Problems of operationalizing the concept of a "cost of living index"

Theoretical and empirical demonstration of the limitations involved in inflation measurement, if it is based on the theory of utility maximisation

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Introduction

In some countries, especially the USA the so called "Cost of Living Index" (COLI, or index of Konüs PK) is preferred as target of inflation measurement over the traditional approach to define a Consumer Price Index (CPI) which is based on a constant "basket" of goods (also known as Cost of Goods Index, or COGI) as for example the well known Laspeyres price index. The COLI is the ratio of minimum costs c required to attain the same utility level $\bar{u}$ under two "price regimes" given by the price vectors $\mathbf{p}_0$ and $\mathbf{p}_t$ respectively, that is

(1) $P^K = c(\mathbf{p}_t, \bar{u})/c(\mathbf{p}_0, \bar{u})$.

The COLI thus compares expenditures required for attaining the same level of utility (well-being) rather than for buying the same quantities goods. In the COGI approach quantities $\mathbf{q}_0$ and $\mathbf{q}_t$ are regarded as exogenous (i.e. independent of prices) and given, as e.g. in

(2) $P^L = \mathbf{p}_t'\mathbf{q}_0/\mathbf{p}_0'\mathbf{q}_0$ (Laspeyres) or

(2a) $P^P = \mathbf{p}_t'\mathbf{q}_t/\mathbf{p}_0'\mathbf{q}_t$ (Paasche)

whereas in the COLI context **q**-vectors are said to be determined or "explained" by (rational) consumer behaviour as conventionally assumed in microeconomic theory and therefore endogenous. For this reason the COLI claims to possess a theoretical (microeconomic) foundation of an index function of a CPI. It is assumed that households are engaged in utility maximisation subject to the restriction of a given total expenditure (or income) M and a given vector **p** of prices. The COLI allows for substitutions among the same goods in response to varying relative prices, while the COGI keeps everything constant except prices. Therefore the COLI will in general display lower inflation rates, which not infrequently may be welcomed (politically), and the difference $P^L - P^K > 0$ is called "substitution bias" (of P^L).

Those who advocate the COLI use to emphasise that the COLI enjoys a theoretical justification or "theoretical underpinning" (in contrast to the COGI which "only" may be justified by representing a weighted average of price relatives and by the "principle of pure price comparison" according to which the index is reflective of price changes only). Note, however, that (1) only *defines* the COLI P^K but it does not give any hints about *how to compile* this index in practice because the c-function (and the utility function from which it is derived) is not observable. Given that we can't know how the "amount" of "utility" is related to the quantity of goods consumed we have to find ways to make the notion of a "constant-utility-index" or COLI nonetheless "operational" or "measurable". There are in principle three ways proposed in order to accomplish this task

1. historically the first approach was to *define upper and lower bounds for a COLI* (it is for example well known that under fairly general conditions $P^P \leq P^K \leq P^L$ holds),

2. then some attempts were undertaken in order to *estimate (econometrically) "demand systems"*, that is systems of n demand equations for n goods from which the theoretical cost-functions as numerator and denominator of the COLI can be derived (and also the shape of the Engel-curves and the estimates of some parameters such as various "elasticities" which may be interesting as regards the economic interpretation of the empirical results). However, as this approach turns out to be extraordinarily difficult to carry out in practice, it became more and more popular, to
3. make use of the theory of "*superlative indices*" developed by W. Erwin *Diewert* (1976, 1978) according to which certain observable price indices such as $P^F = (P^L P^P)^{1/2}$ (Fisher) or P^T (Törnqvist) - each "using the quantities in the base period as well as in the current reference period as weights in a symmetric fashion"[1] - are capable of approximating a COLI derived from a fairly general (or "flexible") demand function.

While the first approach certainly is less promising from a practical point of view of compiling an official CPI on a monthly basis, because it can at best provide intervals only rather than an exact numerical value the second and in particular the third approach may appear more pertinent and successful.

The focus of our paper therefore is on the second and third approach. For this purpose we have undertaken both, an empirical study of demand systems (using data taken from the official German Family Budget Surveys FBS), and a theoretical analysis of the assumptions explicitly (or implicitly) underlying Diewert's theory of superlative indices. To our knowledge there are no studies of demand systems of a comparable broad scope (comparing various systems) to be found in the literature to date. Hence our empirical work is clearly going beyond what hitherto had been done. Also attempts to establish an as complete as possible enumeration of assumptions needed to arrive at Diewert's results are rarely made

Hence the presentation we are interested to give in Warsaw will cover a discussion of

- methods and results of estimating demand systems from which a COLI can be calculated, and of
- assumptions and theoretical implications (and their interpretation) of the use of "superlative indices" as proxies for the theoretical (and unobserved) COLI.

The second part of our detailed and rather comprehensive abstract will introduce some results.

Major results to be presented in the Conference

1. If the COLI theory were correct and "realistic" as regards its assumptions one would expect that all three above mentioned approaches will yield the same result. In particular a COLI derived from empirical estimates of demand systems and a COLI gained by compiling a "superlative index", both based on the same data should coincide. It turned out, however, that this is not the case and to believe, the enormous difficulties involved in the estimation of demand systems could be easily circumnavigated by simply making use of superlative indices is illusionary though a widespread overly optimistic expectation (if not even a sort of wishful thinking). This may come as a surprise because in view of the highly esteemed "theoretical underpinning" of the COLI (as opposed to the COGI) one should expect that there is only one COLI just like there necessarily is also only one utility maximum. In actual fact, however, there are many reasonable functional forms suitable for demand systems and there are also many index formulas known for being "superlative". Moreover demand systems and (superlative) index functions are not linked together in a one-to-one relation. Different index

[1] Peter von der Lippe and W. Erwin Diewert, Introduction to the Special Issue on Index Number Theory and Price Statistics, Jahrbücher für Nationalökonomie und Statistik, Bd. 6/230 (2010), S. 660 - 672 (661).

functions may fit to the same utility function (possibly depending on the exact value of certain parameters) and a particular index function can well be related to different preference orders.

2. There is more than one demand system in accordance with the usual assumptions of the utility maximising household. Hence a choice has to be made among a number of functional forms, each of which renders specific functions for the indirect utility function v(**p**,M), the (minimum) cost c(**p**,u), or the logarithmic costs ln(c(**p**,u)), and above all for the budget shares w_i (this function usually serves as regression function).[2] Each functional form (for the shares w_i) has its particular merits and demerits. This choice may be oriented at various desirable properties of the respective functional forms. The specific regression functions may for example comply with the (microeconomic) "regularity requirements" of utility and demand functions only within a more or less wide range of possible values for the parameters of the function rather than "globally". In addition to this criterion of a sufficiently wide "domain of applicability" there are some other properties that ought to be considered, such as "flexibility" (which is violated for example in the case of a function which can only yield zero [own or cross] price elasticities if applied to empirical data)[3] or "computational facility" and "factual conformity"[4]

- Linear Expenditure System LES (with the Cobb Douglas function as special case), the Generalised Leontieff (GL) and Translog (TL) system, and in more detail (and also empirically estimated) the demand systems
- AIDS (Almost Ideal Demand System)[5] of A. Deaton, D. Muellbauer (1980), and its linear approximation LA-AIDS[6] as well as the more general system QUAIDS (quadratic AIDS) in which AIDS is nested, and finally the
- "homogeneous quadratic" system, and more general, the normalised quadratic system (NQ on the basis of a quadratic mean[7] of order r) brought into play primarily by Diewert and Wales (1988). This last estimation, however, proved difficult and eventually failed.

As regards the criteria for functional forms mentioned above AIDS and QUAIDS represent a fair compromise. Yet these systems were complicated enough and quite difficult to handle even in the case of a very limited number of commodities.

3. The result of the empirical part was that the demand systems estimated had a poor goodness of fit which may possibly indicate the consumption behaviour of households is not well explained by the usual assumptions of utility maximisation in microeconomics as they are materialised in the demand systems. In particular the demand-system and the superlative-index approach obviously don't fit together satisfactorily. The P^F index was consistently considerably lower than the COLI based on a demand system for the same data.

Furthermore we saw that there are many problems concerning the rather demanding data requirements (which are rarely met in the practice of the official statistics of most countries).

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[2] The functions c, ln(c) and v are of course interrelated.

[3] Diewert defines flexibility in his Lecture Notes (ch. 4) as follows: it is a function f(q) that "can provide a second order approximation to an arbitrary function f* around any (strictly positive) point q* in the class of the linearly homogenous functions." And by second order approximation is meant: "A twice differentiable function f(q) … can provide a second order approximation to another such function f*(q) around the point q* if the level and all of the first and second order partial derivatives of the two functions coincide at q*."

[4] According to L. J. Lau (Functional Forms in Econometric Model Building) this is not met if for example the system can only yield *linear* Engel-curves (which e.g. applies to AIDS as opposed to QUAIDS). Also Lau had shown that it is not possible to reconcile the above mentioned criteria so that a compromise is called for.

[5] An abbreviation that in retrospect does not seem to be a good choice.

[6] We get such a "linear AIDS" when we insert the special "price index" (rather an average *absolute* price of n prices) of Stone for P in the term $\beta_i \ln(M/P)$ of the budget share equation for w_i (i = 1, …,n goods) in the AIDS.

[7] More specific: the geometric mean of two power means. The quadratic mean of order r utility function is given by $[\Sigma\Sigma a_{ik} q_i^{r/2} q_k^{r/2}]^{1/r}$ or $\mathbf{q'Aq}^{1/r}$ with $\mathbf{A} = \mathbf{A}'$ (the quadratic mean is the case r = 2).

We could for example make use of only a small part of the FBS micro-data file (just nine selected commodities, solely types of food) because the estimation requires absolute figures of expenditures and quantities thereby indirectly prices actually paid by the households in the sample.

Also the econometric estimation was much more difficult than expected. Not only were there many parameters to estimate, we also saw that many tests for the assumptions of the respective models failed and many results were such that they did not really make sense. It may be suggested that these unsatisfactory findings may perhaps be attributed to the specific German data. Even if this were the case it is still true, that estimating a demand system of more than only some few goods (n = 9 in our case) entails so many insurmountable difficulties that it would by no means be a reasonable option for official statistics to compile a CPI based on estimated demand systems. Thus a monthly COLI-type official CPI based on estimated demand systems comprising a tolerable variety of goods will most probably be impossible.

Even worse, there may be indications that the assumptions underlying the often praised (alleged) microeconomic foundation of the COLI are unrealistic. This can be inferred from the unsatisfactory fit of our (demand) regression equations. In the first place we may conjecture that in reality households can hardly respond so promptly and rapidly to price signals by substituting as assumed in theory. Such theory related arguments should be born in mind when we consider the superlative-index approach next, because this approach to be valid has to rely on the same assumptions (concerning consumer behaviour) and to some more in addition (concerning the notion of "approximating" an "arbitrary" function).

3. As already mentioned in this situation it appears tempting to avoid all those econometric difficulties with demand systems by simply calculating a superlative index combining observable data vectors $\mathbf{p}_t$, $\mathbf{p}_0$, $\mathbf{q}_0$ and $\mathbf{q}_t$ only (where perhaps only the timely availability of $\mathbf{q}_t$ may pose a problem in practice). This, however, is not that easy. The proof of "superlative-ness" of an index function requires a number of restrictive assumptions which are unlikely to hold empirically, and together with the assumptions needed to relate the index to a utility maximum this makes the index no less dependent on restrictive and perhaps unrealistic assumptions than the much simpler COGI approach. We intend to give an as complete as possible list of all these assumptions in our presentation.

The authors

C. C. Breuer M.Sc. wrote his doctoral thesis on the "uses and limitations of the COLI approach". The empirical results as well as the theoretical considerations of the paper are taken from this work. He was assistant of Prof. Dr. Peter von der Lippe, professor of statistics at the University of Duisburg-Essen for many years and he was also invited twice to Canada, by the "Ottawa Group" in 2006, and by the Canadian Statistical office in 2010 (in both cases his work on the COLI was involved).

Professor Dr. Peter von der Lippe published a book on "Index Theory and Price Statistics" and was together with Erwin Diewert editor of a special issue (with the same title) of an academic journal. He also published on "chain indices" and many articles on other index problems. He was supervisor of Breuer's doctoral thesis. For more information see www.von-der-lippe.org.

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