Ten objections against the time reversal test for index numbers

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It is well known that the price index function P_{0t} for which holds $P_{t0} = (P_{0t})^{-1}$ is said to pass the time reversal test (TR).[1] For example the formula of Laspeyres $P_{0t}^{L} = \sum p_{it}q_{i0} / \sum p_{i0}q_{i0}$ fails this test because $P_{t0}^{L} = \sum p_{i0}q_{it} / \sum p_{it}q_{it}$ whereas $\left(P_{0t}^{L}\right)^{-1} = \sum p_{i0}q_{i0} / \sum p_{it}q_{i0}$. Obviously $P_{t0}^{L} = \left(P_{0t}^{P}\right)^{-1} \neq \left(P_{0t}^{L}\right)^{-1}$, so that the Paasche index P^{P} is (in Irving Fisher's terms) the "time antithesis" of the Laspeyres index P^{L}. Fisher's TR (and other reversal tests of Fisher as the factor reversal test FR) continues to play an important part in index theory as a test a good index function (ostensibly) ought to pass. It is widely used in order to justify a preference for Fisher's ideal index and TR also helped to rule out for example Carli's index (as "biased upwards") in the case of "low level aggregation" (unweighted indices). So a closer look at TR is worthwhile. In what follows we present ten heresies. The first three arguments were already advanced by Ladislaus von Bortkiewicz (LvB)[2] and the next seven arguments,[3] despite reflecting our own personal view, may well also be in the spirit of LvB

1. TR is a purely formal (or "mechanistic") test and it is easy to find a formula (however pointless it economically may be) able to pass this test
2. there is no reason why in TR in addition to "reversing" prices $\mathbf{p}_t \leftrightarrow \mathbf{p}_0$ also quantities should be reversed ($\mathbf{q}_t \leftrightarrow \mathbf{q}_0$) simultaneously,
3. reversal tests are motivated by dubious analogies and intuitive appeal only,
4. there are no compelling reasons why independence of the base year should be desirable
5. to argue " what applies to a single price should apply to a price level" is inconclusive,
6. a reversal of time is at odds with common experience; having an "underlying order", as "time" typically has, is only another way of saying that a "reversal" is nonsense;[4]
7. the two periods, 0 and t in the TR test, are not periods of the same kind, and
8. for the TR test but not for real life it makes no difference whether 0 and t are points in time close to one another or widely separated
9. reversal tests, TR and FR instigate a wrong notion of "bias" and of a sort of mirror symmetry between Laspeyres and Paasche erroneously considered the two allegedly equally well reasoned indices (what applies to P^{L} applies to P^{P} with opposite sign),
10. all too typically in most renowned systems of axioms TR/FR are excluded, as such reversal tests seem to be unduly restrictive.

Ad 1: For Irving Fisher both tests, TR and FR were also "finders of formulae". Fisher called the index $P_{0t}^{(T)} = 1/P_{t0}$, "time antithesis" of P_{0t} [5]and he considered the index $P_{0t}^{(T)}$, a byproduct of TR, a new index formula no less useful than P_{0t}. From this it follows that we have two products of index formulas, A and B where $A = P_{0t}P_{0t}^{(T)} = P_{0t}/P_{t0}$ and $B = P_{t0}P_{t0}^{(T)} = P_{t0}/P_{0t}$ so that B is the time reversed term A. Hence $P_{0t}^{(TR)} = \sqrt{A}$ is a time reversible index on the basis

[1] P_{t0} is gained from P_{0t} by interchanging subscripts 0 an t in prices and quantities in the P_{0t} formula.

[2] For more details see von der Lippe 2015.

[3] All my own arguments are taken from a referee report I wrote in 2013 about a paper submitted to the Journal of the Royal Statistical Society.

[4] Conspicuously and not surprisingly TR fits more interregional (with no underlying order) than intertemporal comparisons. Time is irreversible: to suggest $0 \leftarrow t$ were as good as $0 \rightarrow t$ is not "fair" but simply off the track.

[5] and the quantity index $Q_{0t} = V_{0t}/P_{0t}$ is the "factor antithesis" of P_{0t}.

of P_{0t} just like $P_{t0}^{(TR)} = \sqrt{B}$ is a time reversible variant of P_{t0} as $P_{0t}^{(TR)}P_{t0}^{(TR)} = \sqrt{A \cdot B} = \sqrt{1} = 1$. Note that $P^{(T)}$ should be kept distinct from $P^{(TR)}$ because unlike $P_{0t}^{(TR)}$ the index $P_{0t}^{(T)}$ is in general not time reversible ($P_{0t}^{(T)}P_{t0}^{(T)} \neq 1$) unless the underlying index P_{0t} is itself time reversible. The message now is: for *any* index formula irrespective of its meaning, even for a quite nonsensical one, say $P_{0t}^{\#}$ we can get a corresponding time reversible index function with $\sqrt{P_{0t}^{\#}/P_{t0}^{\#}}$. So to pass the TR as such is not really a remarkable feat for an index.[6]

Ad 2: In LvB's view Fisher did not justify why interchanging prices should also automatically (and simultaneously) entail interchanging quantities as well. So LvB suggested a two-steps-procedure for what he considered a "correct" TR:

$$\boxed{P_{0t}^{L} = \frac{\sum p_t q_0}{\sum p_0 q_0}} \rightarrow \boxed{P_{t0}^{P} = \frac{\sum p_0 q_0}{\sum p_t q_0}} \rightarrow \boxed{P_{t0}^{L} = \frac{\sum p_0 q_t}{\sum p_t q_t}} \text{ or } \boxed{P_{0t}^{P} = \frac{\sum p_t q_t}{\sum p_0 q_t}} \rightarrow \boxed{P_{t0}^{L} = \frac{\sum p_0 q_t}{\sum p_t q_t}} \rightarrow \boxed{P_{t0}^{P} = \frac{\sum p_0 q_0}{\sum p_t q_0}}.$$

The first step, that is $p_0 \leftrightarrow p_t$ (or $P_{0t}^{L} \rightarrow P_{t0}^{P}$, and $P_{0t}^{P} \rightarrow P_{t0}^{L}$ respectively) without simultaneously interchanging quantities ($q_0 \leftrightarrow q_t$) is nowadays known as *price reversal test*.[7] Evidently the index-pair Laspeyres-Paasche meets this test, because $P_{0t}^{L}P_{t0}^{P} = 1$, and $P_{0t}^{P}P_{t0}^{L} = 1$, but not TR in Fisher's definition of "time reversal" where also interchanging of quantities is required. In my view, however, LvB's idea of *time reversal as a two-stage process* is not really convincing (and this applies with even more force to a quite similar two-stage process LvB envisaged in his critique of the factor reversal test FR).

Ad 3: In the last analysis in LvB's view Fisher's predilection for reversal tests is based only on clearly misplaced (inappropriate) analogies, viz.

a) the *analogy "index - price relatives"* saying that a price index for n > 1 commodities should behave like a simple price relative $m_{i,0t} = p_{it}/p_{i0}$ for one commodity i only, where of course $m_{i,0t}$ satisfies TR as $m_{i,t0} = p_{i0}/p_{it} = (m_{i,0t})^{-1}$, and
b) an *analogy to justice (impartiality) or "fairness" and symmetry*, implicitly using the "*principle of insufficient reason*": there is no reason why what works forward $0 \rightarrow t$ should not work backward $0 \leftarrow t$ equally well, or (for FR) what applies to prices p_i should mutatis mutandis apply to quantities q_i too.

In the following arguments we go into details concerning points a and b. For a see below number 5, and for b see below number 7 through 9. For Fisher everywhere things seemed to be on a par that, however, for good reasons rather should be kept distinct:

- index numbers expressed in per cent (of a base value) and "absolute" (in €) figures (with no base),
- prices (of individual commodities) and price levels (of aggregates),
- comparisons across countries and across points or intervals in time,
- the base period 0 (kept constant for some years) and the necessarily variable actual period t (in the sequence P_{01}, P_{02}, P_{03} for example we have only one base but t takes on three values 1, 2, and 3),

[6] It can easily be seen that $P^{(TR)}=A^{1/2}$ applied to the price index of Laspeyres P^L and Paasche P^P respectively yields Fisher's "ideal" index $P^F = (P^L P^P)^{-1}$ since P^L is the time antithesis of P^P and vice versa. In this case the new index, that is P^F, is a sensible and meaningful index in its own right. But this need not be the case, and exactly here the critique of LvB and others comes in: Fisher's approach is purely mechanistic or "formal" because it is always possible – for any price index formula $P^{\#}$ whatsoever, whether economically meaningful or not – to find a price index which is time reversible, factor reversible or both, time *and* factor reversible.

[7] See v.d.Lippe 2007, 208f. Its definition is $P(\mathbf{p}_0,\mathbf{q}_0,\mathbf{p}_t,\mathbf{q}_t)\, P(\mathbf{p}_t,\mathbf{q}_0,\mathbf{p}_0,\mathbf{q}_t) = 1$. It is a price reversal taken in isolation with quantities kept constant. To make this plausible imagine a household adapts itself to a new – maybe higher – price level by changing (reducing) its demand (quantities) only with a certain time lag in a second step.

- a pair of periods 0 and t where t and 0 are close to one another and where t is far away from 0.

Hence the problematic assumptions tacitly made by Fisher in his reversal tests are that 0 and t (thus also P^L and P^P), or prices p and quantities q, are in all relevant aspects much the same.

Ad 4 (relative vs. absolute figures): Even in really simple situations many people have difficulties with percentages.[8] We usually transform a series of absolute into relative figures in order to make the figures better comparable. What enhances comparability is just *the common reference to the same base* because each figure is expressed relative to, or "in units of" just this very base (which, however, often is not made explicit). So with an index the choice of the ***base necessarily matters*** as it provides a common denominator. Attempts to circumvent this problem, for example with chain indices or by requiring a strict relation between two bases, 0 and t, like $P_{t0} = (P_{0t})^{-1}$ rather indicate that nature of index numbers is not well understood.[9] The problem of choosing the "correct" base or failing TR should be accepted as price for the better comparability of relative as opposed to absolute numbers.[10]

Ad 5 (analogy price index - price relatives): Given that quite a few people have difficulties with percentages and must have experienced already that with intuition we not always get things right it should be not too annoying to acknowledge, that things may not be so easy with a *price level* of *many* commodities as they are with the *price (or price relative) of a single good*.

A significant difference between the one-good and the many-goods situation is for example that we can handle the *many* commodities situation in two different ways: forming a ratio of averages (ROA) or an average of ratios (AOR).[11] With a *single* commodity situation a ratio of prices p_{it}/p_{i0} *is* a price ratio (relative) – hence also an average of such ratios, that is an AOR – and at the same time also a ROA, a ratio of averages $\bar{p}_t/\bar{p}_0$.

The idea of the time reversal test goes back to the Dutch economist N. G. Pierson (1896) who drew attention to – what he considered – an inconsistency between ROA and AOR in the case of unweighted indices.[12] Some (alleged) ambiguities of Carli's index $P^C_{0t} = \frac{1}{n}\sum(p_{it}/p_{i0})$, deemed so severe to him "that the system of index numbers is untrustworthy" (p. 130) and "is

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[8] I remember how journalists reporting on a lawsuit (about the pay and allowance system for civil servants) at the German Constitutional Court in Karlsruhe (in 2006) could hardly understand why living in Bavaria was not by 20% - as it appeared "logical" to them at first glance (but only 16.7%) cheaper than Munich when prices in Munich are 20% higher than in Bavaria. In a similar vein it is difficult for many to imagine that things have *changed* (to the worse) after a decline by 20% and a subsequent rise by 20% or vice versa (they intuitively tend to think - 20% is offset by + 20%). The problem with percentages obviously is that in quoting such figures we notoriously forget what is meant by 100%. Clearly the same additional X (in absolute terms) will entail a higher percentage with base "Bavaria = 100" than with "Munich = 100" (with a a higher price level than Bavaria).

[9] Moreover it is a misunderstanding on an incredibly low level of Statistics at that: many elementary text books demonstrate at length with numerical examples that time series in *absolute* figures x_1, x_2, … will generate quite different time series of index numbers (necessarily *relative* by their nature) depending on which of the x's serves as base (as if we should get the same graph with each x as base). Worse even: what is merely trivial and unavoidable, i.e. that the base matters, is often dramatized to an ostensibly severe defect of index numbers.

[10] Irrelevance of the base is not in itself favourable and worthwhile to be aimed at. It is sometimes said that chain indices have no base as it is constantly updated and always just the preceding year. However, this applies to the factors, or links $P_{t-1,t}$ only. Characteristic for chain indices is the existence of many links with *many bases* t - 1 (t = 1, 2,…) *multiplied* to form a chain. The focus is not on links but on their product (i.e. the chain indices), and there is neither "no base" nor a *common reference to the same base* (usually seen as major advantage of indices).

[11] There is no such duplicity in the single-good-case, where of course from $p_{it}/p_{i0} = 1{,}2$ follows that $p_{i0}/p_{it} = 1/1.2 = 0.833$ (that is -16.7%) for this only good i. But to what refers 20% or 16.7% in the many-goods-case: to the *change of an average* price or to an *average of* the various *changes* of prices? The difference between ROA and AOR of course vanishes when each price changes at the same rate $p_{it}/p_{i0} = \lambda, \forall$ i.

[12] Note that only Jevons' index P^J (out of the indices above) allows both interpretations, ROA *and* AOR.

not to be reconstructed, but to be abandoned altogether" (p. 127). He studied a fictitious numerical example (tab. 1) with two commodities in three situations comparing price indices of Carli P_{0t}^{C}, Jevons $P_{0t}^{J} = \prod (p_{it}/p_{i0})^{1/n}$, and Dutot $P_{0t}^{D} = \bar{p}_t/\bar{p}_0$:

	situation I		situation II		situation III		situation II*	
	i = 1	i = 2	i = 1	i = 2	i = 1	i = 2	i = 1	i = 2
p_{i0}	50	100	100	100	50	200	200	50
p_{it}	100	50	200	50	100	100	100	100
Carli P^C	1.25		1.25		1.25		1.25	
Jevons P^J	1		1		1		1	
Dutot P^D	150/150 = 1		250/200 = 1.25		200/250 = 0.8		200/250= 0.8	

Pierson favorite index was apparently P^D (a ROA formula). He gave, however, no reasons why P^D should be preferred over an AOR approach like P^C or P^J. Situation III can be viewed as time reversal of situation II (strictly speaking, the time reversed II is rather II*, but none of the indices reflects the difference between III and II*)[13], correctly reflected in $P^J = 1 = 1^{-1}$ but much more eye-catching in P^D where indeed $0.8 = (1.25)^{-1}$. By contrast P^C is clearly inadequate for him as it fails to make a difference between the two situations. From the point of view of TR among the indices P^D and P^J one index should be as good as the other, yet Pierson rejected P^J, because the "geometrical method … leaves the average price unaltered in each of these cases, which is clearly a mistake" (p. 130). By this he obviously meant that the result $P^J = 1$ is independent of the absolute levels $\bar{p}_0$ and $\bar{p}_t$ respectively so that P^J (and P^C) treats I and II alike.[14] Interestingly though index numbers are *relative* figures the case for TR makes recourse to *absolute* prices (and quantities): TR requires that p_{i0} is interchanged with p_{it} (not that $p_{1t}/p_{10} = 2$ is set off by $p_{2t}/p_{20} = p_{10}/p_{1t} = ½$ as for example in I). The focus is also laid on absolute prices when Pierson argues that situations I, II and III should be treated differently despite identical price relatives, $p_{1t}/p_{10} = 2$ and $p_{2t}/p_{20} = ½$ in all three cases.[15]

In sum there are significant differences between the one-good and the many-goods situation in that in the latter case alternatives unknown to the one-good case emerge, such as the dichotomy ROA vs. AOR, or different conclusions we may reach at depending on whether our emphasis is on absolute prices or on price relatives. Hence simple analogies between the one-good and the many-goods case are misplaced and inappropriate and we rather should be prepared to get (and accept) a different picture of a process when looking at it backward from t, that is with the $t \rightarrow 0$ perspective as opposed to forward (from a $0 \rightarrow t$ perspective).

Ad 6 (time is in essence irreversible): Time is usually visualized as arrow with a clear distinction between cause (C) and effect (E) or "before" (C) and "after" (E). What happens *after* E cannot be the cause of E. C and E are clearly *different phenomena* that *deserve to be treated differently*. There is no point in being indifferent to *both* perspectives $C \rightarrow E$ *and* $E \rightarrow C$ (*only either* $C \rightarrow E$ {forecast} *or* $E \rightarrow C$ {explanation} makes sense). In physics it is the increase of "entropy" (disorder) that gives time a direction. We see a cup of water (an object of high order) falling off a table and breaking into pieces, but it is most unlikely (though not *logically* impossible) to see these pieces jumping back to the table and recollecting again to a well

[13] Relating p_{jt} to p_{i0} and p_{it} to p_{j0} rather than p_{it} to p_{i0} and p_{jt} to p_{j0} makes no difference for TR.
[14] For both indices, P^J and P^C, what only counts is that in all four situations we have the same two relatives of 2 and ½ (no matter to which of the two goods each price relative belongs). Absolute prices are irrelevant. This and equality of III and II* is not always the case. Remember, Pierson only studied unweighted indices
[15] That III is in a way a "reversed" II is also owed to our orientation on absolute prices, and the focus is also on absolute rather than relative prices when Pierson argues that P^D should rank higher than P^J or P^C.

formed glass of water. Wave propagation *starts at a source*, we never see a wave travelling back and *ending at its source* where it is absorbed. Because time has an inherent order (sequence) $t_1 \rightarrow t_2$, a reversal to $t_2 \rightarrow t_1$ would be anything but an embodiment of "fairness". It is not intuitively appealing but simply counter-intuitive if not outright nonsense.

A reversal makes sense, however, in the case of two countries; say A (Austria) and F (France). It does so *just because there is no inherent natural ord*er between countries (there is no reason to prefer A to F, or F to A), and for just the same reason it is desirable to have a unique purchasing power parity (PPP) that is P_{AF} (base county A) should be unequivocally related to P_{FA} (base country F) as for example $P_{AF} = (P_{FA})^{-1}$.[16] That countries are unordered also makes it desirable, to have transitivity, that is a consistent order (sequence) of all countries in the sole dimension "PPP".[17]

While we can make *any indirect* comparisons across countries (they are *equivalent*) it is uncommon to make other comparisons between points in time than by way of time series *with* $t_1 < t_2 < t_3$….. It would be queer to study a sequence t_3, t_1, t_5, t_4, t_2 or so. Thus existence of a natural order is tantamount to make reversibility meaningless. Reversibility is a misled concept when applied to time.

Ad 7 (TR ignores that 0 and t are periods of different kind): The base 0 in P_{0t} is usually kept constant for a couple of years[18], whereas t in P_{0t} strictly speaking denotes *a number of* periods (P_{01}, P_{02}, …), not just one period. Fisher was not accounting for this conceptual difference between a (temporarily) *constant* base period 0 and a constantly *varying* period t. There is no point in interchanging 0 and t, not only because time is an arrow ($0 \rightarrow t$ exists, not $0 \leftarrow t$) but also because periods 0 and t serve different purposes in index numbers.

Ad 8 (for TR the distance between 0 and t is irrelevant):. Once an index formula satisfies TR, this *holds for any two periods 0 and t however apart they may be from one another* (provided all relevant data $\mathbf{p}_0$, $\mathbf{p}_t$, and $\mathbf{q}_0$, $\mathbf{q}_t$ are available). However, economically it is highly relevant which periods, 0 and t we refer to, in particular how distant t is from 0. We often hear that weights less frequently than annually updated as no longer "relevant" or "representative" (and therefore a chain index is needed) because nowadays progress is so fast that to compare 2010 to 2015 is like comparing 1900 to 1980 had been in former days. With this in mind, it is strange require TR, since if TR holds it holds for *any two periods,* for $P_{2015,2100} = (P_{2010,2150})^{-1}$ where it might be reasonable, as well as for $P_{1980,1900} = (P_{1900,1980})^{-1}$ (in 1900 we not yet had airplanes while in 1980 it was not unusual to fly to Madeira or so for holydays).[19]

Ad 9 (Fisher's notion of "bias" is based on his reversal tests): Fisher endorsed – as his followers continue to do so to this day – the wrong idea that P^L and P^P are equally well reasoned only with an opposite sign of the "bias" (Laspeyres is said to be "biased" – and

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[16] Country reversibility (CR), as analogon of time reversibility (TR) only makes sense because there is no natural order so that $A \rightarrow F$ or $F \rightarrow A$ makes no difference, or with countries it makes sense to treat things symmetrically. However, this may no longer apply when weights are involved, as for example a "basket of consumer goods" in the case of consumer price indices. It is not unreasonable to insist on using the own country's basket (there are no good reasons not to use the basket of A in P_{AF}), and then nobody expects P_{AF} (where we use the Austrian "basket") to be prima facie somehow related to P_{FA} (based on French household expenditure data). Such expectations were at best justified when in both cases the same "basket" is used.

[17] Transitivity requires consistency between the direct *and all indirect comparisons* between *any two countries*. It is just because countries are unordered that there is no reason to prefer one indirect comparison over another.

[18] I know of course the problem (raised in the chain index discussion) of how many years is "a couple".

[19] TR is usually achieved by making use of both "baskets" (vectors of quantities q, or "weighting schemes"), not only of $\mathbf{q}_0$, but also of $\mathbf{q}_t$. We can't convincingly argue against fixed basket (fixed weighted) indices like P^L in favour of chain indices saying that $\mathbf{q}_0$ becomes progressively irrelevant and unrepresentative (as $t > 0$) and at the same time continually use $\mathbf{q}_t$ in addition to $\mathbf{q}_0$. In other words, I think it is a bit contradictory to advocate on the one hand chain indices (because of rapid changes of consumption patterns) and to require TR on the other hand.

therefore not to recommend – upwards because $P^L_{0t}P^L_{t0} > 1$ and $P^L_{0t}Q^L_{0t} > V_{0t}$ just like Paasche is biased downwards), so that this could best be cancelled out by crossing. Hence for Fisher his reversal tests also serve as argument for his "ideal index" $P^F = (P^LP^P)^{1/2}$. It is a myth, however, that P^L and P^P have an equally well established rationale:

- for P^L inflation takes place to the extent that *to buy the same quantities* (or basket) q_{10}, $q_{20}, \ldots, q_{n0}$ in t will be more expensive, or $\Sigma p_t q_0$ departs from $\Sigma p_0 q_0$, so that elements in a P^L-series differ with respect to prices only $P^L_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0}$, $P^L_{02} = \frac{\sum p_2 q_0}{\sum p_0 q_0}, \ldots$[20]
- while successive elements in a P^P-series $P^P_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1}$, $P^P_{02} = \frac{\sum p_2 q_2}{\sum p_0 q_2}, \ldots$ differ with respect to both, prices and quantities and inflation takes place to the extent that *values* (at current prices, *nominal expenditures*, the numerators in the P^P indices) increasingly exceed *volumes* (at constant prices, *real expenditures*, denominators in the P^P indices)

Accordingly just as P^L is used (at least primarily) to serve for inflation measurement so serves P^P to deflate (i.e. to translate "values" into "volumes") aggregates of National Accounts.[21]

Ad 10 (reversal tests are unduly restrictive): A predilection for TR must be viewed against the backdrop that TR may well rule out many useful index functions (P^L and P^P for example, or also Carli's index P^C for low level aggregation). Even if TR were reasonable as such it should not be achieved at the expense of other reasonable properties or of violating other reasonable axioms. In all inconsistency theorems ("there is no index function that…") I know of either or both, circularity and reversibility (TR/FR) is involved (v.d.Lippe 2007, 184, 215). Typically enough renowned systems of axioms such as the Eichhorn and Voeller system (v.d.Lippe 2007, 220) usually do not mention reversal tests but instead they preferred much less appealing ideas, such as "linear homogeneity" for example. Thus TR must be dispensable, we can and should do without it.

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[20] This is meant by "pure price comparison" (fulfilled by P^L but not by P^P and a fortiori nor by chain indices).

[21] In my view the only reasonable argument in favour of the FR test (and not only the less strict product test) is that the same index P can serve both purposes, inflation measurement and deflation.