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Problems of operationalizing the concept of a "cost-of-living-index"

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Outline:

1. Introduction
2. The superlative index approach: assumptions and problems
3. Demand systems approach DSA
4. Data
5. Estimation
6. Conclusion

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1. Introduction

- **Paradigms**

COGI (cost of goods) fixed basket (Laspeyres) $P_{0t}^{L} = \frac{\mathbf{p}_t^{'}\mathbf{q}_0}{\mathbf{p}_0^{'}\mathbf{q}_0}$

COLI (cost of living) constant utility $P_{0t}^{COLI} = \frac{C(u,\mathbf{p}_t)}{C(u,\mathbf{p}_0)}$

ratio of minimum costs required to attain the same utility level under two "price regimes" given by the price vectors $\mathbf{p}_0$ and $\mathbf{p}_t$

COLI allows for substitution P^{L}-P^{COLI}

- **Three operational concepts**

1. **Bounds**

$$P_{0t}^{L} = \frac{\mathbf{p}_t^{'}q_0}{\mathbf{p}_0^{'}q_0} \geq \frac{C(u_0,\mathbf{p}_t)}{\mathbf{p}_0^{'}q_0} = \frac{C(u_0,\mathbf{p}_t)}{C(u_0,\mathbf{p}_0)} \qquad P_{0t}^{P} = \frac{\mathbf{p}_t^{'}q_t}{\mathbf{p}_0^{'}q_t} \leq \frac{\mathbf{p}_t^{'}q_t}{C(u_t,\mathbf{p}_0)} = \frac{C(u_t,\mathbf{p}_t)}{C(u_t,\mathbf{p}_0)}$$

- **Three operational concepts**

2. **Demand System Approach (DSA)**
   estimate (econometrically) systems of N demand equations for N goods from which the cost-functions of the COLI can be derived (also Engel-curves, elasticities, goodness of fit etc.); difficult in practice, therefore
3. **Superlative Indices Approach (SIA)** became popular
   W. Erwin Diewert (1976, 1978 etc.) observable price indices using symmetrically $\mathbf{q}_0$ and $\mathbf{q}_t$, such as
   Fisher P^F, Törnqvist P^T, Walsh P^W
   are approximations to the COLI

Ideally DSA and SIA should yield the same result, however, our empirical study spectacularly failed to deliver their promises in this respect

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2. The superlative indexes approach (SIA) assumptions and problems

- How the SIA message usually is understood
- Explicit and implicit assumptions needed in the SIA

1. The SIA message

1. Only a few index functions (P^F, P^T, P^W) enjoy the (rare) honour of being "superlative"
2. These indexes are valid for **any** utility function whatsoever and they approximate each other fairly well,
   other indexes (P^L, P^P,…) almost worthless
3. Empirical studies of household substitution behaviour no longer needed, bias simply is $P^L – P^F$
4. With more realistic scenarios (multistage index compilation, many households case) things don't change fundamentally

2. Assumptions to be made in the SIA

- for the COLI in general
- to show that the index is superlative

2.1 For an index to have a COLI interpretation (most fundamental assumptions) households need to

1. engage in **utility maximising** behaviour
   ability and willingness on the part of consumers to collect the necessary information about prices, availability in suitable outlets, immediate reaction in fractional quantities (preferences and flexibility possibly not exogeneously determined)
2. possess a **utility function** (numerical utility u assigned to combi-nations of quantities f(**q**)) with preferences satisfying "regularity conditions"
   continuous, increasing (in elements of **q**), quasi-concave, twice continuously differentiable
   For the SIA it will be necessary to be more specific about the function f(**q**)

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2. The superlative indexes approach assumptions and problems

2.1 For an index to have a COLI interpretation households need to

3. observe **linear budget constraints**
   prices **p** and total expenditure M *given* (households as price takers) and independent of f(**q**) and **q** ; adding up condition; homogeneous of degree zero in M and p
   with non-linear constraints difficulties with a unique utility maximum

4. maintain **preferences** which are **constant over time** and make changes in their consumption *solely in response to changing relative prices* without restrictions on the supply side
   constant set of goods; demand not attributable to changing tastes or activities on the supply side; also no influence of income on preferences

When these four assumption are not given we cannot equate $\Sigma p_s q_s$ to $C(\mathbf{p}_s, u_s)$ and thus relate observed price indices to the COLI

2. The superlative indexes approach
assumptions and problems

Most important and contentious: **homothetic** (≈ linear [r=1] homo-genous) preferences $g(\lambda x_1, \lambda x_2, \ldots, \lambda x_n) = \lambda^r g(x_1, x_2, \ldots, x_n)$

- Unit cost function: $C(u,\mathbf{p}) = u \cdot c(\mathbf{p}) = f(\mathbf{q}) \cdot c(\mathbf{p})$ where $c(\mathbf{p}) = C(1,\mathbf{p})$
- implies that the COLI is independent of the utility level and thus same for all income classes. All goods have unitary income elasticities, Engel curves are straight lines through the origin
- known to be most unrealistic show that the index is, yet important for SIA

2.2 To show that a given index function is superlative another group of four assumptions has to be introduced

2. The superlative indexes approach (SIA)
assumptions and problems

2.2 The group of four assumptions especially for SIA.
We need to

1. find a **functional form** for f(q) (or c(p) respectively) which complies with all assumptions needed to reflect rational consumer behaviour, and derive
2. the price and quantity **index function** corresponding to (being "exact" for) this functional form, and
3. demonstrate that the functional form in question is "**flexible**" in the definition of Diewert, and finally
4. to have some guidance in making a choice among the equally superlative formulas (are they really all approximating each other?) Results of the index formulas are very close.

The four special SIA assumptions (first ass.)

1. A suitable **functional form** for f(**q**) (or c(**p**) respectively)

   When utility maximisation prevails

the value index $V_{0t} = \frac{\sum_i p_{it} q_{it}}{\sum_i p_{i0} q_{i0}} = \frac{\mathbf{p}_t' \mathbf{q}_t}{\mathbf{p}_0' \mathbf{q}_0} = P_{0t} Q_{0t}$ is $V_{0t} = \frac{C(u_t, \mathbf{p}_t)}{C(u_0, \mathbf{p}_0)}$

and with homothetic preferences this simplifies to

$$= \frac{u_t c(\mathbf{p}_t)}{u_0 c(\mathbf{p}_0)} = \frac{f(\mathbf{q}_t)}{f(\mathbf{q}_0)} \cdot \frac{c(\mathbf{p}_t)}{c(\mathbf{p}_0)}$$

COLI quantity index $u_t/u_0 = f(\mathbf{q}_t)/f(\mathbf{q}_0)$

COLI price index $c(\mathbf{p}_t)/c(\mathbf{p}_0)$

2. The superlative indexes approach
assumptions and problems

The first of the four special SIA assumptions: **functional form**

Diewert studied primarily two forms

1. The **quadratic mean of order r** utility and cost function

$$c_r(\mathbf{p}) = \left(\sum_i \sum_j b_{ij} p_i^{r/2} p_j^{r/2}\right)^{1/r} = \left(\mathbf{p}^{'}\mathbf{B}\mathbf{p}\right)^{1/r} \quad \text{where} \quad \mathbf{p} = \left[p_1^{r/2} \ldots p_N^{r/2}\right]$$

This model nests

Homogeneous quadratic r = 2 $\rightarrow$ Fisher P^F
Translog (r$\rightarrow$0) $\rightarrow$ Törnqvist P^T
Generalized linear f(**q**) = **a'p**, $\rightarrow$ Leontief cost
c(**p**) = **b'p** (= CES for specific parameters b)

2. The **normalised quadratic** (NQ) utility and cost function

$$c_{NQ}(\mathbf{p}) = \mathbf{p}'\mathbf{b} + \frac{\frac{1}{2}\mathbf{p}'\mathbf{A}\mathbf{p}}{\boldsymbol{\alpha}'\mathbf{p}}$$

Various assumptions for α possible (α'**p** serves as normalizing scalar) , thus we get a number of additional “superlative” indexes

2. The superlative indexes approach assumptions and problems

(Homogeneous) **Translog** function ($r \to 0$) as a functional form

Homothetic version $\ln c_0(p) = \beta_0 + \sum_i \beta_i \ln p_i + \frac{1}{2}\sum_i \sum_j \beta_{ij} \ln p_j$

for this exact: P^T

Non-homothetic version

$$\ln C_0(u, \mathbf{p}) = \beta_0 + \sum_i \beta_i \ln p_i + \frac{1}{2}\sum_i \sum_j \beta_{ij} \ln p_j + \gamma_0 \ln(u) + \sum_i \gamma_i \ln p_i \ln(u) + \frac{1}{2}\gamma_{00}(\ln(u))^2$$

P^T is exact if $u = u^* = (u_0+u_t)/2$

Indices (P^r or Q^r) based on the quadratic mean of order r functional form are superlative **for all r**, not only for $0 < r \leq 2$

2. The superlative indexes approach
assumptions and problems

The second special SIA assumption: find the exact index

2. Diewert derived the general form a price (P) and quantity (Q) index exact for the homogeneous quadratic form as

$$P_{0t}^{r} = \left[\sum_i s_{i0}\left(\frac{p_{it}}{p_{i0}}\right)^{r/2}\right]^{1/r}\left[\sum_i s_{it}\left(\frac{p_{it}}{p_{i0}}\right)^{-r/2}\right]^{-1/r} \quad Q_{0t}^{r} = \left[\sum_i s_{i0}\left(\frac{q_{it}}{q_{i0}}\right)^{r/2}\right]^{1/r}\left[\sum_i s_{it}\left(\frac{q_{it}}{q_{i0}}\right)^{-r/2}\right]^{-1/r}$$

Evidently $P^{r=2} = P^F$ and $Q^{r=2} = Q^F$. $P^{r=0}$ gives $P_{0t}^{r=0} = \prod_i \left(\frac{p_{it}}{p_{i0}}\right)^{\frac{s_{i0}+s_{it}}{2}} = P_{0t}^{T}$

s_i= expenditure share of good i

An infinite number of superlative indices is defined **as indirect index** (co-factor of the product test)

$$P_{0t}^{r=1} = \left[\sum_i s_{i0}\left(\frac{p_{it}}{p_{i0}}\right)^{1/2}\right]\cdot\left[\sum_i s_{it}\left(\frac{p_{it}}{p_{i0}}\right)^{-1/2}\right]^{-1} = V_{0t}/Q_{0t}^{W} \quad \neq P_{0t}^{W} = \frac{\sum p_{it}\sqrt{q_{i0}q_{it}}}{\sum p_{i0}\sqrt{q_{i0}q_{it}}}$$

2. The superlative indexes approach
assumptions and problems

The third and fourth special SIA assumption

3. Demonstrate that the functional form in question is "**flexible**" in the definition of Diewert
   "flexible if it provides a second order approximation to another function f*(**q**) around the point **q***, meaning that "the level and all of first and second order partial derivatives of the two functions coincide at **q***"
   Instead of "another" function: an "arbitrary *linear homogenous* function"
4. Does choice of equally superlative formulas matter?
   a) They are approximating in the proportional prices point $\mathbf{p}_t = \lambda \mathbf{p}_0$ or equal prices point (λ= 1)
   b) R. Hill: difficulties with extreme values of r of a quadratic mean of order r index.

In **summary**: estimation of demand systems (DSA) not at all superfluous, DSA and SIA results may well differ *substantially*.

3. Demand systems approach

Almost Ideal Demand System, Deaton and Muellbauer (1980):

- cost function of the price independent generalized logarithmic (PIGLOG) class of preferences:

$$\ln c(\mathbf{p}, u) = (1-u)\ln a(\mathbf{p}) + u \ln b(\mathbf{p})$$

$$\ln a(\mathbf{p}) = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln p_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \gamma^{*}_{ij} \ln p_i \ln p_j$$

$$\ln b(\mathbf{p}) = \ln a(\mathbf{p}) + \beta_0 \prod_{i=1}^{N} p_i^{\beta_i}$$

$$\rightarrow \quad \ln c(\mathbf{p}, u) = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln p_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \gamma^{*}_{ij} \ln p_i \ln p_j + u\beta_0 \prod_{i=1}^{N} p_i^{\beta_i}$$

3. Demand systems approach

- budget share equation:

$$w_i = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln p_j + \beta_i \ln\left(\frac{M}{P}\right)$$

- price aggregator:

$$\ln P = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln p_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij} \ln p_i \ln p_j, \quad \text{with} \quad \gamma_{ij} = \frac{1}{2}\left(\gamma_{ij}^{*} + \gamma_{ji}^{*}\right)$$

- parameter restrictions:

$$\sum_{i=1}^{N} \alpha_i = 1, \sum_{i=1}^{N} \beta_i = 0, \sum_{i=1}^{N} \gamma_{ij} = 0 \forall j, \gamma_{ij} = \gamma_{ji}$$

→ to ensure adding-up, homogeneity of degree 0 in **p** and M of the budget share equation and symmetry of the Slutsky matrix

3. Demand systems approach

some facts about the AIDS:

- locally flexible functional form
  - cost function can be interpreted as a second-order approximation to an "arbitrary" unknown function in a single point
  - no a priori restrictions on the possible elasticities at one point
- number of parameters of the budget share system to estimate:

$$N(N\text{-}1)/2+2N\text{-}2$$

- theoretical restrictions can be tested empirically
- linear Engel curves, but not necessarily through the origin
- often very small regions of theoretical regularity

3. Demand systems approach

Quadratic Almost Ideal Demand System, Banks et al. (1997):

- extension of the AIDS that allows for nonlinear Engel curves
- cost function:

$$\ln c(\mathbf{p},u) = \alpha_0 + \sum_{k=1}^{N} \alpha_k \ln p_k + \frac{1}{2}\sum_{k=1}^{N}\sum_{j=1}^{N} \gamma^*_{kj} \ln p_k \ln p_j + \frac{\ln u \prod_{i=1}^{N} p_i^{\beta_i}}{1 - \ln u \sum_{i=1}^{N} \lambda_i \ln p_i}$$

- budget share equation:

$$w_i = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln p_j + \beta_i \ln\left(\frac{M}{P}\right) + \frac{\lambda_i}{\prod_{k=1}^{N} p_k^{\beta_k}}$$

- price aggregator:

$$\ln P = \alpha_0 + \sum_{k=1}^{N} \alpha_k \ln p_k + \frac{1}{2}\sum_{k=1}^{N}\sum_{j=1}^{N} \gamma_{kj} \ln p_k \ln p_j,$$

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3. Demand systems approach

- parameter restrictions:

$$\sum_{i=1}^{N} \alpha_i = 1, \sum_{i=1}^{N} \beta_i = 0, \sum_{i=1}^{N} \gamma_{ij} = 0 \forall j, \sum_{i=1}^{N} \lambda_i, \sum_{j=1}^{N} \gamma_{ij} = 0 \forall i, \gamma_{ij} = \gamma_{ji}$$

some facts about the QUAIDS:

- effectively globally regular flexible functional form
  - regular regions that include (almost) all data points in the sample
- number of parameters of the budget share system to estimate:

  N(N-1)/2+3N-3
- allows quadratic Engel curves
  - the expenditure and price elasticities can vary with the level of total expenditure *M*
- The AIDS is nested in the QUAIDS

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4. Data

- German income and expenditure survey (EVS)
  - every five years (1988, 1993, 1998, 2003, (2008))
  - 0.2% of the German households covered (about 75,000)
  - detailed log book for food-, beverage- and tobacco-commodities
- nine staple foods (milk, cream, eggs, butter, margarine, apples, bananas, mineral water) chosen
  - high data availability
  - no quality change
- only couples with children under eighteen to ensure homogeneity of the sample
- for 1988 we have 2581 observations
- price data from the official German CPI statistics

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5. Estimation

- equation system with contemporaneous correlation between the error terms
- inter-equation dependency due to the imposed parameter restrictions
- SUR model of Zellner (1962)
- problem of non-linearity
- iterated FGLS estimator

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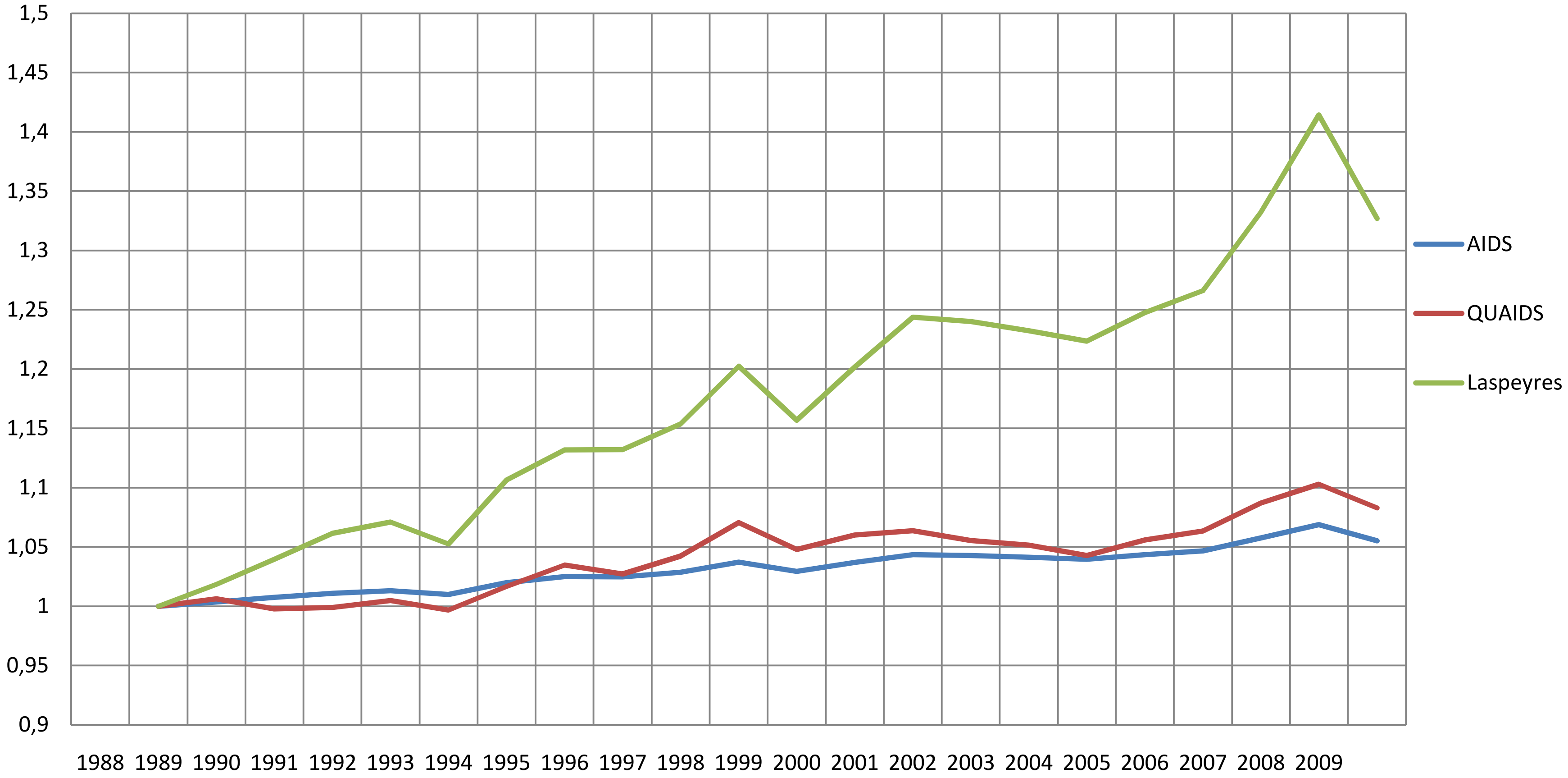
5. Estimation

COLIs and Laspeyres price index, base year 1988



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5. Estimation

- COLIs compared to superlative index numbers, base year 1988:

year	Laspeyres	Paasche	Fisher	Törnqvist	AIDS	QUAIDS
1993	1.0526	1.0566	1.0546	1.0547	1.0099	0.9968
1998	1.2024	1.2052	1.2038	1.2038	1.0371	1.0706
2003	1.2323	1.2562	1.2442	1.2439	1.0413	1.0515

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5. Estimation

- LR test of the **symmetry** and **homogeneity** restrictions
  - null hypotheses, that the restricted model has the same goodness of fit as the unrestricted model, can be rejected for the AIDS and the QUAIDS for nearly all years
  - observed household data sets are not allowing the conclusion that the neoclassical assumptions of demand functions that are homogeneous of degree *0* in prices and expenditure and symmetric Slutsky matrices were fulfilled
- testing the **concavity** condition by checking the negative semidefiniteness of the Slustky matrix (non-positivity of all the eigenvalues of the matrix)
  - concavity condition broadly rejected, but cost functions have a weakly non-concave shape
- LR ratio test AIDS vs. QUAIDS
  - for all four data sets the null hypotheses, that the AIDS has the same goodness of fit as the QUAIDS, can be rejected on a level of significance smaller than 0.05

6. Conclusion

- the assumptions underlying the microeconomic foundation of the COLI are (at least for our data set) unrealistic
- the demand system approach has no practical value in the everyday CPI production
- more an analytical tool to get more information about consumer behaviour
- some light beyond the assumptions that have to be made by following the economic approach
- maybe the “simple” COGI is not such a bad choice…

THANK YOU FOR YOUR ATTENTION!