Unit Value Bias Reconsidered
Price and Unit Value Indices in Germany

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Abstract

The paper describes the conceptual and empirical differences between customs-based unit value indices (UVIs) of exports and imports as opposed to survey-based price indices (PIs). It is not uncommon for the former to be confounded with what should preferably be called "Drobisch's index", an index quite different from unit value indices as regards its axiomatic performance and the way in which it is derived from aggregation over unit value ratios. The focus of the paper is on relating the discrepancy between UVIs and PIs (the "unit value bias") to a (well known) Laspeyres (or substitution) effect or "L-effect" and a structural effect or "S-effect". Both effects can be expressed in terms of covariances, and it is shown that the less understood S-effect will occur when the amount by which quantities within a group of related (more or less homogeneous) commodities change is correlated with the prices of these commodities in the base period.

Key words: Price index, unit value index, unit values, axioms, foreign trade statistics, Bortkiewicz,

JEL: C43, C80, E01, F10

Preliminary remark

This paper is a revised version of a joint paper of Peter von der Lippe and Jens Mehrhoff (von der Lippe and Mehrhoff (2008)) which we submitted to the Ottawa Group. We have not yet got the permission for dissemination of the paper by the Bundesbank, the affiliation of Jens Mehrhoff. Moreover we decided to give less emphasis to the empirical study which was primarily drawn up by Jens Mehrhoff in order to have more time to discuss formal aspects of the discrepancy between unit value indices (UVIs) and (genuine) price indices. We did so because we wanted to benefit from the invaluable chance of discussing some not yet fully developed theoretic points of what we called "structural effect" (or S-effect for short) with many most famous index experts in Neuchâtel. So we decided to conceive a new paper with a different (less empirical and more formal focus) which retained, however, the original title and which is drafted by von der Lippe who also added some findings concerning the determinants of the S- effect he has gained only very recently (see section 6 of the paper).[1]

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[1] Notwithstanding we still intend to publish appropriately our joint paper in addition to the present paper.

1. Introduction

Only few countries (among which Germany and Japan) are able to provide on a monthly basis both, a unit value index (UVI) and a true price index (PI) for measuring the price development in export and import. This offers the opportunity to study empirically the impact of the methodological differences between these two indices. These differences and in particular some considerable shortcomings of UVIs gave rise to concerns as they are internationally much more common and can be viewed only as a (unsatisfactory) surrogate of PIs. In the process of composing the "Export and Import Price Index Manual" of the IMF (XMPI Manual, 2008) Mick Silver (2007) provided an extensive empirical study (using data of Germany and Japan) as well as an examination of axiomatic properties of UVIs.

The problem with UVIs is, however that the term is used for quite different indices. On the one hand there are indices actually compiled in official statistics as for example the German export and import[2] UVIs where unit values as a sort of average prices (for a *group* of goods) take the part prices of individual goods have in the case of a price index (which thus uses data on a much more disaggregated level). On the other hand the term UVI is also in use for what we would rather call "Drobisch's index" which is of theoretical interest only because this index requires the calculation of a total unit value of all goods (and maybe also services) at two points in time, 0 (base period) and t (present period). This is, however, in practice not possible for the simple reason that a summation over all quantities is not possible.

Much of the literature to be found under the key word "unit value index" is dealing with the UVI in the sense of Drobisch's index. This applies for example to Balk 1994, 1998, 2005 and Diewert 1995, 2004. The present author has written an unpublished text (quoted by Silver) in 2006 which is available, however, in the internet[3] and in 2008 he has set up a new paper in cooperation with Jens Mehrhoff with a completely new empirical study (worked out by J. Mehrhoff) which also is not yet published. In section ("sec." for short) 3.2 of this paper we briefly refer to the results of his statistical analysis.

Sec. 2 of the paper aims at making clear the difference between those UVIs which are actually compiled by official statistics of Germany for example and the UVI in the sense of Drobisch's index. In sec. 3 we give some information concerning the German official statistics as well as our empirical study. Sec. 4 properties of unit values are examined. They have implications for the axioms the indices in question will fail or fulfil. In sec. 5 a decomposition of the "discrepancy" between a Paasche UVI and the "normal" Laspeyres PI is derived. It introduced two components of the discrepancy, a "Laspeyres" or substitution effect (henceforth "L-effect") and a "structural" or "S-effect" respectively. While the former is already well known and sufficiently understood it was a challenge to give an interpretation to the S-effect. Sec. 6 makes an attempt to find determinants of this effect which is apparently closely related to the heterogeneity of the aggregate underlying the calculation of unit values. The interpretation of S is still not quite satisfactory. This applies in particular to an explication in terms of utility maximizing behaviour. Such a microeconomic theoretical underpinning we are familiar with long since in the case of the L-effect appears to be desirable also here with the S-effect. So sec. 7 concludes with making some suggestions for further work.

[2] The method of a UVI is also quite common in the case of indices of wages or prices for certain services (air transport for example).

[3] http://mpra.ub.uni-muenchen.de/5525/1/MPRA _paper_5525.pdf. He also gave an account of the conceptual differences between UVIs an PIs in his book, "Index Theory and Price Statistics" (2007).

2. Unit value index and Drobisch's index

2.1. Definition of unit values and Drobisch's index

It is important to realize that unit values are defined only four several goods grouped together in a sub collection of goods defined by a classification of product (eg commodities for production or foreign trade statistics). The relevant unit of the classification is called "commodity number" (CN) and the unit value is a kind of average price of the n_k goods in the k^{th} CN ($k = 1, \dots, K$)

(1) $$\tilde{p}_{k0} = \frac{\sum p_{kj0} q_{kj0}}{\sum q_{kj0}} = \sum_{j=1}^{n_k} p_{kj0} \frac{q_{kj0}}{Q_{k0}} = \sum p_{kj0} m_{kj0}$$ in period 0 and correspondingly

(1a) $$\tilde{p}_{kt} = \frac{\sum p_{kjt} q_{kjt}}{\sum q_{kjt}} = \sum_{j=1}^{n_k} p_{kjt} \frac{q_{kjt}}{Q_{kt}} = \sum p_{kjt} m_{kjt}$$ in period t.

The summation takes place over the $j = 1, \dots, n_k$ ($n_k < n$) goods of a CN. Only in the case of a commodity number (CN), like the k-th CN sums $Q_{k0} = \sum_{j=1}^{n_k} q_{kj0}$ or $Q_{kt} = \sum q_{kjt}$ of quantities have a meaningful interpretation.

It is, however, in general not possible - let alone meaningful - to summate over the quantities of all $n = \Sigma n_k$ commodities, that is to calculate $Q_t = \sum_k \sum_j q_{kjt} = \sum_k Q_{kt}$ (Q_0 analogously) and compile

(2) $$P_{0t}^{UD} = \frac{\sum_k \sum_j p_{kjt} q_{kjt} \Big/ \sum_k \sum_j q_{kjt}}{\sum_k \sum_j p_{kj0} q_{kj0} \Big/ \sum_k \sum_j q_{kj0}} = \frac{Q_0}{Q_t} \frac{\sum_k \sum_j p_{kjt} q_{kjt}}{\sum_k \sum_j p_{kj0} q_{kj0}} = \frac{V_{0t}}{Q_t / Q_0} = \frac{\tilde{p}_t}{\tilde{p}_0}$$

This is (or rather should be called) **Drobisch's index** (because of Drobisch (1871)), an index, however, unfortunately also quite often called "unit value index".[4]

This index can be viewed as being aggregated over "low level" unit value ratios as follows

(2a) $$P_{0t}^{UD} = \left(\sum_k \frac{\tilde{p}_{kt}}{\tilde{p}_{k0}} \frac{\tilde{p}_{k0} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{k0}} \right) \cdot \frac{\sum_k Q_{k0}}{\sum_k Q_{kt}}.$$

Note that the term in brackets is just the value index and that the weights $\tilde{p}_{k0} Q_{kt}$ do not add up to $\sum \tilde{p}_{k0} Q_{k0}$. Hence this term is not a mean value of unit value ratios in the same way that the value index is not a mean of price relatives. The manner in which Drobisch's index is related to $\tilde{p}_{kt} / \tilde{p}_{k0}$ is not straightforward.

2.2. Unit value indices in official statistics

The "unit value" index as it is in actual fact calculated in official statistics of some countries differs from eq. 2 in that unit values are established only for CNs. There are no "total" or all-items unit values $\tilde{p}_t$ and $\tilde{p}_0$. UVIs are necessarily compiled in two steps, in the first $\tilde{p}_{kt}$ and $\tilde{p}_{k0}$ are calculated and in the second they - or ratios of them that is $\tilde{p}_{kt} / \tilde{p}_{k0}$ - are incorporated (instead of prices) in the Paasche price index formula

[4] This is possibly so because the label "Drobisch's" index is already in use for another index also advocated by Moritz Wilhelm Drobisch (1802 – 1894), viz. the *arithmetic* mean of a Laspeyres and a Paasche index.

(3) $$PU_{0t}^{P} = \sum_k \frac{\tilde{p}_{kt}}{\tilde{p}_{k0}} \frac{\tilde{p}_{k0} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}} = \frac{\sum \tilde{p}_{kt} Q_{kt}}{\sum \tilde{p}_{k0} Q_{kt}}$$

In contrast to the Drobisch index (eq. 2a), this index is evidently a weighted arithmetic average of unit value ratios $\tilde{p}_{kt} / \tilde{p}_{k0}$ on a first (or "low") level of aggregation.

The first step maybe called "low level" aggregation. There are, however, some differences to the usual notion of low level (as opposed to upper level)

	no information about quantities available	using information about quantities
the same commodity in different outlets	"normal" usage of the term "low level"	
different commodities grouped by a classification		situation of a UVI (Σq needed for unit value)

As to step 2 there is of course no obvious reason why the Paasche formula should be preferred to the Laspeyres formula

(3a) $$PU_{0t}^{L} = \frac{\sum \tilde{p}_{kt} Q_{k0}}{\sum \tilde{p}_{k0} Q_{k0}}$$

which would be equally useful. Assuming that a price index is comprising all K CNs with $\Sigma n_k = n$, that is all n commodities (i = 1, …, n) the Laspeyres *price* index (as opposed to the Laspeyres *unit value* index) is

(4) $$P_{0t}^{L} = \frac{\sum_{i=1}^{n} p_{it} q_{i0}}{\sum_{i=1}^{n} p_{i0} q_{i0}} = \frac{\sum_{k=1}^{K} \sum_{j=1}^{n_k} p_{kjt} q_{k0}}{\sum_{k=1}^{K} \sum_{j=1}^{n_k} p_{kj0} q_{kj0}} = \frac{\sum_{k}^{K} \sum_{j}^{n_k} p_{kjt} q_{kj0}}{\sum_k \tilde{p}_{k0} Q_{k0}}$$

and the Paasche price index is defined accordingly. Strictly speaking the assumption is not justified, however, because price indices are based on a sample survey whereas unit value indices are resulting from a comprehensive customs statistics. This inaccuracy may be acceptable because our focus is on the formal aspects not the differences between the two types of indices with respect to the conceptual basis and the methods of data collection

2.3. Formulas of indices of export and import in Germany

Unit values $\tilde{p}_{kt}$, $\tilde{p}_{k0}$ take the part of prices in both price- and quantity indices; hence we have unit value indices on the level of price and of quantity indices respectively (the latter is less common, however). Moreover in theory at least $2^4 = 16$ indices exist due to four dichotomies:

1. UVI/PI concept (level of aggregation in price data),
2. index describing movement of prices vs. quantities (volumes),
3. Laspeyres vs. Paasche formula and
4. prices of exports vs. imports.

In Germany there exists a unit value index of exports and imports of the Paasche form in addition to genuine Laspeyres price indices of export and import respectively. There are also countries in which use is made of both, prices and unit values in the same (price) index.[5]

[5] According to the Internet Canada is an example. The export/import price index (= International Merchandise Trade Price index IMTPI) makes use of both unit values processed by the International Trade Division (on the basis of customs data) and when unit values are not accurate (heterogeneous aggregates) or unavailable price

Figure 1: The structure of indices on the basis of unit values*

Indices on the basis of

unit values (UV)		prices	
UV price indices	UV quantity indices	true price indices	true quantity indices
summation over k = 1, ... ,K groups $\sum_k$		summation over n commodities $\sum_k \sum_j$	
formula of Laspeyres		formula of Laspeyres	
$PU_{0t}^{L} = \frac{\sum \tilde{p}_t Q_0}{\sum \tilde{p}_0 Q_0}$	$QU_{0t}^{L} = \frac{\sum Q_t \tilde{p}_0}{\sum Q_0 \tilde{p}_0}$	$P_{0t}^{L} = \frac{\sum\sum p_t q_0}{\sum\sum p_0 q_0}$	$Q_{0t}^{L} = \frac{\sum\sum q_t p_0}{\sum\sum q_0 p_0}$
formula of Paasche		formula of Paasche	
$PU_{0t}^{P} = \frac{\sum \tilde{p}_t Q_t}{\sum \tilde{p}_0 Q_t}$	$QU_{0t}^{P} = \frac{\sum Q_t \tilde{p}_t}{\sum Q_0 \tilde{p}_t}$	$P_{0t}^{P} = \frac{\sum\sum p_t q_t}{\sum\sum p_0 q_t}$	$Q_{0t}^{P} = \frac{\sum\sum q_t p_t}{\sum\sum q_0 p_t}$

* The universe of n commodities is partitioned into K groups (sub-collections) of related commodities; the subscript k = 1, 2, ..., K denotes the number of the group and the subscript j the j^{th} commodity of the k^{th} group.

3. Price and unit value indices in German foreign trade statistics

In addition to differences in the formula of UVIs and PIs respectively there are also a number of conceptual and methodological differences (sample survey vs. complete enumeration of customs documents, time of recording prices etc.) which give rise to testing some hypotheses. This section is briefly dealing with these issues although the bulk of the paper is devoted to formal aspects of UVIs vs. PIs.

3.1. Data basis (survey based price indices vs. customs based unit value indices)

The unit value indices (UVIs) and price indices (PIs) of export and import differ also because of a different conceptual and data basis (see table 1).

Table 1: Indices of prices in foreign trade (export and import) in Germany

	Price index (PI)	Unit value index (UVI)
Data	Survey based (monthly), sample; more demanding (weights!)	Customs based (by-product), census, in the case of Intrastat a survey
Formula	Laspeyres	Paasche
Prices, aggregates	Prices of specific goods at time of contracting (lead of price index?)	Average value of CNs; time of crossing border (lag of UVI?)
New or disappearing goods	Included only with a new base period; vanishing goods replaced by *similar* ones constant selection of goods *	Immediately included; price quotation of disappearing goods is simply discontinued variable universe of goods
Quality	Quality adjustment are performed	No quality adjustment (not feasible?)

* All price determining characteristics are deliberately kept constant

To compile a PI is more demanding. It requires special surveys addressing exporting and importing establishments as well as compliance with the principle of "pure price comparison". This implies making adjustments (of reported prices) for quality changes in the traded goods

data provided by other (Canadian and foreign, e.g. the BLS of the USA) sources. Both direct index formulas, Laspeyres and Paasche are used. For internal use also a chained Fisher index is being compiled.

or avoiding changes in the collection of goods, reporting firms or in the countries of origin (in the case of imports) or destination involved. By contrast there is no need for satisfying such requirements in the production of UVIs, which thus are much less commendable from a theoretical point of view.

To sum up the PI appears to be theoretically more ambitious and fits better to the general methodology (and the principle of pure price comparison in particular) of official price statistics whereas UVI might be a low budget "second best" solution and surrogate for PIs as they are more readily available and less demanding as regards data collection.

3.2. Hypothesis on the basis of the conceptual differences between price (P) and unit value (U) indices

The conceptual and methodological characteristics of the two types of indices, UVI and PI respectively give rise to formulate some hypothesis about possible empirical differences we expect to observe. We[6] studied altogether six hypotheses (see table 2 summarizing the main results) using German data (Jan. 2000 through Dec. 2007).

Table 2: Summary of tests about differences between unit value indices (U) and price indices (P) based on the empirical study of Jens Mehrhoff

Hypothesis	Argument	Method	Result
1) **U < P**, **growing discrepancy**	Laspeyres (P) > Paasche (U) Formula of L. v. Bortkiewicz	Theil's inequality coeff. applied to growth rates of the series	largely confirmed
2) **Volatility** U > P	U no pure price comparison (U reflecting changes in product mix [structural changes])	Dispersion (RMSE) of detrended (HP Filter) series (of P and U in exports and imports)	confirmed [a)]
3) **Seasonality** U > P	U no adjustment for seasonally non-availability	Standard dev. of seasonal component (Census X-2ARIMA)	similar to hypothesis no. 2
4) **U** suffers from **heterogeneity**	Variable vs. constant selection of goods, CN less homogeneous than specific goods	average correlation (root of mean R^2) of subindices (if small heterogeneity)	U only slightly more heterogeneous [b)]
5) **Lead** of P against U	Prices refer to the earlier moment of contracting (contract-delivery lag; exchange rates)	Correlation between ΔP (shifted forward) against ΔU	no systematic pattern [c)]
6) **Smoothing** in the case of P	Quality adjustment in P results in smoother time series	special data analysis [d)] of the German Stat Office	confirmed

a) Hypothesis largely confirmed, P is integrated, U stationary (depending on the level of (dis)aggregation)
b) more pronounced in the case of imports than of exports
c) in line with Silver's results
d) concerning desktops, notebooks, working storage and hard disks; coefficient of variation was in all cases sizeably smaller after quality adjustment than before.

The hypotheses were obvious given the conceptual differences and most of them proved true. Above all UVIs- and PIs of export and import respectively differ with regard to their level and volatility. UVIs indices tend to display a relative to PIs more moderate rise of prices combined with more accentuating oscillations. An altogether smoother pattern of the time series can also be attributed to the process of quality adjustment of PIs whereas UVIs are habitually not adjusted (in no small measure also due to the fact that details about the quality of the goods are lacking in customs data). Conspicuously and contrary to our expectations there was no clear evidence for the expected lead of PIs relative to the UVIs.

[6] This part of the paper rests particularly upon the work of Jens Mehrhoff, cp. our work von derLippe, Mehrhoff (2008)

4. Properties of unit values, unit value indices and Drobisch's index

4.1. Properties of unit values

The outstanding feature of unit values is that they are *not reflecting a pure price movement* because they are also affected by the quantities involved (in addition to the prices). According to eq. 1a $\tilde{p}_{kt}$ can well reflect a rise (decline) of an average price compared to $\tilde{p}_{k0}$ even if no individual price within the aggregate were changing. It all depends on the structure of quantities in 0 and t (that is on the coefficients $m_{kjt} \neq m_{kj0}$). Assume only two commodities in group (CN) k with constant prices so that $p_{k10} = p_{k1t} = p$ and $p_{k20} = p_{k2t} = \lambda p$ and equal quantity shares $m_{k10} = m_{k20} = ½$ in the base period. Then the difference of unit values of this k-th CN is depending on $\mu = m_{k2t}/0.5 = 2m_{k2t}$ $(0 \leq \mu \leq 2)$

(5) $$\Delta = \tilde{p}_{kt} - \tilde{p}_{k0} = \frac{p}{2}(1+\mu\lambda-\mu-\lambda) = \frac{p}{2}(1-\lambda)(1-\mu)$$

so that we have for a positive base period price $p = p_{10}$ four quadrants as follows

Table 3

$\lambda > 1$	II $\lambda > 1$ and $\mu < 1 \rightarrow \Delta < 0$ less of the more expensive good 2 unit value ***declining***	I $\lambda > 1$ and $\mu > 1 \rightarrow \Delta > 0$ more of the more expensive good unit value ***rising***
$\lambda < 1$	III $\lambda < 1$ and $\mu < 1 \rightarrow \Delta > 0$ less of the cheaper good 2 unit value ***rising***	IV $\lambda < 1$ and $\mu > 1 \rightarrow \Delta < 0$ more of the cheaper good 2 unit value ***declining***
	$\mu < 1$	**$\mu > 1$**

We will come back to this (first) two-commodities example in sec. 6.7.

4.2. Properties of ratios unit values

The ratio of unit values $\tilde{p}_{kt}/\tilde{p}_{k0}$ is (just as Drobisch's index) *not* a mean value of price relatives p_{kjt}/p_{kj0} as the weights are $p_{kj0}q_{kjt}/\tilde{p}_{k0}Q_{kt}$ summing up to

(6) $$\sum_k \frac{p_{kj0}q_{kjt}}{\tilde{p}_{k0}Q_{kt}} = \frac{Q_{k0}}{Q_{kt}} \cdot Q_{0t}^{L(k)} = \frac{Q_{0t}^{L(k)}}{\tilde{Q}_{0t}^{k}}$$

where $Q_{0t}^{L(k)}$ is the Laspeyres quantity index of the k^{th} CN. Hence the ratio may well violate the mean value condition. This can also easily be seen assuming $n_k = 2$ commodities only

(6a) $$\frac{\tilde{p}_{kt}}{\tilde{p}_{k0}} = \frac{p_{k1t}}{p_{k10}}\left(\frac{p_{k10}}{\tilde{p}_{k0}} \cdot \frac{q_{k1t}}{Q_{kt}}\right) + \frac{p_{k2t}}{p_{k20}}\left(\frac{p_{k20}}{\tilde{p}_{k0}} \cdot \frac{q_{k2t}}{Q_{kt}}\right)$$

and with a numerical example as follows

p_0	p_t	p_t/p_0	q_0	q_t
50	60	1.2	3	6
4	6	1.5	12	6

The individual price relatives are 1.2 and 1.5 respectively and the unit values are $\tilde{p}_t = (60 \cdot 6 + 6 \cdot 6)/(6+6)$ = 33.0 (in period t) which is 250% of 198/15 = 13.2 (the unit value in period 0).

Total quantities Q declined from $Q_0 = 15$ to $Q_t = 12$. Hence the sum of weights is 1.89 + 0.15 = (15/12)* 1.6363 = 2.045 (with the Laspeyres quantity index of this aggregate amounting to 1.6363). It is also obvious that the ratio $\tilde{p}_{kt}/\tilde{p}_{k0}$ can violate identity in which case $\tilde{p}_{kt}/\tilde{p}_{k0}$ simply amounts to the sum of the weights (in the example 204.5% although no price changed).

As aforesaid the ratio may also be conceived as first step of a two stage index compilation in the case of an UVI (by contrast to a one-step compilation in the case of a PI). Moreover $\tilde{p}_{kt}/\tilde{p}_{k0}$ can also be written as

(7) $$\frac{\tilde{p}_{kt}}{\tilde{p}_{k0}} = Q_{k0}\sum\nolimits_j p_{kjt}q_{kjt} \Big/ Q_{kt}\sum\nolimits_j p_{kj0}q_{kj0} = \frac{Q_{k0}}{Q_{kt}}\cdot\frac{V_{kt}}{V_{k0}} = \frac{V_{0t}^{k}}{\tilde{Q}_{0t}^{k}}$$

V_{kt}/V_{k0} is the value ratio (index) of the k^{th} CN ($V_{kt}/V_{k0} = 2$ in the numerical example) that is $\Sigma p_{kjt}q_{kjt}/\Sigma p_{kjt}q_{kjt}$ which is an interesting result in so far as in the special case $Q_{kt} = Q_{k0}$ is the unit value ratio "boils down" to the value ratio.

4.3. Unit value index and Drobisch's index

Table 4 summarizes the axiomatic properties of P_{0t}^{UD} (eq. 2) and PU_{0t}^{P} (or PU_{0t}^{L}). It reinforces once more our desire to make a clear distinction between the two types of indices.

Table 4: Axiomatic performance (Drobisch vs. unit value index)

Axiom	Defintion	Drobisch P^{UD} **eq. 2**	PU^{P} **eq. 3**
Proportionality	$U(\mathbf{p}_0, \lambda\mathbf{p}_0, \mathbf{q}_0, \mathbf{q}_t) = \lambda$ (identity = 1)	no	no
Commensurability	$U(\mathbf{\Lambda p}_0 0, \mathbf{\Lambda p}_t, \mathbf{\Lambda}^{-1}\mathbf{q}_0, \mathbf{\Lambda}^{-1}\mathbf{q}_t) = U(\mathbf{p}_0, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t)$	no	no
Linear homogeneity	$U(\mathbf{p}_0, \lambda\mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t) = \lambda\, U(\mathbf{p}_0, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t)$	yes	yes
Additivity (in current period prices)	$U(\mathbf{p}_0, \mathbf{p}_t^*, \mathbf{q}_0, \mathbf{q}_t) = U(\mathbf{p}_0, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t) +$ $U(\mathbf{p}_0, \mathbf{p}_t^+, \mathbf{q}_0, \mathbf{q}_t)$ for $\mathbf{p}_t^* = \mathbf{p}_t + \mathbf{p}_t^+$,	yes	yes
Additivity (in base period prices)	$[U(\mathbf{p}_0^*, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t)]^{-1} = [U(\mathbf{p}_0, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t)]^{-1}$ $+ [U(\mathbf{p}_0^+, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t t)]^{-1}$ for $\mathbf{p}_0^* = \mathbf{p}_0 + \mathbf{p}_0^+$	yes	yes

Product test	Implicit quantity index of Drobisch or PU^{P}	$\Sigma q_t/\Sigma q_0$	$QU^{L\,*)}$
Time reversibility	$U(\mathbf{p}_t, \mathbf{p}_0, \mathbf{q}_t, \mathbf{q}_0) = \mathbf{U}^{\leftarrow}$ $= [U(\mathbf{p}_0, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t)]^{-1} = [\mathbf{U}^{\rightarrow}]^{-1}$	yes (?)*	$(PU^{P\leftarrow}) = 1/(PU^{L\rightarrow})$
Transitivity	$U(\mathbf{p}_0, \mathbf{p}_2, \mathbf{q}_0, \mathbf{q}_2) = U(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1)^*$ $U(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}_1, \mathbf{q}_2)$	yes	no

* Laspeyres quantity index of unit value type (see fig. 1)

5. Decomposition of discrepancy between unit value index and price index

5.1. Theorem of Bortkiewicz and decomposition formula

The basis of the following decomposition is

(8) $$V_{0t} = PU_{0t}^{L}QU_{0t}^{P} = PU_{0t}^{P}QU_{0t}^{L} = \frac{\sum p_t q_t}{\sum p_0 q_0},$$

a relationship patterned after the well known identity $V_{0t} = P_{0t}^{L}Q_{0t}^{P} = P_{0t}^{P}Q_{0t}^{L}$.

In combination with the formula of Ladislaus von Bortkiewicz for the covariance between price and quantity relatives weighted with expenditure shares $p_0q_0/\Sigma p_0q_0$

(9) $C = Q_{0t}^{L}\left(P_{0t}^{P} - P_{0t}^{L}\right)$, due to the fact that

(9a) $$C = \sum_i \left(\frac{p_{it}}{p_{i0}} - P_{0t}^{L} \right) \left(\frac{q_{it}}{q_{i0}} - Q_{0t}^{L} \right) \frac{p_{i0} q_{i0}}{\sum p_{i0} q_{i0}} = V_{0t} - Q_{0t}^{L} P_{0t}^{L} = Q_{0t}^{L} P_{0t}^{P} - Q_{0t}^{L} P_{0t}^{L}$$

using eq. 8 leads to the following multiplicative decomposition of the discrepancy D

(10) $$D = \frac{PU_{0t}^{P}}{P_{0t}^{L}} = \left(\frac{C}{Q_{0t}^{L} P_{0t}^{L}} + 1 \right) \left(\frac{Q_{0t}^{L}}{QU_{0t}^{L}} \right) = \frac{P_{0t}^{P}}{P_{0t}^{L}} \cdot \frac{PU_{0t}^{P}}{P_{0t}^{P}} = L \cdot S \,.$$

D has two components or distinct "effects" which may work in the same or in opposite direction, so that they may be positively or negatively correlated.

The term L is referred to as Laspeyres- or simply ***L-effect*** reflecting the fact that $P^P \neq P^L$. A negative covariance ($P^P < P^L$) may arise from rational substitution among goods in response to price changes on a given (negatively sloped) demand curve. The less frequent case of a positive covariance is supposed to take place when the demand curve is shifting away from the origin (due to an increase of income for example). This is since long a well known and well understood effect (much in contrast to the second effect.

The second component of the discrepancy will henceforth be called structural component or ***S-effect*** for short. It refers to changing quantities within a group of goods k = 1,…,K (for which unit values are established). S is related to the composition ("structure") of the CNs.

Both effect, L and S can be expressed in terms of quantity indices as well as in terms of price indices

(11) $$L = \frac{C}{Q_{0t}^{L} P_{0t}^{L}} + 1 = \frac{Q_{0t}^{P}}{Q_{0t}^{L}} = \frac{P_{0t}^{P}}{P_{0t}^{L}} = \frac{Q_{0t}^{P}}{S \cdot QU_{0t}^{L}} = \frac{PU_{0t}^{P}}{S \cdot P_{0t}^{L}}$$

(12) $$S = \frac{Q_{0t}^{L}}{QU_{0t}^{L}} = \frac{PU_{0t}^{P}}{P_{0t}^{P}} = \frac{Q_{0t}^{P}}{L \cdot QU_{0t}^{L}} = \frac{PU_{0t}^{P}}{L \cdot P_{0t}^{L}}$$

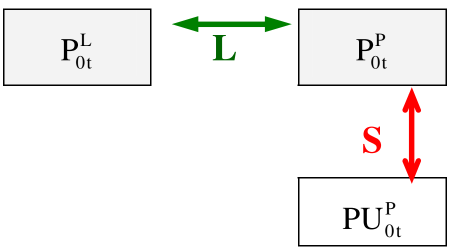
In an earlier version of this paper we also introduced an additive decomposition which turned out, however, to be less interesting

(10a) $$D^* = D - 1 = \frac{PU_{0t}^{P}}{P_{0t}^{L}} - 1 = \left(\frac{C}{QU_{0t}^{L} P_{0t}^{L}} \right) + \left(\frac{Q_{0t}^{L}}{QU_{0t}^{L}} - 1 \right) = L^* + S^*$$

where S* = S-1 and L* = (L-1)S.

The distinction between L and S springs from the fact that it is difficult to compare P^L to PU^P directly. It is useful to divide the comparison into two parts as depicted in figure 2.

Figure 2



L = relation between P^L an P^P
S = relation between P^P and PU^P

In general both effects, S and L respectively, will coexist. It is also possible that either or both effects vanish (the latter situation is L = S = 1). Table 5 displays various inequalities which can easily be inferred from a closer inspection of eqs. 11 and 12 or of figure 2. In quadrants I and III the effects S and L are working in the same direction (in which case we can combine two inequalities), generating thereby a positive (I) or negative (III) discrepancy D. By contrast in quadrants II and IV they take the opposite direction so that the sign of D is indeterminate.

Table 5

	L < 1 (C < 0)	**L = 1** (C = 0)	**L > 1** (C > 0)
S > 1	**II:** D is indefinite	$PU^P > P^L = P^P$	**I:** $PU^P > P^P > P^L \Rightarrow$ **D > 1**
S = 1	$PU^P = P^P < P^L$	$PU^P = P^P = P^L$	$PU^P = P^P > P^L$
S < 1	**III:** $PU^P < P^P < P^L \Rightarrow$ **D < 1**	$PU^P < P^L = P^P$	**IV:** D is indefinite

Our empirical study revealed that the most frequently observed case is quadrant III where both effects are negative and reinforce each other to yield a negative discrepancy D <1 and $PU^P < P^P < P^L$ (or equivalently $Q^P < Q^L < QU^L$).

5.2. How individual commodities contribute to the L-effect

It is useful to study the covariance (as the decisive term in L) broken down to the level of individual commodities I = 1, …, n. Following by Canadian statisticians (Chaffé et al. (2007)) Bogdan Szulc has to be given credit to the following formula (quoted in their notation)

$$R = \frac{P_P - P_L}{P_L} = \sum_i \left[\left(\frac{p_i^1 / p_i^0 - P_L}{P_L} \right) \cdot \left(\frac{q_i^1 / q_i^0 - Q_L}{Q_L} \right) \cdot \left(\frac{p_i^0 q_i^0}{\sum p_i^0 q_i^0} \right) \right]$$

This, however, is nothing else but $C / P_{0t}^L Q_{0t}^L$ (a sort of a "centred" covariance s_{xy}, that is s_{xy} divided by the respective means $\bar{x}$ and $\bar{y}$). Hence according to the famous eq. 8 we are owing to Bortkiewicz R is simply L – 1. Thus we can express L as follows

$$\text{(13)} \quad L = R + 1 = \sum_i \left(\frac{p_{it}/p_{i0} - P_{0t}^L}{P_{0t}^L} \right) \left(\frac{q_{it}/q_{i0} - Q_{0t}^L}{Q_{0t}^L} \right) \frac{p_{i0} q_{i0}}{\sum p_{i0} q_{i0}} + 1,$$

that is as a sort of covariance between relative deviations from an average price or quantity plus one.

The eq. for R of Szulc or Bortkiewicz is interesting in the first place because it provides a link between the aggregate parameter L and the prices and quantities of each of the n individual goods. It shows in detail how a single good contributes to a positive or negative L-effect. In what follows we try to find a similar equation in order to explain the S-effect. Furthermore the theorem of L. von Bortkiewicz states in essence that it is a covariance that determines sign and amount of L. In a similar vein we look in sec. 6 for a covariance able to explain S.

Eq. 12 also shows that the L-effect will disappear (L = 1) when one or more of the following conditions apply:

- all price relatives are equal $p_{it}/p_{i0} = P_{0t}^L$ or unity (no price changes)[7] $p_{it}/p_{i0} = P_{0t}^L = 1$
- the same applies mutatis mutandis to quantity relatives
- the covariance disappears.

[7] For this reason the first two-commodities example of sec. 2.1 is not suitable to demonstrate the L-effect, because there is no L-effect (prices did not change), however, the S-effect does exist I this case.

Hence for the L-effect to exist it is essential that price and quantity relatives are correlated. We start our attempts to derive formulas for S in the next section by showing in quite the same manner under which conditions the L-effect will vanish (or equivalently S = 1).

6. Some attempts to give an interpretation to the S-effect

In this section we propound a theory of determinants of the S-effect by developing formulas for the contribution of a single CN to S (patterned after eq. 13 in the case of L) and using a generalized theorem of linear indices of which eq. 9 of Bortkiewicz is a special case. As the S-effect is not yet well understood it appears useful to start with showing in which situations concerning prices and quantities this effect will *not* materialize.

6.1. General remarks

It should be noted right at the outset that the structural effect owes its existence to the two-stage compilation of the unit-value index (UVI). If summation would take in one stage over the individual commodities (not grouped into CNs) the S-effect would disappear.

The S-effect will also vanish (S = 1) if one or more of the following conditions is given

- $n_k = 1$ (a perfectly homogenous CN), such that $m_{kjt} = m_{kj0} = 1$ (unlike the L-effect the S effect only exists when commodities are grouped together in CNs)[8]
- for all j = 1, …, n_k holds $m_{kjt} = m_{kj0}$ (no structural change within a CN), or
- all n_k base period prices of a CN k are equal $p_{kj0} = \tilde{p}_{k0} \quad \forall j = 1,...,n_k$, because in this case (as will be shown later in more detail) the following holds

(14) $$\tilde{Q}_{0t}^{k} = \sum_j \frac{q_{kjt}}{q_{kj0}} \frac{q_{kj0}}{\sum_j q_{kj0}} = Q_{0t}^{L(k)} = \sum_j \frac{q_{kjt}}{q_{kj0}} \frac{q_{kj0}p_{kj0}}{\sum_j q_{kj0}p_{kj0}} .$$

The statements of the first two bullets will be expounded in sec. 6.2 and 6.3. The third point is particularly interesting because it leads over to a more general theory about the determinants of S, according to which it is not necessary for the S-effect to occur that prices in t are different from prices in the base period 0. Unlike the L-effect the S-effect is possible even though no price is changing.[9] Quite different price relatives (and a different Laspeyres price index P_{0t}^{L} as their average) may yield the same of the S-effect (in amount and sign).

What matters are solely the base period prices since they will entail a different structure of weights $w_{kj}^{L} = q_{kj0}p_{kj0} \big/ \sum q_{kj0}p_{kj0}$ as opposed to $\tilde{w}_{kj} = q_{kj0} \big/ \sum q_{kj0}$. The decisive relevance of the price structure in 0 is also visible in the term

(15) $$\frac{w_{kj}^{L}}{\tilde{w}_{kj}} = \frac{p_{kj0}}{\tilde{p}_{k0}},$$

showing that an above average price $p_{kj0} > \tilde{p}_{k0}$ tends to contribute to $S = Q_{0t}^{L} \big/ QU_{0t}^{L} > 1$ and a below average price to a negative S-effect (S < 1). Beginning with sec. 6.3 it will be shown that factors influencing S are not only the price structure at the base period but also certain covariances these base period prices and quantity relatives.

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[8] There can be no S-effect when there is no heterogeneity and/or structural change within the CNs. It appears therefore sensible to study the S-effect by examining the situation *within* the CNs.

[9] We can therefore use the first two-commodities example of sec. 4.1 to demonstrate the S-effect although we don't have an L-effect in this case.

6.2. No S-effect occurs when commodities are not grouped in CNs or when the CNs are perfectly homogeneous

The S-effect, given by $S = PU_{0t}^{P} / P_{0t}^{P}$ owes its existence to the two stage compilation of a unit value index (UVI), where $n_k > 1$ commodities are lumped together in CNs. When the summation takes place over each individual commodity in numerator and denominator as in $P_{0t}^{P} = \frac{\sum_k \sum_j p_{kjt} q_{kjt}}{\sum_k \sum_j p_{kj0} q_{kjt}}$ the result will in general differ from $PU_{0t}^{P} = \frac{\sum_k \sum_j p_{kjt} q_{kjt}}{\sum_k \frac{Q_{kt}}{Q_{k0}} \sum_j p_{kj0} q_{kj0}}$ unless all prices in 0 are equal $p_{kj0} = p_0$ in which case both, $\frac{Q_{kt}}{Q_{k0}} \sum_j p_{kj0} q_{kj0}$ as well as $\sum_j p_{kj0} q_{kjt}$ reduce to $p_0 Q_{kt}$ so that S = 1. Instead of homogeneity with respect to prices in 0 one can also assume equal quantities of all n_k goods q_{k0} or q_{kt} for all j in 0 and in t with $Q_{k0} = n_k q_{k0}$ and $Q_{kt} = n_k q_{kt}$ because we then get again $S = PU_{0t}^{P} / P_{0t}^{P} = 1$ due to

$$PU_{0t}^{P} = \frac{\sum_k q_{kt} \sum_j p_{kjt}}{\sum_k \frac{Q_{kt}}{n q_{k0}} q_{k0} \sum_j p_{kj0}} = \frac{\sum_k q_{kt} \sum_j p_{kjt}}{\sum_k q_{kt} \sum_j p_{kj0}} = P_{0t}^{P}.$$

S = 1 is also true if each CN would consist of $n_k = 1$ commodity only with price $p_{kt} = \tilde{p}_{kt}$ and quantity $q_{kt} = Q_{kt}$ in t (likewise $p_{k0} = \tilde{p}_{k0}$, $q_{kt} = Q_{kt}$ in 0) because then $\sum_j p_{kjt} q_{kjt} = p_{kt} q_{kt}$ and

$$P_{0t}^{P} = PU_{0t}^{P} = \frac{\sum_k p_{kt} q_{kt}}{\sum_k p_{k0} q_{kt}} = \frac{\sum_k \tilde{p}_{kt} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}}.$$

To sum up: S =1 when there are no CNs (or equivalently if each CN consists of only $n_k = 1$ good), or when in the case of $n_k > 1$ all n_k prices or all quantities in the base period are equal.

6.3. No S-effect occurs when the structure of quantities within CNs does not change

Both indices PU_{0t}^{P} as well as P_{0t}^{P} can be written as weighted sums of partial (*within* a CN) Paasche indices $P_{0t}^{P(k)}$ using coefficients $m_{kjt} = q_{kjt} / \sum_j q_{kjt} = q_{kjt} / Q_{kt}$ (m_{kj0} analogously defined) describing the structure of quantities

$$\text{(16)} \qquad PU_{0t}^{P} = \frac{\sum_k \tilde{p}_{kt} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}} = \left(\sum_k \frac{\tilde{p}_{k0}}{\tilde{p}_{kt}} \frac{\tilde{p}_{kt} Q_{kt}}{\sum \tilde{p}_{kt} Q_{kt}} \right)^{-1} = \sum_k P_{0t}^{P(k)} \frac{Q_{kt} \sum_j p_{kj0} m_{kjt}}{\sum_k Q_{kt} \sum_j p_{kj0} m_{kj0}}$$

$$\text{(17)} \qquad P_{0t}^{P} = \frac{\sum_k \sum_j p_{kjt} q_{kjt}}{\sum_k \sum_j p_{kj0} q_{kjt}} = \left(\sum_k \sum_j \frac{p_{kj0}}{p_{kjt}} \frac{p_{kjt} q_{kjt}}{\sum_j p_{kjt} q_{kjt}} \right)^{-1} = \sum_k P_{0t}^{P(k)} \frac{Q_{kt} \sum_j p_{kj0} m_{kjt}}{\sum_k Q_{kt} \sum_j p_{kj0} m_{kjt}}$$

In the case of no structural change *within* each of the CNs ($m_{kjt} = m_{kj0}$) we get

$$\text{(17a)} \qquad PU_{0t}^{P} = \sum_k P_{0t}^{P(k)} \frac{Q_{kt} \sum_j p_{kj0} m_{kj0}}{\sum_k Q_{kt} \sum_j p_{kj0} m_{kj0}} = P_{0t}^{P}, \text{ such that again } S = 1.$$

Instead of studying the ratio $S = PU_{0t}^{P} / P_{0t}^{P}$ and using the eqs 16 and 17 one can also use $S = Q_{0t}^{L} / QU_{0t}^{L}$ and

(18) $$S = \frac{Q_{0t}^{L}}{QU_{0t}^{L}} = \frac{\sum_k Q_{kt} \sum_j m_{kjt} p_{kj0}}{\sum_k Q_{kt} \sum_j m_{kj0} p_{kj0}}$$

which demonstrates again that leads to S = 1.

That weights in eq. 16 and 18 do not add up to unity unless for all sums $\Sigma p_{kj0} m_{kjt} = \Sigma p_{kj0} m_{kj0}$ holds is also responsible for the fact that UVIs violate the mean value property. The existence of structural change $m_{kjt} \neq m_{kj0}$ entails a number of violations of axioms. It can easily be seen for example that $\tilde{p}_{kt} / \tilde{p}_{k0}$ does not comply with proportionality. Given that for each j holds $p_{kjt} = \lambda p_{kj0}$ then it follows from eq. 6 and 7

(19) $$\frac{\tilde{p}_{kt}}{\tilde{p}_{k0}} = \lambda \frac{\sum_j p_{kj0} m_{kjt}}{\sum_j p_{kj0} m_{kj0}}$$

so that $\tilde{p}_{kt} / \tilde{p}_{k0}$ fails proportionality and thereby identity too ($\lambda = 1$).

6.4. S need not vanish when the structure between CNs remains constant

Assume that total quantities of each CN remains constant, that is $Q_{kt} = Q_{k0}$ for all k = 1, …, K. This case is not sufficient for the S-effect to disappear. According to eq. 7 the ratio of unit values (or Drobisch's index) then is equal to the value index $\tilde{p}_{kt} / \tilde{p}_{k0} = \sum_j p_{kjt} q_{kjt} / \sum_j p_{kj0} q_{kj0}$ within the k^{th} CN and because of eqs. 3 and 3a we also get in this case

$V_{0t} = PU_{0t}^{P} = PU_{0t}^{L}$ and $QU_{0t}^{L} = QU_{0t}^{P} = 1$, $S = Q_{0t}^{L}$.

Hence the S-effect reduces to the Laspeyres quantity index and the discrepancy to $D = Q_{0t}^{P}$.

6.5. How an individual CN contributes to the S-effect

For a better understanding of the nature of the S-effect we now try to find out to which extent a given CN tends to raise (or lower) S. To this end the following formulas – introduced already in sec. 6.1 (eq. 14) - proved useful (they easily follow from the respective definitions)

(20) $$QU_{0t}^{L} = \sum_k \tilde{Q}_{0t}^{k} s_{k0} \text{ and}$$

(21) $$Q_{0t}^{L} = \sum_k Q_{0t}^{L(k)} s_{k0}.$$

where $\tilde{Q}_{0t}^{k} = Q_{kt} / Q_{k0}$ and $Q_{0t}^{L(k)} = \sum_j q_{kjt} p_{kj0} / \sum_j q_{kj0} p_{kj0}$ and expenditure shares s_{k0} (or w_0) are defined as $s_{k0} = Q_{k0} \tilde{p}_{k0} / \sum_k Q_{k0} \tilde{p}_{k0} = \sum_j p_{kj0} q_{kj0} / \sum_k \sum_j p_{kj0} q_{kj0}$. Hence S can be expressed as

(22) $$S = \frac{Q_{0t}^{L}}{QU_{0t}^{L}} = \sum_k \frac{Q_{0t}^{L(k)}}{\tilde{Q}_{0t}^{k}} \cdot \frac{\tilde{Q}_{0t}^{k} s_{k0}}{\sum_k \tilde{Q}_{0t}^{k} s_{k0}} = \sum_k \frac{Q_{0t}^{L(k)}}{\tilde{Q}_{0t}^{k}} \cdot \frac{\tilde{p}_{k0} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}}.$$

The ratio $Q_{0t}^{L(k)} / \tilde{Q}_{0t}^{k}$ is reflecting the contribution of the k_{th} CN to S. This particular CN tends to raise (lower) the S-effect when $Q_{0t}^{L(k)} > \tilde{Q}_{0t}^{k}$ ($Q_{0t}^{L(k)} < \tilde{Q}_{0t}^{k}$).The "within-CN" indices $Q_{0t}^{L(k)}$

and $\tilde{Q}_{0t}^{k}$ are are not only two different ways of measuring the development of quantities in the k^{th} CN, they are also *linear* quantity indices we can again make use of Bortkiewicz's reasoning. According to the *generalized* theorem of Bortkiewicz for two linear indices (See von der Lippe (2007), pp. 194 - 196)

$X_t = \frac{\sum x_t y_t}{\sum x_0 y_t}$ and $X_0 = \frac{\sum x_t y_0}{\sum x_0 y_0}$. The ratio between them is given by $\frac{X_t}{X_0} = 1 + \frac{s_{xy}}{\overline{X} \cdot \overline{Y}}$ with the covariance $s_{xy} = \sum \left(\frac{x_t}{x_0} - \overline{X} \right) \left(\frac{y_t}{y_0} - \overline{Y} \right) w_0 = \frac{\sum x_t y_t}{\sum x_0 y_0} - \overline{X} \cdot \overline{Y}$ and weights $w_0 = x_0 y_0 / \sum x_0 y_0$.

The mean of the x_t/x_0 terms is with these weights $\sum \frac{x_t}{x_0} w_0 = \overline{X} = X_0$.

Prices p and quantities q can be assigned to x and y in a number of different ways. What is known as theorem of Bortkiewicz (eq. 9) is the special case

$$x_0 = p_0,\ x_t = p_t,\ y_0 = q_0,\ y_t = q_t,\ w_0 = p_0 q_0 / \Sigma p_0 q_0 \text{ and } X_0 = P_{0t}^{L},\ X_t = P_{0t}^{P},\ \overline{Y} = Q_{0t}^{L}.$$

It turns out that there are two ways of sensibly defining X_t and X_0 (and thus x_t, x_0, y_t, y_0 and w_0) in order to explain either $Q_{0t}^{L(k)} / \tilde{Q}_{0t}^{k}$ in S (according to eq. 22) or $\tilde{Q}_{0t}^{k} / Q_{0t}^{L(k)}$ as elements of S^{-1} given by

$$S^{-1} = \frac{QU_{0t}^{L}}{Q_{0t}^{L}} = \sum_k \frac{\tilde{Q}_{0t}^{k}}{Q_{0t}^{L(k)}} \cdot \frac{Q_{0t}^{L(k)} s_{k0}}{\sum_k Q_{0t}^{L(k)} s_{k0}}. \tag{23}$$

How prices and quantities are assigned to x and y terms is summarized in the following table

Table 6

explain	indices, averages	x and y terms, weights*	covariance
$Q_{0t}^{L(k)} / \tilde{Q}_{0t}^{k}$ $\rightarrow$ S	$X_t = Q_{0t}^{L(k)}$, $X_0 = \tilde{Q}_{0t}^{k}$ $\overline{X} = X_0 = \tilde{Q}_{0t}^{k}$, $\overline{Y} = \tilde{p}_{k0}$	$x_0 = q_0$, $x_t = q_t$, $y_0 = 1$, $y_t = p_0$ $w_0 = q_0/\Sigma q_0$	$s_{xy}^{(1)}$ $\rightarrow$ eq. 24
$\tilde{Q}_{0t}^{k} / Q_{0t}^{L(k)}$ $\rightarrow$ S^{-1}	$X_t = \tilde{Q}_{0t}^{k}$, $X_0 = Q_{0t}^{L(k)}$ $\overline{X} = X_0 = Q_{0t}^{L(k)}$, $\overline{Y} = (\tilde{p}_{k0})^{-1}$	$x_0 = q_0$, $x_t = q_t$, $y_0 = p_0$, $y_t = 1$ $w_0 = p_0 q_0/\Sigma p_0 q_0$	$s_{xy}^{(2)}$ $\rightarrow$ eq. 25

* interchanging of x an y that is $x_0 \rightarrow y_0$ and $x_t \rightarrow y_t$ yields the same result

The first way of defining X_t and X_0 leads to

$$s_{xy}^{(1)} = \sum \left(\frac{q_{kjt}}{q_{kj0}} - \tilde{Q}_{0t}^{k} \right) \left(p_{kj0} - \tilde{p}_{k0} \right) \frac{q_{kj0}}{\sum q_{kj0}} = \frac{\sum_j q_{kjt} p_{kj0}}{\sum_j q_{kj0}} - \tilde{p}_{k0} \tilde{Q}_{0t}^{k} = \tilde{p}_{k0} \left(Q_{0t}^{L(k)} - \tilde{Q}_{0t}^{k} \right) \tag{24}$$

It can easily be verified that

$$\frac{X_t}{X_0} = 1 + \frac{s_{xy}^{(1)}}{\overline{X} \cdot \overline{Y}} = 1 + \frac{\tilde{p}_{k0} \left(Q_{0t}^{L(k)} - \tilde{Q}_{0t}^{k} \right)}{\tilde{p}_{k0} \tilde{Q}_{0t}^{k}} = \frac{Q_{0t}^{L(k)}}{\tilde{Q}_{0t}^{k}}. \tag{24a}$$

The second way yields

$$s_{xy}^{(2)} = \sum \left(\frac{q_{kjt}}{q_{kj0}} - Q_{0t}^{L(k)} \right) \left(\frac{1}{p_{kj0}} - \frac{1}{\tilde{p}_{k0}} \right) \frac{p_{kj0} q_{kj0}}{\sum p_{kj0} q_{kj0}} \tag{25}$$

$$= \frac{\sum_j q_{kjt}}{\sum_j p_{kj0} q_{kj0}} - Q_{0t}^{L(k)} \cdot \frac{1}{\tilde{p}_{k0}} = \left(\tilde{p}_{k0}\right)^{-1} \left(\tilde{Q}_{0t}^{k} - Q_{0t}^{L(k)}\right).$$

It can easily be seen that

(25a) $$\frac{X_t}{X_0} = 1 + \frac{s_{xy}}{\bar{X} \cdot \bar{Y}} = 1 + \frac{\left(\tilde{p}_{k0}\right)^{-1}\left(\tilde{Q}_{0t}^{k} - Q_{0t}^{L(k)}\right)}{\left(\tilde{p}_{k0}\right)^{-1} Q_{0t}^{L(k)}} = \frac{\tilde{Q}_{0t}^{k}}{Q_{0t}^{L(k)}}.$$

Both covariances have merits and demerits. From eq. 24 and 25 follows

(26) $$\left(\tilde{p}_{k0}\right)^2 s_{xy}^{(2)} = -s_{xy}^{(1)}.$$

Thus the covariances necessarily have different signs. The covariance $s_{xy}^{(1)}$ is useful because it relates to S rather than S^{-1}, however, on the other hand $s_{xy}^{(2)}$ can more readily be compared to the covariance responsible for the L-effect, which according to eq. 9a, may be written as

(9b) $$s_{xy}^{(L)} = \sum \left(\frac{q_{kjt}}{q_{kj0}} - Q_{0t}^{L(k)} \right) \left(\frac{p_{kjt}}{p_{kj0}} - P_{0t}^{L(k)} \right) \frac{p_{kj0} q_{kj0}}{\sum p_{kj0} q_{kj0}} = Q_{0t}^{L(k)} \left(P_{0t}^{P(k)} - P_{0t}^{L(k)} \right).$$

In $s_{xy}^{(2)}$ and $s_{xy}^{(L)}$ the same differences $\frac{q_{kjt}}{q_{kj0}} - Q_{0t}^{L(k)}$ between quantity relatives and their mean and the same weights $\frac{p_{kj0} q_{kj0}}{\sum p_{kj0} q_{kj0}}$ are used. However $s_{xy}^{(2)}$ explains 1/S rather than S.

Giving an interpretation in terms of the second covariance it can be stated

$s_{xy}^{(2)} < 0 \rightarrow \tilde{Q}_{0t}^{k} / Q_{0t}^{L(k)} < 1$ or equivalently $Q_{0t}^{L(k)} / \tilde{Q}_{0t}^{k} > 1$ contributing to a positive S effect (that is S > 1), and conversely

$s_{xy}^{(2)} > 0 \rightarrow$ entails a negative S effect (or S < 1). rise to

What matters in $s_{xy}^{(2)}$ are the inverse base period prices and their weighted average $1/\tilde{p}_{k0}$. The admittedly somewhat complicated table 7 (next page) provides a synopsis of all 2^3 = 8 possibilities. Our empirical study reached the conclusion that L < 1 an S < 1 seems to be the most frequent combination. Situations in which this takes place are highlighted in table 7 (for example the left lower field: above average price increases and base period prices go with below average rising or with decreasing quantities).

6.6. Comments on Párniczky

It was only when I presented this paper in Neuchâtel that I became aware of the fact that Párniczky (1974) had already mentioned the formula given in eq. 24. Moreover, he did so with explicit reference to Bortkiewicz. However, he neither showed how the covariance $s_{xy}^{(1)}$ in eq. 24 can be derived from the general theorem, nor considered another covariance $s_{xy}^{(2)}$, the existence of an L-effect in addition to the S-effect or the aggregation of "within-CN covariances" to a unit value index rather than the Drobisch index. Only the S-effect is addressed as "unit value bias" and contributions of sub-aggregates (CNs) to S are aggregated to the Drobisch index.[10]

[10] His analysis is, therefore, valid only for the low level of aggregation. We also do not agree with his main result "that disaggregation in general is not likely to improve the accuracy of the unit value index" (he used the term in

There is also no need to distinguish between a "within-group covariance" ($s_{xy}^{(1)}$) and a "between-group covariance" as Párniczky did, since according to eq. 22 the "contributions" of the CNs to S, that is the terms $Q_{0t}^{L(k)} / \tilde{Q}_{0t}^{k}$, are simply averaged. S is a weighted mean of these terms with weights $\tilde{p}_{k0}Q_{kt} / \sum \tilde{p}_{k0}Q_{kt}$ depending on quantities (not prices) in t. Eq. 22 can also be expressed as follows

(22a) $$S = \frac{\sum_k v_0^k Q_{0t}^{L(k)}}{\sum_k v_0^k \tilde{Q}_{0z}^{k}},\ v_0^k = \sum_j p_{kj0} q_{kj0}\ .$$

So what Párniczky found as the nature and determinants of the "unit value bias" is quite different from our analysis even though our covariance of eq. 24 is already mentioned in his (largely unknown) paper dating back to 1974.

6.7. Illustration with a single CN and two-commodities only

As mentioned already the example of sec. 4.1 cannot display the L-effect because prices in 0 and t are identical, so the covariance $s_{xy}^{(L)} = 0$ and therefore L = 1. However the S-effect occurs although no price changed and we get (we use the notation of sec. 4.1.2 the subscript k or superscript (k) is dropped however)

(27) $$Q_{0t}^{L} = \frac{\tilde{Q}_{0t}(2-\mu-\lambda\mu)}{1+\lambda} = \tilde{Q}_{0t}\left(1+\frac{(1-\lambda)(1-\mu)}{1+\lambda}\right)$$

so that the (contribution to the) S-effect is given by

(28) $$S = \frac{Q_{0t}^{L}}{\tilde{Q}_{0t}} = \frac{2-\mu(1-\lambda)}{1+\lambda} = 1+\frac{(1-\lambda)(1-\mu)}{1+\lambda},$$

Table 7: Signs of L- and S- effect depending on two covariances
Distinctions are made as follows

QR +	quantity relatives above average $q_{kjt}/q_{kj0} > Q_{0t}^{L(k)}$	**QR -**	quantity relatives below average $q_{kjt}/q_{kj0} < Q_{0t}^{L(k)}$
PR +	price relatives above average $p_{kjt}/p_{kj0} > P_{0t}^{L(k)}$	**PR -**	price relatives below average $p_{kjt}/p_{kj0} < P_{0t}^{L(k)}$
P_0 +	base period prices above average $p_{kj0} > \tilde{p}_{k0}$ or $1/p_{kj0} < 1/\tilde{p}_{k0}$	**P_0 -**	base period prices below average $p_{kj0} < \tilde{p}_{k0}$ or $1/p_{kj0} > 1/\tilde{p}_{k0}$

	price relatives **PR +**		price relatives **PR -**	
	P_0 +	**P_0 -**	**P_0 +**	**P_0 -**
QR +	$s_{xy}^{(L)} > 0 \rightarrow \mathbf{L > 1}$ $s_{xy}^{(2)} < 0 \rightarrow \mathbf{S > 1}$	$s_{xy}^{(L)} > 0 \rightarrow \mathbf{L > 1}$ $s_{xy}^{(2)} > 0 \rightarrow \mathbf{S < 1}$	$s_{xy}^{(L)} < 0 \rightarrow \mathbf{L < 1}$ $s_{xy}^{(2)} < 0 \rightarrow \mathbf{S > 1}$	$s_{xy}^{(L)} < 0 \rightarrow \mathbf{L < 1}$ $s_{xy}^{(2)} > 0 \rightarrow \mathbf{S < 1}$
QR -	$s_{xy}^{(L)} < 0 \rightarrow \mathbf{L < 1}$ $s_{xy}^{(2)} > 0 \rightarrow \mathbf{S < 1}$	$s_{xy}^{(L)} < 0 \rightarrow \mathbf{L < 1}$ $s_{xy}^{(2)} < 0 \rightarrow \mathbf{S > 1}$	$s_{xy}^{(L)} > 0 \rightarrow \mathbf{L > 1}$ $s_{xy}^{(2)} > 0 \rightarrow \mathbf{S < 1}$	$s_{xy}^{(L)} > 0 \rightarrow \mathbf{L > 1}$ $s_{xy}^{(2)} < 0 \rightarrow \mathbf{S > 1}$

the sense of Drobisch's index). This is clearly at odds with the conventional wisdom that splitting CNs into smaller (and thus more homogeneous) groups of commodities will reduce the S-effect.

because in the case of a single CN $QU^L_{0t} = \tilde{Q}_{0t}$. Using eq. 5 we get

(28a) $$S = 1 + \frac{2\Delta}{p(1+\lambda)} = 1 + \frac{\Delta}{\tilde{p}_0}.$$

Hence S and Δ are directly related and the S-effect tends to be positive S-1 > 0 when Δ is positive and conversely S -1 tends to be negative when Δ is negative.

The covariance $s^{(1)}_{xy}$ between quantity relatives and base period prices explaining the ratio $Q^{L(k)}_{0t} / \tilde{Q}^k_{0t}$ of the single CN and thus $Q^{L)}_{0t} / QU^L_{0t}$ now amounts to

(29) $$s^{(1)}_{xy} = \sum_j \left(\frac{q_{jt}}{q_{j0}} - \tilde{Q}_{0t} \right) (p_{j0} - \tilde{p}_0) \frac{q_{j0}}{\sum q_{j0}} = \tilde{Q}_{0t} \frac{p_{10}}{2} [(1-\lambda)(1-\mu)] = \tilde{Q}_{0t}\Delta .$$

If total quantities in the CN would remain constant ($\tilde{Q}_{0t} = 1$) and no price changes the difference between the unit values Δ = $\tilde{p}_t$ - $\tilde{p}_0$ is in fact equal to the covariance $s^{(1)}_{xy}$. It should be noted that there is no L-effect in any of the four cases distinguished in table 8 because it is assumed that no price changes.

Table 8 (in analogy to tab. 3)

$\lambda > 1$	II $\lambda > 1$ and $\mu < 1 \rightarrow \Delta < 0$ $s^{(1)}_{xy} < 0 \rightarrow S < 1$	I $\lambda > 1$ and $\mu > 1 \rightarrow \Delta > 0$ $s^{(1)}_{xy} > 0 \rightarrow S > 1$
$\lambda < 1$	III $\lambda < 1$ and $\mu < 1 \rightarrow \Delta > 0$ $s^{(1)}_{xy} > 0 \rightarrow S > 1$	IV $\lambda < 1$ and $\mu > 1 \rightarrow \Delta < 0$ $s^{(1)}_{xy} < 0 \rightarrow S < 1$
	$\mu < 1$	**$\mu > 1$**

Just like $s^{(1)}_{xy}$ the other covariance $s^{(2)}_{xy}$ also does not depend on current period prices p_t but only on quantity relatives and on (reciprocal) base period prices. Using eqs 25 and 27 we get

(30) $$s^{(2)}_{xy} = \frac{\tilde{Q}_{0t}}{\tilde{p}_{k0}} \left(1 - \frac{2 - \mu + \lambda\mu}{1+\lambda} \right) = \frac{\tilde{Q}_{0t}(\lambda - 1)(1-\mu)}{p(1+\lambda)^2/2} = -\frac{\Delta}{(\tilde{p}_0)^2}.$$

In order to bring L ≠ 1 into the play and to study a more general situation prices have got to change and they should change to a different extent. Denote the price relative of good 1 by $\pi = \frac{p_{1t}}{p_{10}}$ and let $\frac{p_{2t}}{p_{20}} = \eta \frac{p_{1t}}{p_{10}} = \eta\pi$ be the price relative of good 2.

It then follows $\tilde{p}_0 = \frac{p}{2}(1+\lambda)$ and $\tilde{p}_t = \frac{p}{2}\pi(2 - \mu + \mu\eta\lambda)$, so that $\tilde{p}_t$ - $\tilde{p}_0$ no longer equals Δ as used in sec. 4.1, but rather

(31) $$\Delta^* = \tilde{p}_t - \tilde{p}_0 = \frac{p}{2}[\pi(2 - \mu(1 - \eta\lambda)) - (1+\lambda))].$$

It can easily be seen that in the case of constant prices $p_{1t} = p_{10} = p$ and $p_{2t} = p_{20} = \lambda p$ considered in sec. 4.1 that is if π = η = 1 the difference Δ* reduces to $\Delta^* = \Delta = \frac{p}{2}(1-\mu)(1-\lambda)$.

By contrast to the S-effect which remains unchanged as expressed in eqs 28 and 28a the price indices and L will no longer amount to unity. As prices are no longer constant we now get

(32) $$P_{0t}^{L} = \frac{\pi(1+\eta\lambda)}{1+\lambda},$$

(33) $$P_{0t}^{P} = \frac{\tilde{p}_t Q_t}{\sum p_o q_t} = \frac{Q_t p\pi(2-\mu+\mu\eta\lambda)}{2Q_t\left(p(1-m_{2t})+\lambda pm_{2t}\right)} = \frac{\pi(2-\mu+\eta\lambda\mu)}{2-\mu+\lambda\mu} \text{ and}$$

(34) $$L = \frac{P_{0t}^{P}}{P_{0t}^{L}} == \frac{2-\mu+\eta\lambda\mu}{1+\eta\lambda}\cdot\frac{1+\lambda}{2-\mu+\lambda\mu} = \frac{2-\mu+\eta\lambda\mu}{(1+\eta\lambda)S} \text{ which implies}$$

(35) $$D = L\cdot S = \frac{PU_{0t}^{P}}{P_{0t}^{L}} == \frac{2-\mu(1-\eta\lambda)}{1+\eta\lambda}.$$ Finally the relevant covariance is

(36) $$s_{xy}^{(L)} = \frac{2\tilde{Q}_{0t}\lambda\pi(1-\mu)(1-\eta)}{(1+\lambda)^2}.$$

Many additional results such as $PU_{0t}^{P} = \frac{\pi(2-\mu+\eta\lambda\mu)}{1+\lambda} = S\cdot P_{0t}^{P}$ or $V_{0t} = \frac{\pi\tilde{Q}_{0t}(2-\mu+\mu\eta\lambda)}{1+\lambda}$ can be derived and may be useful for checking and crosschecking the relations quoted above. It is also interesting to verify that the assumptions $\pi = \eta = 1$ lead to $P_{0t}^{L} = P_{0t}^{P} = 1$ and thus again (as in sec. 4.1) to L = 1, $s_{xy}^{(L)} = 0$, and $V_{0t} = Q_{0t}^{L} = \frac{\tilde{Q}_{0t}(2-\mu+\mu\lambda)}{1+\lambda} = \tilde{Q}_{0t}PU_{0t}^{P}$.

6.8. An alternative view at the determinants of the S-effect

In sec 6.5 the contribution of an individual CN to S defined as S = Q_{0t}^{L}/QU_{0t}^{L} (Laspeyres type indicators of quantity movement) was examined. However, S can also be expressed in terms of Paasche type price indicators S = PU_{0t}^{P}/P_{0t}^{P} which in turn may be conceived as a function of individual CN's contributions to S. P_{0t}^{P} is a weighted mean of the K CN-specific Paasche indices $P_{0t}^{P(k)}$ as shown in eq 12,[11] however, PU_{0t}^{P} cannot be seen this way because weights in eq 11 do not add up to unity. Moreover prices p_{kjt} disappear in the ratio

(37) $$\frac{PU_{0t}^{P}}{P_{0t}^{P}} = \frac{\sum_k \tilde{p}_{kt}Q_{kt}}{\sum_k \tilde{p}_{k0}Q_{kt}}\cdot\frac{\sum_k\sum_j p_{kj0}q_{kjt}}{\sum_k\sum_j p_{kjt}q_{kjt}} = \frac{\sum_k\sum_j p_{kj0}q_{kjt}}{\sum_k \tilde{p}_{k0}Q_{kt}}$$

because $\sum_k \tilde{p}_{kt}Q_{kt} = \sum_k\sum_j p_{kjt}q_{kjt}$ and the resulting expression for PU_{0t}^{P}/P_{0t}^{P} is the same as $S = Q_{0t}^{L}/QU_{0t}^{L}$ in eq. 22 since $\frac{\sum_k\sum_j p_{kj0}q_{kjt}}{\sum_k \tilde{p}_{k0}Q_{kt}} = \frac{Q_{0t}^{L}\sum_k\sum_j p_{kj9}q_{kjo}}{QU_{0t}^{L}\sum_k \tilde{p}_{k0}Q_{k0}}$.

Hence eq 37 does not provide any new insights.

7. Conclusions and final remarks

The discussion of the paper in Neuchâtel revealed that the distinction between the Drobisch formula (eq. 2) and the unit value index (of prices according to a Paasche formula; eq. 3) was indeed not a familiar one. It is true – as was pointed out during the discussion – that both concepts differ only with respect to the higher (second) level of aggregation. A unit value index may be gained from K "low level" Drobisch indices $\tilde{p}_{kt}/\tilde{p}_{k0}$ according to eq. 3. However,

[11] Insofar analogous to eq 21 where Q^L was described as weighted mean of individual $Q^{L(k)}$ indices..

comparing eq. 2a and 3 shows that only the unit value index and not the Drobisch index can be interpreted as a weighted mean of $\tilde{p}_{kt}/\tilde{p}_{k0}$ terms. Furthermore, there are many other aspects (for example, the axiomatic properties), as pointed out in this paper, which require the two indices to be looked at as two distinctive types of price indices.

A clear distinction is also necessary with regard to both the S-effect and the L-effect. These are two quite different phenomena. While the L-effect can be viewed as a substitution between quantities in response to changing prices, the same does not apply to the S-effect. The contribution of a CN to the S-effect is positive on average (S > 1) when there is an increase in the quantities of the commodities which were relatively expensive in the base period. Conversely, when the quantities of low-priced commodities increase, the S-effect tends to be negative (or S < 1). While prices must be changing for the L-effect to occur, the S-effect is possible even with constant prices, provided that the structure of quantities is changing. The fact that S is determined by the covariance $s_{xy}^{(1)}$ in eq. 24 means that the structural change in the quantities has to be correlated with the prices in the base period. It is difficult to imagine the sort of economic behaviour (in terms of utility maximisation) which gives rise to a negative and a positive covariance in each case. However, table 3 indicates the sort of behaviour which entails a positive or negative Δ (the difference in unit values).[12] Furthermore, as $s_{xy}^{(1)}$ is a function of Δ (see eq. 29), table 3 may be understood as a description of the behaviour leading to S > 1 and S < 1 respectively if there are no price changes. With constant prices, S < 1 (Δ < 0) amounts to switching to a structure with lower unit values ($\tilde{p}_{kt} < \tilde{p}_{k0}$). Yet it is difficult to think of a microeconomic theory that is able to explain a change in quantities demanded as a function of base period prices, a change which takes place even when prices remain constant.

In addition to the formal aspects of unit value indices as opposed to genuine price indices, on which the present paper focuses, there are many other aspects that should be considered when an assessment of unit value indices has to be made. As Silver showed in his contribution to the 10th meeting of the Ottawa Group in 2007, there are strong reservations about unit value indices, although they are standard practice in many countries. The reservations may be summarised as follows: these indices do not compare like with like; they violate the principle of pure price comparison. They are not based on the observation of carefully specified goods under comparable conditions using methods for adjustments on quality changes, taking temporary (seasonal) unavailability into account, or for outlier detection and deletion etc. They may be justified – if at all – only as low-budget proxies for survey-based price indices. Moreover, there are reasons to expect ever more difficulties with customs-based statistics, of which a unit value index is an example. We observe an increasing proportion of trade in services rather than in goods that physically cross borders or e-trade and intra-area trade within customs unions without customs documents on which statistics could be based. Such developments tend to reduce the scope and reliability of customs statistics. Hence, in addition to the bias, as a result of the S-effect in particular there are quite a few reasons to distrust unit value indices and to attempt to replace them more and more with survey-based price indices.

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[12] If S is taken in isolation (with no change in prices), otherwise Δ* has to be considered.

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