1$ (unequal prices in the base period) and $\mu \neq 1$ (structural change within the CN) is given
b) C = 0 when the quantity structure has not changed, that is $m_{21} = \mu/2 = m_{20} = ½$ or $\mu = 1$
c) there is only one such covariance as K = 1
d) obviously neither η nor π is relevant for the covariances c_k and c_k* which explain the S-effect

Table 4 (part b, S effect when assuming constant prices, thus L = 1)

		μ **< 1** (less of good 2)		μ **> 1** (more of good 2)
λ **> 1** good 2 more expensive	II	$\lambda > 1$ and $\mu < 1 \rightarrow \Delta < 0$ less of the more expensive good 2 $c_k < 0 \rightarrow S < 1$	I	$\lambda > 1$ and $\mu > 1 \rightarrow \Delta > 0$ more of the more expensive good 2 $c_k > 0 \rightarrow S > 1$
λ **< 1** good 2 less expensive	III	$\lambda < 1$ and $\mu < 1 \rightarrow \Delta > 0$ less of the cheaper good 2 $c_k > 0 \rightarrow S > 1$	IV	$\lambda < 1$ and $\mu > 1 \rightarrow \Delta < 0$ more of the cheaper good 2 $c_k < 0 \rightarrow S < 1$

5. Conclusions and final remarks

In sec 4.1 the contribution of an individual CN to S = Q_{01}^{L}/QU_{01}^{L} (Laspeyres type indicators of quantity movement) was examined. However, S can also be expressed in terms of Paasche type price indicators S = PU_{01}^{P}/P_{01}^{P}. While P_{01}^{P} is a weighted mean of the K CN-specific Paasche indices[24] $P_{0t}^{P(k)}$ unfortunately the index PU_{0t}^{P} cannot be seen this way.[25] Moreover prices p_{kjt} disappear in the ratio $\frac{PU_{0t}^{P}}{P_{0t}^{P}} = \frac{\sum_k \tilde{p}_{kt}Q_{kt}}{\sum_k \tilde{p}_{k0}Q_{kt}} \cdot \frac{\sum_k \sum_j p_{kj0}q_{kjt}}{\sum_k \sum_j p_{kjt}q_{kjt}} = \frac{\sum_k \sum_j p_{kj0}q_{kjt}}{\sum_k \tilde{p}_{k0}Q_{kt}}$ which therefore does not provide any new insights compared to $S = Q_{0t}^{L}/QU_{0t}^{L}$.

It can also be shown that a theory of the structural effect S is much more straightforward if one considers a Paasche type unit value index of the PU_{01}^{P}. The analogous analysis of the PU_{01}^{L} type unit value index would be more difficult.

As to the relationship between a unit value index (PU_{01}^{P}) and Drobisch's index (P_{01}^{UD}), this is not merely a matter of the level of aggregation. Comparing eq. 7a and 8 shows that the unit value index may be viewed as a weighted mean of "low level" Drobisch indices $\tilde{p}_{kt}/\tilde{p}_{k0}$ while this is not true for Drobisch's index. Furthermore, as pointed out in this paper, there are many other aspects (for example, the axiomatic properties), which require the two indices to be looked at as two distinct types of price indices.

A clear distinction is also necessary between the S-effect and the L-effect. The L-effect can be viewed as resulting from a substitution between quantities in response to changing prices, and it may be desirable for a price index to reflect this phenomenon. This, however, does not apply to the S-effect, which rather seems to be an unwanted disturbance. Moreover, while prices must be changing for the L-effect to occur, the S-effect is possible even with constant prices, provided only that the structure of quantities is changing.

After all it is difficult to think of a microeconomic theory able to explain the sign of the covariance c_k as this covariance relates changes in quantities from period 0 to 1 to the structure of base period prices irrespective of prices in period 1, and a change in quantities may even take place although all prices remain constant. Thus the change in quantities cannot be viewed as response to changing prices.

It seems therefore difficult to "explain" the sort of economic behaviour which gives rise to a negative and a positive covariance c_k in terms of utility maximizing behaviour ion a similar fashion to the well known microeconomic theoretical underpinning of the L-effect.

In addition to the formal aspects regarding the difference between PU^P and P^L on which this paper focuses, there are many other aspects that should be considered when an assessment of unit value indices has to be made. Although they are standard practice in many countries there are strong reservations about unit value indices for the principal reason that they do not compare like with like; they violate the principle of pure price comparison,[26] and we agree with Silver (2007, 2008) that they may be justified – if at all – only as low-budget proxies for sur-

[24] Insofar analogous to eq. 14b where Q^L was described as weighted mean of individual $Q^{L(k)}$ indices.
[25] The reason is that the weights in eq. 14c do not add up to unity.
[26] This has already been established by the SNA 1993 which states that unit values are "affected by changes in the mix of items as well as by changes in their prices. Unit value indices cannot therefore be expected to provide good measures of average price change over time" (§ 16.13). Interestingly the SNA did not seem to realize that the same argument (no pure price comparison) would apply also to chain indices.

vey-based price indices. The following appendix will present some more details regarding the deficiencies of unit value indices.

Appendix

A1. Formulas of indices of export and import in Germany

Unit values $\tilde{p}_{kt}$ take the part of prices in both price- and quantity indices; hence we have unit value indices on the level of price and of quantity indices respectively (the latter is less common, however). So in theory at least $2^4 = 16$ indices exist due to the four dichotomies:

1. unit value index (UVI) vs. price index (PI) concept (level of aggregation in price data),
2. index describing movement of prices vs. quantities (volumes),
3. Laspeyres vs. Paasche formula and
4. prices of exports vs. those of imports.

German official statistics provides Paasche *unit value* indices in addition to genuine Laspeyres type *price* indices (both of export and import respectively). There are also countries in which use is made of both, prices and unit values in the same (price) index.[27]

Figure A.1: The structure of indices on the basis of unit values*

Indices on the basis of

unit values (UV)		prices	
UV price indices	UV quantity indices	true price indices	true quantity indices
summation over k = 1, ... ,K groups $\sum_k$		summation over n commodities $\sum_k \sum_j$	

formula of Laspeyres		formula of Laspeyres	
$PU_{01}^{L} = \frac{\sum \tilde{p}_1 Q_0}{\sum \tilde{p}_0 Q_0}$	$QU_{01}^{L} = \frac{\sum Q_1 \tilde{p}_0}{\sum Q_0 \tilde{p}_0}$	$P_{01}^{L} = \frac{\sum\sum p_1 q_0}{\sum\sum p_0 q_0}$	$Q_{0t}^{L} = \frac{\sum\sum q_1 p_0}{\sum\sum q_0 p_0}$
formula of Paasche		**formula of Paasche**	
$PU_{01}^{P} = \frac{\sum \tilde{p}_1 Q_1}{\sum \tilde{p}_0 Q_1}$	$QU_{01}^{P} = \frac{\sum Q_1 \tilde{p}_1}{\sum Q_0 \tilde{p}_1}$	$P_{01}^{P} = \frac{\sum\sum p_1 q_1}{\sum\sum p_0 q_1}$	$Q_{01}^{P} = \frac{\sum\sum q_1 p_1}{\sum\sum q_0 p_1}$

* The universe of n commodities is partitioned into K groups (sub-collections) of related commodities; the subscript k = 1, 2, ..., K denotes the number of the group and the subscript j the j^{th} commodity of the k^{th} group.

A2. Data basis (survey based price indices vs. customs based unit value indices)

Unit value indices (UVIs) are based on a complete statistics of customs documents rather than on the observation of a sample of carefully specified goods under comparable conditions. Thus UVIs also refrain from using appropriate methods for adjustments of quality changes, temporary (seasonal) unavailability, or outlier detection and deletion. Moreover there are reasons to expect ever more difficulties in the future as regards customs statistics. We observe an increasing proportion of trade in services rather than in goods that physically cross borders.

[27] According to the Internet Canada is an example. The export/import price index (= International Merchandise Trade Price index IMTPI) makes use of both unit values processed by the International Trade Division (on the basis of customs data) and when unit values are not accurate (heterogeneous aggregates) or unavailable price data provided by other (Canadian and foreign, e.g. the BLS of the USA) sources are taken. Both direct index formulas, Laspeyres and Paasche are used. For internal use also a chained Fisher index is being compiled.

Likewise e-trade and intra-area trade within customs unions without customs documents on which statistics could be based gain importance. In sum unit value indices are less commendable from a theoretical point of view.

Table A.2: Indices of prices in foreign trade (export and import) in Germany

	Price index (PI)	Unit value index (UVI)
Data	Survey based (monthly), sample; more demanding than UVI (empirical weights!)	A by-product of customs statistics, census, in the case of Intrastat* survey
Formula	Laspeyres	Paasche
Prices, aggregates	Prices of specific goods at time of contracting (lead of price index?)	Average value of CNs; time of crossing border (lag of UVI?)
New or disappearing goods	Included only with a new base period; vanishing goods replaced by *similar* ones constant selection of goods *	Immediately included; price quotation of disappearing goods is simply discontinued; variable universe of goods
Quality	Quality adjustment are performed	No quality adjustment (not feasible?)

* intra European Community (or Union)

** All price determining characteristics are deliberately kept constant

By contrast to compile a sample survey based PI is more demanding. It requires special surveys addressing exporting and importing establishments as well as compliance with the principle of "pure price comparison". This implies making adjustments (of reported prices) for quality changes in the traded goods or avoiding changes in the collection of goods, reporting firms or in the countries of origin (in the case of imports) or destination involved.

To sum up PIs appears to be theoretically more ambitious and to fit better to the general methodology (and the principle of pure price comparison in particular) of official price statistics whereas UVI might be a low budget "second best" solution and surrogate for PIs as they are more readily available and less demanding as regards data collection.

A3. Hypothesis on the basis of the conceptual differences between P and U indices

The conceptual and methodological differences mentioned give rise to testing empirically some hypotheses. In what follows we refer to an unpublished paper the present author has written in cooperation with Jens Mehrhoff (von der Lippe, Mehrhoff (2008)).[28] We studied altogether six hypotheses (see table A.3 summarizing the main results) using German data (Jan. 2000 through Dec. 2007). The hypotheses were quite obvious given the conceptual differences and most of them proved true. Above all UVIs and PIs of export and import respectively differ with regard to their level and volatility. UVIs tend to display a relative to PIs more moderate rise of prices combined with more accentuating oscillations. An altogether smoother pattern of the time series can also be attributed to the process of quality adjustment of PIs whereas UVIs are habitually not adjusted (which is in no small measure also due to the fact that details about the quality of the goods are lacking in customs data). Conspicuously and contrary to our expectations there was no clear evidence for the expected lead of PIs relative to the UVIs.

[28] Compared to von der Lippe (2007b) it contains a completely new empirical study (worked out by J. Mehrhoff).

Table A.3: Summary of tests about differences between unit value indices (U = UVI) and price indices (P = PI) based on empirical calculations of Jens Mehrhoff

Hypothesis	Argument	Method	Result
1) **U < P**, **growing discrepancy**	Laspeyres (P) > Paasche (U) Formula of L. v. Bortkiewicz	Theil's inequality coeff. applied to growth rates of the series	largely confirmed
2) **Volatility** U > P	U no pure price comparison (U reflecting changes in product mix [structural changes])	Dispersion (RMSE) of detrended (HP Filter) series (of P and U in exports and imports)	confirmed [a)]
3) **Seasonality** U > P	U no adjustment for seasonally non-availability	Standard dev. of seasonal component (Census X-2ARIMA)	similar to hypothesis no. 2
4) **U** suffers from **heterogeneity**	Variable vs. constant selection of goods, CN less homogeneous than specific goods	average correlation (root of mean R^2) of subindices (if small heterogeneity)	U only slightly more heterogeneous [b)]
5) **Lead** of P against U	Prices refer to the earlier moment of contracting (contract-delivery lag; exchange rates)	Correlation between ΔP (shifted forward) against ΔU	no systematic pattern [c)]
6) **Smoothing** in the case of P	Quality adjustment in P results in smoother time series	special data analysis [d)] of the German Stat Office	confirmed

a) Hypothesis largely confirmed, P is integrated, U stationary (depending on the level of (dis)aggregation)
b) more pronounced in the case of imports than of exports
c) in line with Silver's results
d) concerning desktops, notebooks, working storage and hard disks; coefficient of variation was in all cases sizeably smaller after quality adjustment than before.

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