PRICE INDICES AND UNIT VALUE INDICES IN GERMAN FOREIGN TRADE STATISTICS

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1. INTRODUCTION

Very few countries are able to provide both, a unit value index (PU) and a true price index (P) on a regular (mostly monthly) basis. Fortunately Germany is one of those countries which offer the opportunity to study the impact of the still not well understood methodological differences of the two tools of measuring the price development in export and import. While a PU-Index is basically resulting from foreign trade statistics as a kind of by-product, the compilation of a true price index is much more demanding. It requires special surveys addressing exporting and importing establishments as well as compliance with some principles of price statistics among which aiming at "pure price comparisons" is most prominent. This implies in turn making adjustments (of reported prices) for quality changes in the traded goods or avoiding changes in the collection of goods, reporting firms or in the countries of origin (in the case of imports) or destination involved. By contrast there is no need of satisfying such requirements in the production of PU-indices. Hence the PU type of index is popular though much less commendable from a theoretical point of view. This gives rise to relate the main empirical differences between the PU-index and the P-index observed using German data to their respective conceptual and methodological characteristics. Above all PU- and P-indices of export and import respectively differ with regard to their level and volatility. PU indices tend to display a relative to P-indices more moderate rise of prices combined with more accentuating oscillations as shown in figure 1.

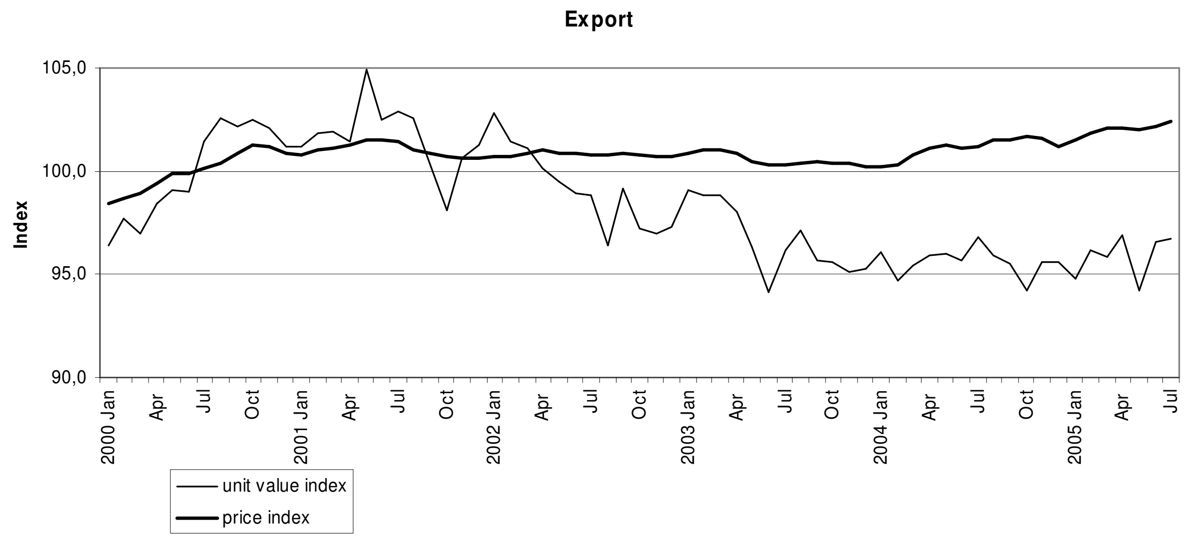
The present paper tries to relate the index formulas used in the case of the German PU- and P-index respectively in order to describe the differences in terms of quantifiable "effects". Another motivation may be expressed in questions like

- what can we learn from the observed differences between the two gauges of price movement?
- can we make use of the more readily available unit values as building blocks in order to facilitate the complicated compilation of P-indices?

Finally a general concern of the paper is to demonstrate the limitations of PU-indices which to date are unfortunately not often played down.

The paper is organised as follows. Section 2 defines the notion of "unit values" and the index formulas based on them as opposed to the traditional "true" price and quantity indices usually well known from the relevant statistic textbooks. In addition to different formulas, the indices to be compared, differ also with respect to concepts, data sources and definitions (of prices for example) giving rise to further empirical investigations presented briefly in section 5.

Section 3 summarizes some empirical findings, showing in particular the influence of the type of goods in question. Seasonal fluctuations reflected in PU indices but not in P-indices are for example more effective in the case of apparel, than in the case of chemical products.



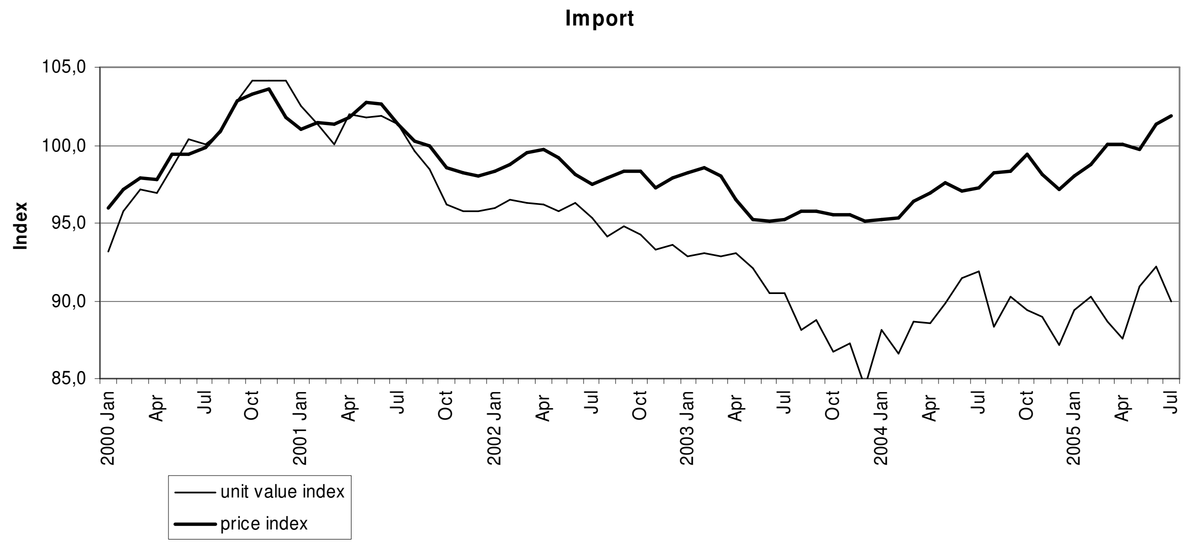


Figure 1 Unit value indices and price indices in German foreign trade

Section 4 introduces what might be called a formal theory of PU-indices that is in terms of formulas and "axioms" and section 5 and 6 briefly report some results of econometric estimations inspired from an investigation in conceptual differences in sec. 2. Section 7 concludes.

2. DEFINITIONS AND FUNDAMENTAL RELATIONSHIPS

2.1. Unit values and indices made thereof

Let k = 1,…K denote the k-th group of goods (GG for short) a collection of related (preferably homogeneous) goods as regards use and/or the manufacturing process for which a common unit of quantity (e.g. kilograms) is used and meaningful. The so called unit value of the k-th GG at period t (a kind of average price) is given by

$$(1) \qquad \tilde{p}_{kt} = \frac{\sum p_{kjt} q_{kjt}}{\sum q_{kjt}} = \frac{\sum p_{kjt} q_{kjt}}{Q_{kt}}$$

where the summation takes place over all goods included in the kth GG.[1]

In the case of foreign trade statistics[2] unit values for more or less broadly defined GGs are easily calculated as both "values" $V_{kt} = \sum p_{kjt} q_{kjt}$ (numerators of unit values) as well as quantities (denominators) $Q_{kt} = \sum q_{kjt}$ are elements of foreign trade statistics. Note that no separate statistical inquiry of prices of individual goods p_{kjt} is necessary. Indices of the unit value type designed to measure price movements are comparing present unit values $\tilde{p}_{kt}$ with base period unit values $\tilde{p}_{k0}$. Correspondingly a quantity index of the unit value type is made of aggregated) quantities Q_{kt} and $Q_{ko} = \sum q_{kjo}$ as opposed to a (true) quantity index composed of individual quantities.

Assuming, in contradiction to the actual facts of index computations in Germany though, that prices and quantities of all n commodities enter the formulas of "true" index numbers, summation over i = 1,…, n would be equivalent to a double summation over all K groups of goods (GGs) and their respective commodities j = 1, …, m_k ($\sum m_k = n$). In theory we would arrive at a system of eight index formulas as presented in figure 2 where superscripts L and P stand for Laspeyres and Paasche respectively.

	Index - concept	
	(true) Price index concept	Unit value index concept
Piceindex	P^L , P^P	PU^L , PU^P
Quantityindex	Q^L , Q^P	QU^L , QU^P

Figure 2 System of eight indices

In actuality, "true" formulas are not comprehensive but based on a fixed sample of selected goods. In addition to the value index (or value ratio) $V_{0t} = V_t/V_0$, only 3 out of the 8 formulas of fig. 2 are in use in Germany, that is

$$(2) \quad PU_{0t}^{P} = \frac{\sum_k \tilde{p}_{kt} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}} = \frac{\sum_k^K \sum_j^{m_k} p_{kjt} q_{kjt}}{\sum_k^K Q_{kt} \left(\sum_j^{m_k} \frac{p_{kj0} q_{kj0}}{Q_{k0}} \right)},$$

$$(3) \quad P_{0t}^{L} = \frac{\sum_k \sum_j p_{jkt}\, q_{jk0}}{\sum_k \sum_j p_{jk0}\, q_{jk0}} = \frac{\sum_i p_{it}\, q_{i0}}{\sum_i p_{i0}\, q_{i0}}, \text{ and}$$

[1] In general unit values as such (rather than unit values as elements of an index formula) are not an object of interest. It sounds rather strange when some Austrian authors (Glatzer et al. 2006) state that import prices in Austria amount to about 20 € per kilogram with the explicitly mentioned consequence that a reduction in weight is equivalent to a rise in "prices" (p. 11, 17).

[2] Statistics of wages and salaries is another field in which frequently use is made of unit values. Often wage sums paid for a group of similarly qualified employees in charge of comparable type of work are readily available allowing the calculation of average wages whereas it would be difficult if worthwhile to derive a statistic of wages based on truly comparable qualifications.

(4) $$QU^L_{ot} = \frac{\sum_k \tilde{p}_{k0}\ Q_{kt}}{\sum_k \tilde{p}_{k0}\ Q_{k0}},$$

Under the assumptions made values can be derived in both ways using unit values as well as individual prices so that

(5) $$V_{0t} = \frac{\sum_k \tilde{p}_{kt} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{k0}} = \frac{\sum_k^K \sum_j^{m_k} p_{kjt} q_{kjt}}{\sum_k^K \sum_j^{m_k} p_{kj0} q_{kj0}} = \frac{V_t}{V_0}.$$

This leads to the following identities

(6) $$V_{0t} = \frac{\sum p_t q_t}{\sum p_0 q_0} = PU^L_{0t} QU^P_{0t} = PU^P_{0t} QU^L_{0t} = P^L_{0t} Q^P_{0t} = P^P_{0t} Q^L_{0t}$$

serving as our starting point in section 4. It should be noted that the interesting comparison is not between P^L_{0t} and P^P_{0t} -where much theory already exists- but between P^L_{0t} and PU^P_{0t} where new ground is to be broken.

The observed differences between the time series of unit value indices and true price indices in German exports and imports are not only attributable to (idealized or simplified) differences in the formulas, but also stem, in no small measure, from conceptual and procedural differences in index compilation.

2.2. Conceptual differences

Table 1 exhibits some of the most influential conceptual differences between the PU- and the P-index of foreign trade (export and import) in German statistics. They are best understood by considering the type of measurement the two approaches are taking. A price index, such as P^L_{0t} in particular, intends to achieve a "pure" price comparison where the index reflects the changes of prices only. It therefore should not be "contaminated" by simultaneous changes in the qualities and quantities of goods as well as other price determining characteristics (such as reporting firms, countries involved, etc.). Price indices are compiled on the basis of the selection of preferably identical goods which may, however, with the passage of time, become progressively less relevant or "representative" of all traded goods (requiring the updating of this selection at certain time intervals). In contrast to P-indices a PU-index encompasses all goods and is hence affected from a number of influences and structural changes[3].

Notice that the principles of "pure price comparison" on the one hand, and "representativity" on the other are almost antagonistic and difficult to reconcile. It is therefore not surprising that the merits of one approach coincide with the demerits of the other. This observation strongly lends support to our view that both index approaches, P-indices as well as PU-indices are justifiable in their own right. It is not contradictory, however, to consider P-indices as a superior and more refined type of price measurement from an axiomatic point of view[4]. According to table 1 some consequences may readily be hypothesised. So it seems plausible that PU-indices may be lagging behind P-indices because prices at crossing the border of a country are

[3] From the point of view of pure comparisons such changes should be represented in the quantity dimension rather than price dimension.

[4] Criteria of index theory (in the way of axioms) as well as the justification of the idea "pure price comparison" are discussed in detail in P. v. d. Lippe, Chain Indices, A Study in Price Index Theory, Wiesbaden 2001.

referring to a later point in time than prices agreed upon in contracts. We may also conjecture that the omission of quality adjustments of PU-indices may contribute to their comparatively high volatility. Conclusions of this sort can be derived from table 1, and tested empirically in section 5.

Table 1: Comparison of true price and unit value (price) indices

	(True) Price index (P-index)	Unit value price index (PU-index)
What is measured?	How the prices of ideally the same products of a given (fixed) collection of products are developing over time	Unit value (average value) of all products of a certain type (e.g. all exported goods) at two points in time
New and disappearing goods	Price of new goods are included only when a new base period is defined (i.e. the index is rebased); vanishing good should, if possible, be replaced by similar goods.	New goods enter immediately the formula. The price quotation of disappearing goods is simply discontinued. No "corrections" are made in the case of incomparability.
Prices	Prices refer to the time of contracting; they express the valuation agreed upon in the contract.	Prices are implicitly given by cross-border values (at the time of crossing the frontier of a country)
Merits	P-indices guarantee pure price comparison by keeping the selection constant and making adjustments for quality changes	PU-indices satisfy "representativity" by inclusion of *all* products (complete coverage instead of a selection); no quality adjustments are made.
Demerits	Representativity is said to be impaired; a lot more demanding as far as price collection, empirical foundation of weights and quality adjustment is concerned	PU-indices are influenced by changes in the composition of the products in the group. A structural change may be reflected in the average price rather than in the quantity (volume) dimension*

* A mere switch from cheaper to more expensive products within a group of commodities for which a unit value is established is producing a rise in the PU-index (and thus in the price dimension which thereby is overstated since prices remained unchanged); using PU_{0t} (instead of P_{0t}) as deflator therefore may overstate price and understate volume change.

3. EMPIRICAL FINDINGS: THE CASE OF GERMANY

The section starts with a short description of the data and the statistics designed to highlight the characteristic differences between the time series of PU- and P-indices. We then study in more detail the already mentioned two major differences, viz the lower level and greater volatility of PU-indices compared to P-indices.

3.1. Sample and descriptive statistics

Our data, taken from the database of the Deutsche Bundesbank[5], cover T = 67 monthly observations of index numbers starting with January 2000. Both indices are structured according to a commodity classification. It permits comparisons of export and import prices between dif-

[5] This paper is summarizing a first pilot study of an ongoing project in cooperation with the Bundesbank, reporting some results gained from calculations that a student from the University of Duisburg made when he was a trainee of the bank in 2005.

ferent groups of goods (services are of course not object of foreign trade statistics). Three measures of "discrepancies" between a PU-index and the corresponding P-index, are calculated

(7) $$D1 = \frac{1}{T}\sum_t \left(PU_{0t} - P_{0t}\right) = \sum d_t / T,$$

(8) $$D2 = \sqrt{\sum (d_t)^2 / T},$$

i.e. the arithmetic and quadratic mean of the deviations d, and finally a relative indicator not depending on the level of the indices in addition to the absolute indicators.

(9) $$D3 = \left[\frac{1}{T}\sum_t (d_t / P_{0t})^2\right]^{1/2}.$$

The advantage of D1 is its ability to show the direction of the deviation, not only the amount. As D2 and D3 tell much the same story as D1 we will report D1-results only in what follows. Obviously $(D2)^2 - (D1)^2$ is the variance of the d-differences.

A striking phenomenon is the much greater volatility of the PU-indices as compared to the P-indices. We follow the common practice of taking the coefficient of variation (CV) as the gauge of dispersion or "volatility".

3.2. Relevance of goods and discrepancies between price indicators

In order to focus on the most significant groups of goods table 2 displays the five divisions (two-digit-codes) of the German commodity classification[6] which contribute most to the export and import values in our sample. Interestingly these five most important groups of exported goods (divisions), are also the most important groups of imports.

Table 2: The six important groups of goods (divisions of GP 2002)[7] and their discrepancies

Division	VS (X)	VS (M)	D1 (X)	D1 (M)
29 Machinery and equipment	19.78	10.14	- 4.13	- 5.50
24 Chemicals and chemical products	12.85	11.25	- 4.71	- 4.10
15 Food products and beverages	6.90	8.27	- 1.62	- 5.50
31 Electrical machinery and apparatus	6.69	5.06	- 3.62	- 8.87
27 Basic metals (iron, steel etc.)	6.14	7.06	- 1.49	0.06

VS = Value share, D1 = measure of discrepancy between PU- and P-indices (see eq. 7 multiplied by 100 that is D3 measured in percent)

Divisions 34 and 33 (motor vehicles and medical, precision and optical instruments) are ranking next with regard to exports, while Division 01 and 32 (agriculture, radio and television etc.) do so in the case of imports. Differences between the two measures of price dynamics,

[6] Güterverzeichnis für Produktionsstatistiken 2002 (GP2002), a German adaptation of the so-called PRODCOM-list a European classification which in turn is closely related to international standard classifications of goods.
[7] We take here divisions (two-digit classification units) as groups of goods (GGs). In practice index calculations in terms of unit values are generally making use of more narrowly defined GGs, each of which is comprising still a number of goods (otherwise the difference between P- and PU-indices would fade away).

using D1, are not spectacular but rather in the vicinity of the overall mean of D1 (export: 2.52, import: 4.37). Sizeable differences in D1, particular in the case of exports, were as follows

40 energy (electricity, gas etc.) – 33.59
11 crude petroleum and natural gas - 15.51
14 other mining and quarrying[8] - 13.20,
and in the case of imports the following divisions
11 petroleum etc. (see above) + 12.68[9]
30 office machinery and computers - 10.64
31 electrical machinery (see above) - 8.87

In accordance with our expectations the statistic D1 is overwhelmingly negative, as a consequence of the fact that the PU-index has a tendency to become progressively lower, over time, than the P-index. Classified in broader categories such as "consumption goods", "machinery and equipment", "materials and supplies", etc., it is remarkable that discrepancies (D-measures) are highest in the case of consumer durables where buyers are more likely to switch to lower priced models in response to a price increase. The fact that PU-indices reflect changes in the composition of goods (a factor which later will be called the "structural component") may also explain a second major finding: the volatility of PU-indices is significantly greater than that of the corresponding P-indices which in general are much smoother.

3.3. Volatility and seasonality of indices

As mentioned before, we use the coefficient of variation, denoted CV_U and CV_P respectively applied to PU- and P-indices to express the amount of volatility. The greater volatility of PU (that is $CV_U > CV_P$ holds fairly generally) is due to the fact that PU-indices are based on constantly changing sets of goods that pass the border, while P-indices are compiled using only an invariant sample of those goods, which is kept as constant as possible.

Table 3: The three divisions with most volatile PU- and P-indices of export and import

	first	second	third
CV_U (X)	40: 0.1672*	23: 0.1316	18: 0.1312
CV_P (X)	40: 0.2707	23: 0.1342	11: 0.1302
CV_U (M)	40: 0.1885	11: 0.1819	32: 0.1733
CV_P (M)	40: 0.1818	23: 0.1769	30: 0.1496

* Number of the division (40) : value of the coefficient of variation (0.1672)

23 = refined petroleum products etc., 18 = apparel, dressing,
other codes are already explained above

By the same token, PU-indices are much more reflecting seasonal variations or changes in other aspects as e.g. the regional composition of exports and imports. An example of the first aspect (seasonal fluctuations) is division 18 "wearing apparel, clothing" (as contrasted with

[8] Except energy producing materials and metal ores, hence mainly covering stone, chalk and the like.
[9] This figure for division 11 (petroleum etc.) is one of only two examples in which D3 is (distinctly i.e. > 1) positive indication that the P-index is on average lower rather than higher than the P-index. The second case is motor vehicles (division 40) where D1 is amounting to 4.04.

17 = textiles) where PU is much more volatile than P in both in the case of exports[10] as shown in figure 3. Prices of tobacco products (division 16) are volatile especially if measured in unit values due to the fact that the countries of origin or destination vary a lot. Volatility of PU-indices (CV_U) may possibly cause divergence between PU- and P-indices, given that the latter are less volatile. It is therefore appropriate to look at CV_P (volatility of true price indices) too as done in table 3 or to study systematically the ratio CV_U/CV_P. This ratio is taking on high values in divisions like wearing apparel (18) in exports, motor vehicles (34) and furniture (36) in imports, and textiles (17) publishing, printing and reproduction (22) in both directions, imports as well as exports.

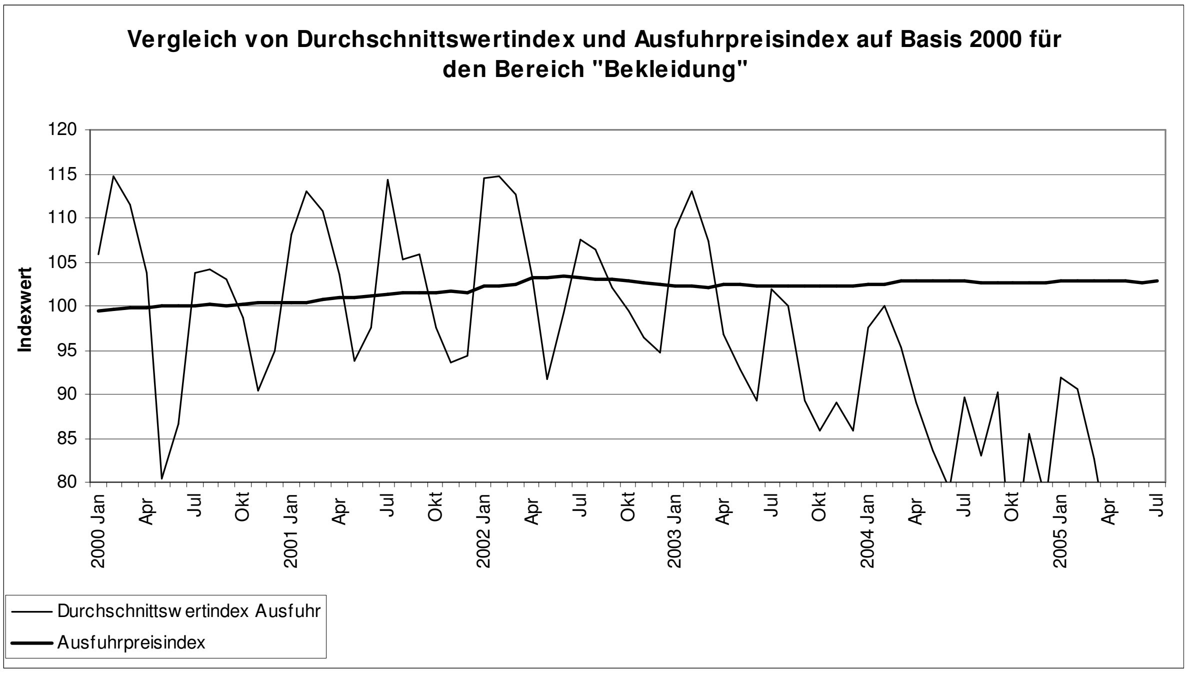


Figure 3 German exports in division 18 “wearing apparel, clothing”

None of these divisions are also mentioned in table 2 such that widely differing PU- and P-values can in general not be accounted to excessively volatile PU indices. It is therefore better to keep the level- and the volatility issue separate. A satisfactory "theory" of unit value indices should be able to explain both frequently observed phenomena, viz. discrepancy (PU ≠ P) and the more volatile PU-index. In the following an attempt is made to trace the first phenomenon back to the different index-formulas used.

4. PROPERTIES OF UNIT-VALUE-INDICES

4.1. Components of the discrepancy

One might be tempted to explain the fact that $PU_{0t}^{P} < P_{0t}^{L}$, that is the official German unit value index (PU_{0t}^{P}) is as a rule falling short of the corresponding (time) price index $\left(P_{0t}^{L}\right)$ with

[10] In division 18 CV_U (X) = 0.1312 is about 12 times CV_P (X) = 0.0109 because the type of wearings represented by PU is in summer much different from winter. The same applies to a smaller extent, however, to imports, that is CV_U (M) > CV_P (M) of wearing apparels.

a recourse to a formula found by Ladislaus v. Bortkiewicz, according to which the covariance C between p_{it}/p_{i0} and q_{it}/q_{i0} respectively, the price and quantity relatives is given by

(10) $C = V_{0t} - P^L_{0t}Q^L_{0t} = P^L_{0t}(Q^P_{0t} - Q^L_{0t})$

Hence the Paasche formula yields lower values than the Laspeyres formula whenever the covariance is negative. As in practice (and for example in the so called "economic theory of index formulas") the situation C < O is exclusively considered it is often said that the Laspeyres formula tends to overrate the price movement (much like Paasche is underrating it), which is referred to as Laspeyres- or simply ***L-effect***. A negative covariance may arise from rational substitution among goods in response to price changes on a given (negatively sloped) demand curve.
It should be borne in mind, however, that the comparison in question is not between P^P_{0t} and P^L_{0t} but rather between PU^P_{0t} and P^L_{0t}. Under such conditions a second component of the discrepancy is coming into play which may well reinforce but would also counteract the L-effect. This factor is called structural component or ***S-effect*** for short and refers to changing quantities within a GG (for example switching from a high-priced to a relatively cheap good belonging to the same GG). The two effects, L and S will both show up in

(11) $$D = \frac{PU^P_{0t}}{P^L_{0t}} - 1 = \left(\frac{C}{QU^L_{0t}P^L_{0t}}\right) + \left(\frac{Q^L_{0t}}{QU^L_{0t}} - 1\right) = L + S$$

an equation easily derived from eq. 6 using eq. 10. Hence although $C < 0$ and therefore necessarily $P^P_{0t} < P^L_{0t}$ (negative L-effect) the unit value index PU^P_{0t} (instead of P^P_{0t}) may still equal or exceed P^L_{0t} (that is $D \geq 0$)[11] simply because a negative L is offset or outstripped by a positive S[12]. The problem of eq. 11 is, however, that we are lacking data as there is no Q^L_{0t} index compiled in practice, which prevents us from carrying out empirical studies. We therefore have to confine ourselves to a numerical example which will also serve as an illustration of what is meant by L and S.

4.2. A fictitious numerical example

Assume two groups of goods (GGs), A and B, each composed of two goods, 1 and 2 in the case of A and 3 and 4 in the case of B. Total quantities in both GGs are kept constant such that $Q_{k0} = Q_{kt} = 10$. The quantities q_A and q_B are introduced in order to simulate structural changes with in the GGs. Table 4 is putting together all figures needed to calculate the index formulas of eq. 11 as functions of q_A and q_B.

Note that things rapidly will become more complex and intricate once we abandon the simplifying assumptions K = 2, $m_1 = m_2 = 2$, and in particular $Q_{k0} = Q_{kt}$ which yields $QU^L_{0t} = 1$ and therefore[13]

[11] Note that D in eq. 11 is a relative measure of discrepancy similar to D3 (eq. 9) with the difference that no averaging over time (in terms of a quadratic mean) is taking place here.
[12] This is in fact a situation which is taking place in the right wedge of fig. 4 included to illustrate the numerical example presented in section 4.2.
[13] Here and in what follows it may be more convenient to drop the subscripts 0 for the base period, and t for the actual observation period.

Table 4: Numerical example

	p_0	p_t	q_0	q_t
1 (A)	8	10	5	q_A
2 (A)	4	7	5	$10 - q_A$
3 (B)	7	9	5	q_B
4 (B)	6	4	5	$10 - q_B$

(11a) $\underbrace{Q^P - 1}_{=D} = \underbrace{Q^P - Q^L}_{=L} + \underbrace{Q^L - 1}_{=S}$

is taking the part of eq. 11. As a consequence we get two straight lines for q_B as a linear function of q_A, a positively sloped line (left part of fig. 4) separating positive from negative C-values (and thus positive from negative L-effects), and a negatively sloped line delimiting positive D-values (upper right area) and negative D-values (lower left are).

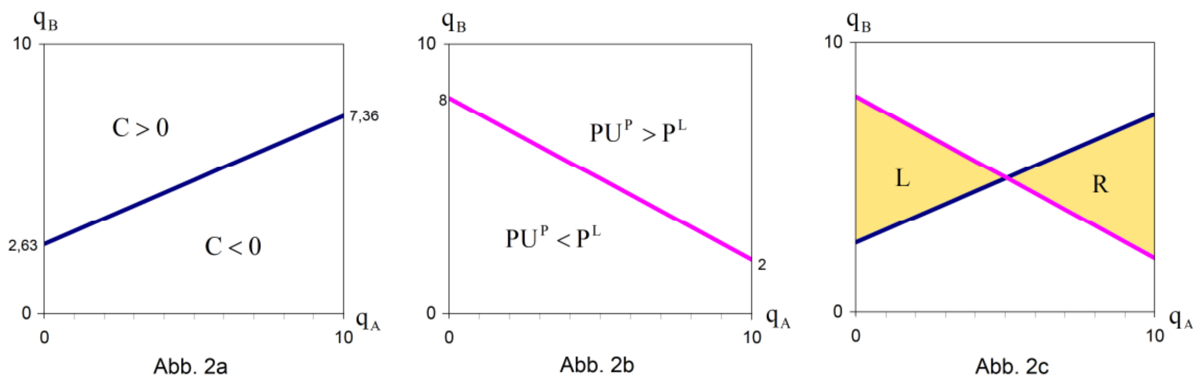


Figure 4 Impact of L-effect and S-effect on D (eq. 11a) in the numerical example of table 4

When putting both lines together a configuration with a left (L) and right (R) "wedge" is created, in which the effects L and S are acting in opposition to one another. In the two trapezoid areas, however, not highlighted in the right part of fig. 4, the effects are tending both in the same direction

4.3. Interpretation of L and S

Situations in which S vanishes are firstly each commodity group (GG) consists of one good only ($m_k = 1$ for all k, the maximum possible homogeneity of GGs) or secondly the structure of goods within a CG remains constant. This easily follows from

(12) $QU_{0t}^L = \sum_k Q_{kt}\tilde{p}_{k0} \Big/ \sum_k Q_{k0}\,\tilde{p}_{k0} = \sum_k Q_{kt} \sum_j m_{kj0} p_{kj0} \Big/ \sum\sum q_{kj0} p_{kj0}$

where $m_{kj0} = q_{kj0} \Big/ \sum q_{kj0}$ as opposed to

(13) $Q_{0t}^L = \sum_k Q_{kt} \sum_j m_{kjt} p_{kj0} \Big/ \sum\sum q_{kj0} p_{kj0}$ where $m_{kjt} = q_{kjt} \Big/ \sum_j q_{kjt}$ giving

(14) $$Q^L_{0t} - QU^L_{0t} = \sum_k Q_{kt} \sum_j p_{kj0}\left(m_{kjt} - m_{kj0}\right) \Big/ V_0$$

Hence a constant structure $m_{kjt} = m_{kj0}$ for all k, j results in S = 0 which may justify the term structural component (a phenomenon, by definition, non-existent in Q^L). In the absence of a structural change we also have $PU^P_{0t} = P^P_{0t}$ and D boils down to $D = P^P_{0t}/P^L_{0t} - 1 = L$.

If on the other hand L = 0 eq. 11 shows that PU^P can differ from P^L although $P^P = P^L$ (as C = 0), which in turn is possible only if QU^L differs from Q^L, that is, if S is effective. A point on the left straight line with a positive slope, for example on the lower boundary of the left wedge, is given by $q_A = 10/3$, $q_B = 80/19$. The reader may easily verify that we then get C = L = 0 and $PU^P = 1.128$ while $P^L = P^P = 1.2$, furthermore $Q^L = Q^P = 0.94$ while $QU^L = 1$. The difference between Q^L and QU^L, to be interpreted as a weighted sum of differences between the m-coefficients according to eq. 14 (i.e. reflecting the structural change within the GGs) amounts to -6 %. As the covariance vanishes, the fact that PU^P is falling short of P^L by 6 percent is only because of the S-effect, by virtue of which Q^L is 6 % less than QU^L. In the example S amounts to Q^L-1 (as $Q_{kt} = Q_{k0}$). The structural changes $m_{kjt} - m_{kj0} = \Delta_{kj}$ are given by -1/6, 1/6, - 3/38, and 3/38. In combination with prices p_{kj0}, acting as weights, the terms $\sum p_{kj0}\Delta_{kj}$ are generating a negative difference amounting to $Q^L - 1 = -0.06$.
The absence of structural changes within the GGs (that is S = 0) cause $Q^L < QU^L = 1$ as well as $PU^P < P^P$ both relative differences being equal. Structural changes can also be responsible for unit values showing a price movement although no price has changed.

4.4. Axiomatic defects of unit value indices

The fact that price and quantity gauges of the unit value type are reflecting a structural change (and do not therefore represent a pure price or quantity comparison respectively) can well yield awkward results which are not tolerable from an axiomatic point of view. Assume that no price changes that is $p_{kjt} = p_{kj0}$ $(\forall k, j)$. Nonetheless, as the following equation shows

(15) $$d_k = \tilde{p}_{kt} - \tilde{p}_{k0} = \sum\nolimits_j p_{kj0}\left(\frac{q_{kjt}}{Q_{kt}} - \frac{q_{kj0}}{Q_{k0}}\right) = \sum\nolimits_j p_{kj0}\left(m_{kjt} - m_{kj0}\right)$$

unit values need not remain constant, that is d_k may differ from 0. Thus unit values do not reflect price movements only. They violate the identity axiom of price index theory. For a similar reason they do not necessarily satisfy the mean value property. Expressed as weighted sum of price relatives p_{jkt}/p_{kj0} PU^L is given by

(16) $$PU^L_{0t} = \sum_k^K \sum_j^{m_k} \frac{p_{kjt}}{p_{kj0}}\left(p_{kj0}q_{kj0}\frac{m_{kjt}}{m_{kj0}}\right) \Bigg/ \sum_k^K \sum_j^{m_k} p_{kj0}q_{kj0}\,,$$

and a similar formula applies to PU^P. The weights (in brackets) in the numerator will in general not add up to $V_0 = \sum p_{kj0}q_{kj0}$ in the denominator of the right hand side of eq. 16 unless we have no structural change (S = 0 because $m_{kjt} = m_{kj0}$).
These defects of indices in terms of unit values are already well understood, at least on the part of price indicators such as PU^P and PU^L. The System of National Accounts (SNA)[14] therefore rightly made some reservations as to the use of unit value (price) indices as an alternative to P^P or P^L. They were rightly rejected with the argument

[14] The System of National Accounts 1993 is a bulky manual prepared by the "Inter-Secretariat Working Group", the members of which were the Commission of the European Communities, the IMF, OECD, World Bank and the UN in order to harmonize National Accounts worldwide.

> Unit value indices are "affected by changes in the mix of items as well as by changes in their prices. Unit value indices cannot therefore be expected to provide good measures of average price change over time" (SNA 1993, paragraph 16.13).[15]

PU- and QU-indices may generally be viewed as crude measures of price and quantity levels as they are based on average prices and sums of quantities rather than individual prices and quantities. Not surprisingly…
Assume that each GG is consisting of one good only or the structure of the GG is remaining constant the equation

(17) $$D^* = \frac{P_{0t}^P}{P_{0t}^L} - 1 = \frac{C}{Q_{0t}^L P_{0t}^L} = L^*$$

Note how D* and $D = PU_{0t}^P / P_{0t}^L - 1$ in eq. 11 differ. Moreover L* is related to L as follows L = L*(1+S) where S is given by

(18) $$S = \frac{Q_{0t}^L}{QU_{0t}^L} - 1 = \frac{PU_{0t}^P}{P_{0t}^P} - 1 = \frac{\sum_k Q_{kt} \sum_j p_{kj0} (m_{kjt} - m_{kj0})}{\sum_k Q_{kt} \sum_j p_{kj0} m_{kj0}} ,$$

where the denominator can also be expressed as $V_t / PU_{0t}^P = V_0 QU_{0t}^L = \sum_k Q_{kt} \tilde{p}_{k0}$.

The situation described (homogeneous GGs or GGs represented by one good only and therefore $m_{kj0} = m_{kj1} = 1$)[16] amounts to S = 0, $PU^P = P^P$ and $QU^L = Q^L$ or equivalently = eq. 11 "reduces" to eq. 17. The difference between the two equations is owed to the inhomogeneity of GGs used as building blocks of indices of the unit value type.

5. ADDITIONAL EMPIRICAL RESULTS

5.1. Homogeneity of groups of goods

From the above follows that one reasonably might expect a growing discrepancy in the vales of D (or D1 through D3 of eq. 7 – 9) to the extent that the GGs to which PU (or P respectively) is referring are increasingly heterogeneous[17]. We therefore conjecture that a sub-index of PU_{0t}^P (compiled for a division of the commodity classification) should differ more from the corresponding P_{0t}^L sub index if the division in question is less homogeneous. The 31 divisions (two-digit classificatory units) are not only comprising a widely different number of subdivisions (commodity codes) in the classification but also of price quotations (or contracts or "series") in the official P-index ranging from 3 (in the case of division 13 which is metal ores) to 1067 (machinery and equipment, division 29). Furthermore it should be noticed that each division is represented by a much greater (and basically unknown and varying) number of actual models and varieties being exported or imported. Hence it is difficult if not almost impossible to state the true degree of heterogeneity of the elementary GGs used for the compilation of a PU-index. Moreover, there is no formula available for a variable H, the degree of

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[15] However the SNA apparently (and amazingly) did not realise that the same type of objections can also be raised against chain indices, advocated with great vigour by the SNA.

[16] This may apply approximately also in the case of homogeneous goods ($m_k > 1$) or in the case $m_{kjt} = m_{kj0}$.

[17] The argument set out above implies, however, that the PU-index may differ more of less from a P-index depending on the classification defining the GGs, whether it offers more or less variety and degree of fineness. Yet this hypothesis can hardly ever be tested empirically as we only have data of one PU-index based on one classification rather than of various PU-indices based on a number of classifications.

homogeneity of a division. We decided to take the average correlation (over T = 67 months) $\bar{r}$ between any two series belonging to the same division[18] so that $H = \bar{r}$. In order to keep calculations manageable[19] we confined ourselves to some series referring to intracommity trade (the so - called Intrastat statistics) and to series with noticeable price movement.

H was particularly high in imports of division 11 (crude petroleum and natural gas) with an average correlation between the 99 series of + 0.7219 and in exports of other transport equipment (division 35) with M (i.e. the average correlation) amounting to 0.6594. Interestingly, imports in division 11 are rather quite divergent (D1 = 12.7 %) although homogeneity is high. Correlations tend to be negative in particular in divisions like 2 (imports of forestry products) and 22 (exports of publishing, printing and reproduction) with -0.0585 and -0.0085 respectively. Divergence there was not conspicuously high.

To sum up regressing divergence (D3 eq. 9) on H was disappointing from the point of view of our hypothesis, yielding R^2_{adj} of 0.0742 only. It should be borne in mind, however, that the available data is not appropriate (and will continue to be so) to test the notwithstanding highly plausible hypothesis.

5.2. Lead of the price index?

The same possibly applies to the hypothesis that P^L_{0t} is a leading indicator while PU^P_{0t} is lagging one or more periods behind due to the different time of recording prices (see table 1). Correlating PU^P_{0t} with $P^L_{0,t-\Delta}$ $(\Delta > 0)$ did, however, not result in a systematic improvement of correlations as Δ is increasing. Only in some cases, had the shift of the price index a considerable impact on the divergence measures (D2 and D3)[20] Looking at the relative reduction of D2 when choosing $\Delta = 2$ instead of $\Delta = 0$ we find a noticeable "improvement" after lagging P in particular in the following cases: exports of basic non iron metals[21] and imports of electricity, gas etc. In general, D-measures are not significantly reacting to time lags, so that our hypothesis of P preceding PU does not really find support in the data.

5.3. The smoothing effect of quality adjustment

We made the assumption that making quality adjustments (that is reduction of the quoted price in the case of an improvement in quality) will result in a smoother price movement. This could explain the relatively (compared to the true price index P) high volatility of unit value indices (PU-indices) where such adjustments are not made. Fortunately we were in a position to verify (partly at least) this supposition because the Federal Statistical Office (FSO) thankfully carried out a special analysis of their price data in the field of data processing goods. The ordinary user of official statistical data can only make use of data after quality adjustment. The raw data are in general not accessible to him. The FSO gave us data concerning four

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[18] Homogeneity is maximum as H approaches +1 and minimum in the case of on average negatively correlating series, where H tends to -1.

[19] In the case of division 29 (machinery) we have as said above 1067 series (contracts) requiring the calculation of 568.711 correlation coefficients.

[20] As negative and positive deviations d_t may cancel out in the care of D1 a lower absolute value of D1 in response to a shift of the P-index is not necessarily indicative of less discrepancy between the two time series.

[21] Division 27. D2 is in the case of $\Delta = 2$ about 26 % as compared to $\Delta = 0$. In all other cases we found reductions, if any, not greater than 4 or 5 % only.

products of the GG "information and communication technology (ICT)", viz. desktops, notebooks, working storage and hard disk. Each of these products was in turn represented by a number of models of different producers, ranging from 84 to 190. As ICT products are characterised by remarkable price reductions accompanied by quality improvements it was not surprising that the amount of price reduction was uniformly higher after quality adjustment than before[22]. Volatility was also reduced substantially by quality adjustments.

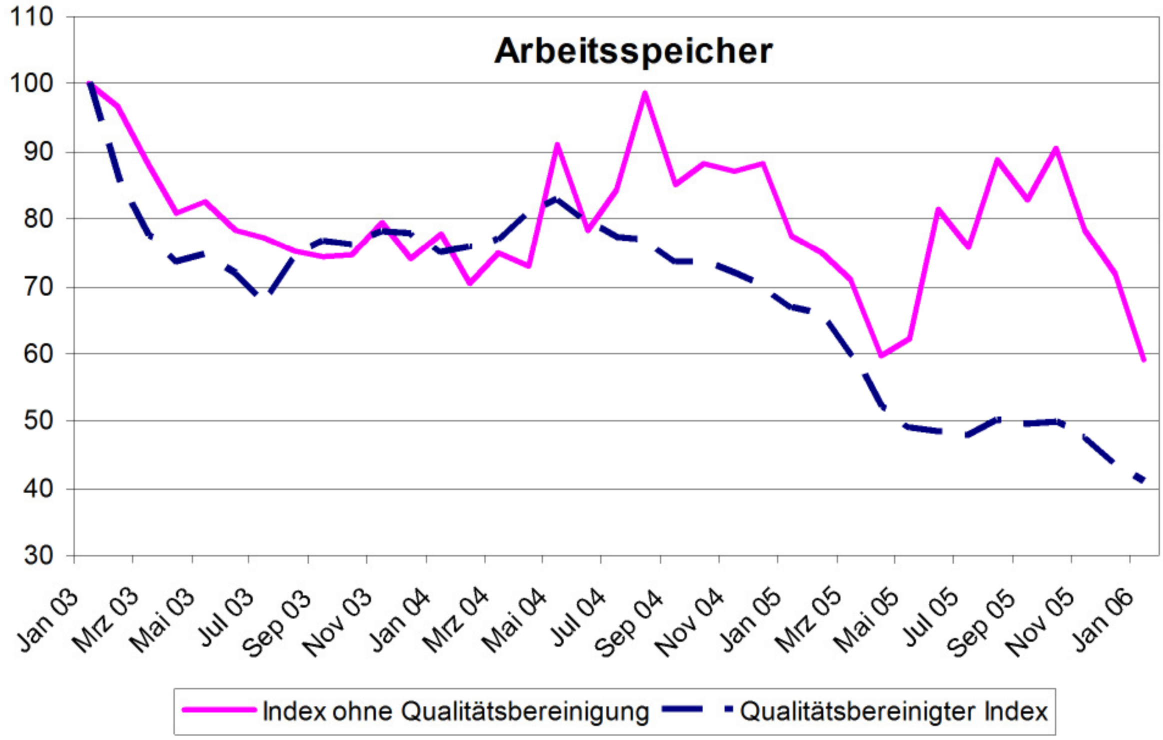

Figure 5 Prices of working storages as collected by the German Statistical Office (FSO), both before and after quality adjustment

Figure 5 may serve as illustration for this point showing that prices of working storages (190 models) were considerably less volatile after quality adjustments. We may therefore conclude that quality adjustment made in the case of P-indices in contrast to PU-indices is likely to account for the smoother time profile of the former price movement as compared to the latter.

6. INCOME EFFECTS, AN ATTEMPT TO TEST A BEHAVIOURAL HYPOTHESIS

Imports are known to depend on income. As income rises people not only tend to buy more of the same, they also will switch to qualitatively superior and more expensive goods within a GG serving basically the same purpose. It therefore is a reasonable assumption that upon increasing wealth a negative S-effect (possibly income rather than price induced) is put into force by which according to eq. 18 PU_{0t}^{P} is coming closer to P_{0t}^{P} and due to eq. 11 to P_{0t}^{L} as well. It is therefore assumed that the difference d between the price and the unit value index of German imports will be negatively correlated with the increasing income level y of Ger-

[22] For example in the case of working storage prices fell by 2.3 % after quality adjustment, compared to 0,9 % before. In this group of products, reduction of volatility also topped the other goods. The coefficient of variation was, after quality adjustment, only nearly one sixth of its value before.

man consumers[23]. The correlation was, however, positive r_{dy} = 0,881 indicating that the (moderate though) income increase is on average not reducing but enlarging the difference $P^L_{0t} - PU^P_{0t}$. There must be other factors overriding the structural component in PU, that is, the fact that unit value indices are reflecting mere changes in the structure of goods, even if not induced by prices. Our formula of eq. 11 allows for an L-effect in the opposite direction: the response to rising prices in favour of cheaper goods[24] may be more effective than a structural change in favour of more expensive goods due to a rising income level.
Despite the impressive $R^2_{adj} = 0.772$ the relation between y and d is not a candidate for a co-integrating relationship for the simple reason that Δy_t has a stochastic trend while Δd_t has not.

7. CONCLUSION

Unit value indices in foreign trade are not amenable to the "normal" or usual interpretation of price indices. They differ from the latter by a number of reasons not only the formula but also concepts and data collection procedures. The difference between the two approaches to price measurement is hitherto not well understood. Notably an integrating theory of the combined effect of various positively as well as negatively correlated influences remains to be developed. Moreover, many "effects" that probably could explain the difference, are difficult to capture empirically. The present paper, therefore, is only an attempt to improve our understanding of the nature of the two index designs and to exhibit some empirical findings.

In no small measure the paper can only make suggestions for a more thorough analysis. To name but a few of such issues: first more emphasis should be laid on a clear-cut list of determinants of L and S, preferably without overlaps and identifiable empirically. Secondly the difference between the two types of indices should be explained in terms of microecomic theory, that is, by tracing decisions back to utility maximizing behaviour. This should be useful in order to assess the relative strength of factors influencing L and S. Thirdly for the most part arguments advanced to explain specific traits of unit value indices are not well suited to understand their volatility in particular. In other words, there must be some other reasons than the omission of quality adjustments to explain volatility, for example possibly the frequent change in the composition of the traded goods.[25] Finally, there is no doubt that homogeneity of the commodity codes matters but it proved difficult to measure homogeneity, and thus to demonstrate this empirically.

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[23] Income y is measured as disposable income of private households from National Accounts. The series were seasonally adjusted and the logarithm of incomes was taken.
[24] Another influence affecting PU but not P is he appearance of new and cheaper goods (the so-called "new goods bias") in the ICT-sector above all contributing to the tendency of Laspeyres indices to overstate inflation.
[25] In this case, by the way, the analysis of chain indices would benefit from a closer look at unit value indices since we have in both situations a reduced comparability over time due to a changing basis of observations.

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