v. d. Lippe: TES Course exercises (Axioms)

1	Axioms	Definitions

The following seven statements

A) if all prices rise the index should not remain unity

B) if all prices change k-fold the index should be k

C) if no price changes the index should remain unity

D) if all prices change k-fold and quantities remain constant the index should be k

E) if all base and current prices are multiplied by k the index should remain constant

F) if only one price rises (the other prices being unchanged) the index should rise as well (possess a value > 1)

G) if prices move up and down and will finally (all) return to their original level at time 0, the index P_{ot} should be 1

should be assigned to the following axioms (by indicating one or more, or also none of the symbols A,B,...)

	A	B	C	D	E	F	G
a) monotonicity							
b) linear homogeneity							
c) (weak) proportionality							
d) (price) dimensionality							
e) identity							
f) circularity and identity							
g) time reversal test							
h) product test							

2	Axioms	Implications

Indicate the relevant consequence in the sense of "if A then B" (or **A** → **B**), but the converse is not necessarily true

A (if)	→	**B** (then also...)
proportionality		
additivity (of index formula)		
identiy + circularity		
factor reversal test		
linear homogeneity and identity		
additivity (linearity) of index fct.		
strict monotonicity and proport.		

Uniqueness -, existence- and inconsistency theorems: Indicate cases in which there is only one index (which?), none or several indices satisfying the condition(s) in question:

condition(s)	only one	none (inconsistent)	several
volumes (deflation) are additive (struct. consistency)			
variable weights, circularity			
factor reversal test			
identity, circularity, and product test			
circular test and five-axiom system (Eichhorn + Voeller)			
index function consistent in aggregation but not addiitive			
proportionality but not linear homogeneity			
identity, lin. homogeneity but not proportionality			

Fisher's ideal index (price index) is unable to fulfill the following useful properties from a deflation point of view:

a) aggregative consistency of the index function (disaggregation of P^F into "sub-indices", or aggregation to an "all-item-index")

b) structural consistency (additivity) of volumes

c) resulting volume index is linear in quantities

d) all of the above mentioned criteria are not fulfilled.

Deflation with a chain Fisher's ideal index (price index) will result in the following additional defect (as compared with a direct Fisher index, see above):

a) factor reversal test is not met;

b) if all quantities and prices in t > 2 have regained their initial (t = 0) value the resulting quantity index Q_{0t} may well show rise ($Q_{0t} > 1$) or decline ($Q_{0t} <1$) in quantites (volume); .

c) both answers a) and b) correct;

d) both answers a) and b) incorrect.

Which of the following axioms ("tests" or other requirements) is necessarily violated when an index is compiled using "out-dated" weights?

a) identity;

b) mean value property;

c) desirable aggregation properties;

d) none of the (mathematical) axioms commonly postulated. .

Ausgewählte weitere MC-Fragen

1	1.2b	1	

A price index in general is

a) the ratio of the consumption expeditures at times 0 and t of some specified households of considerable policy concern (e.g. retired persons, farmers, workers earning minimum wages etc);

b) to be expressed as a weighted average of price relatives with prices that relate to truely identical goods at times 0 and t;

c) a function of price-vectors or price-vectors and quantity-vectors defined for times 0 and t with prices/quantities related to some meaningful aggregate (e.g. consumer prices) and a function that satisfies some "axioms" (mathematical characteristics);

d) none of the definitions given under a) to c) is sufficiently general: any single figure that describes comprehensively the change of all prices is a price index.

3	1.3b	2	

Sampling considerations in price statistics

a) are involved in the following stages: households to define a "market basket", locations and types of stores (including e.g. mail order business), representative (most frequently chosen) products or variants of products etc;

b) require a random sample whilst "purposive sampling" is invalid;

c) both answers a) and b) are correct;

d) both answers a) and b) are wrong.

5	1.3b	4	

The principle of "pure price comparison" is violated in the following case(s):

a) share price indices provide "corrections" due to the foundation of new firms, mergers, capital increase and other events of similar kind affecting the market conditions;

b) the rates of inflation of various countries are compared by comparing their respective national consumer price indices;

c) corrections are made to account for increase or decrease of quality (improvements require appropriate reductions of observed prices);

d) all answers are wrong, that is all answers (a to c) constitute procedures that have to be complied with in order to satisfy the requirements of the above mentioned principle.

13	2.3	1	

It is a very common statement, that the Laspeyres and the Paasche form are completely equivalent (they both "rest on the same solid logical ground" (Mudgett))

a) this is true only in the following sense: what applies (concerning weights) to the base period 0 in case of the Laspeyres formula applies to period t in case of the Paasche index;

b) this is true only in a binary (two situations) comparison of 0 and t, however, it should be recognised that 0 is only <u>one</u> period (kept constant for some time), whilst t represents many (a series of adjacent) periods;

c) the Laspeyres formula conforms with the concept of pure price comparison and defines a rise of prices in terms of "a fixed basket now is more expensive to buy", whereas a rise in prices in Paasche's index is inferred indirectly (value rising more quickly than volume);

d) all answers are correct.

14	2.4	1	

Expenditures at current prices increased at a rate of 40% between 0 and t. In the same intervall the expenditures at constant prices (i.e. the volumes) and the Laspeyres price index have both changed by +20%. The variances of price and quantity relatives are 0.16 and 0.09 respectively. The following conclusions can be drawn:

a) the Paasche price index increased by +1/6, thus the index now is 116.67 and the covariance between price and quantity relatives is 1.4 -1.2·1.2 = -0.04;

b) in addition to a) we may also say that the Paasche quantity index is 116.67 (thus equals the Paasche price index) and that the correlation between price and quantity changes is negative;

c) in addition to b): the correlation between price and quantity changes is -1/3 = -0.333 and the Laspeyres price index interchanging base and reporting period, that is $P^L_{t0} = 0.85714$ (hence 85.7 or -14.3% whereas $P^L_{0t} = 1.2$ is indicating an increase of 20%);

d) all conclusions above are correct and we may also say that the difference between Paasche and Laspeyres indices (price and quantity) is likely to increase as time goes on because then the variances of price and quantity relatives will automatically increase.

15	2.4	2	

Suppose the price index comprises only two commodities A and B and the price-relatives are as follows: for A: 1.2 and for B: 0.9; consumers spent 2/3 of their total expenditure for A at base time 0. The Paasche price index will be unity, that is showing neither rise nor fall of prices. Expenditure (at current prices) raised by 20% such that $V_{0t} = 1.2$. What can be inferred?

a) Laspeyres quantity index $Q^L = 1.2$;

b) Paasche quantity index $Q^P = 1.0909$;

c) Covariance between price and quantity relatives C = -0.22;

d) all answers correct.

16	2.3	2	

Suppose the price index comprises only two commodities A and B and the price-relatives are as follows: for A: 1.2 and for B: 0.8; consumers spent 2/3 of their total expenditure for A at base time 0

a) the Laspeyres price index will be 1.0667 and the Paasche price index will be less than 1.0667 whenever the share of the expenditures at period t (and at prices of t) devoted to A is less than 2/3, that is $p_{A\,t}q_{A\,t} / \Sigma p_t q_t < 2/3$;

b) like a) but the Paasche price index will be less than 1 and thereby indicate a decline of prices whenever the expenditure share of A at constant prices (that is $p_{A\,0t}q_{A\,t} / \Sigma p_0 q_t$) will be less than one half (< 1/2);

c) the Paasche price index will only be unity or less when the absolute quantity consumed of commodity A is declining;

d) It is not possible that P^L shows a rise in prices and P^P at the same time a decline.

20	2.3	3a

An aggregate at current prices increased by + 40% and the same aggregate at constant prices by + 25% (see also exercise no. 18). Hence the Laspeyres quantity index is $Q^L = 1.25$ and the Paasche price index is $P^P = 1.12$, and the value change of 40% has a quantity component of 25% and a price component of 12% summing up to 37%, unfortunately not to 40%. The reason is that Laspeyres- and Paasche indices are violating the factor reversal test

a) yes, such a result is not possible when a pair of "factor reversible" ("ideal") indices, for example Fisher's indices were used (we could get for example $P^F = 1.25$ and $Q^F = 1.15$ such that 25% + 15% = 40%)

b) no, if the covariance between price and quantity relatives were C = 0 we would get precisely the same result: $Q^F = Q^L = 1.25$ (quantity component 25%), and $P^F = P^P = 1.12$ (price component 12%) though Fisher indices pass factor reversal test;

c) answer b) is correct, and in addition: if the covariance were C < 0 we have $P^F > P^P = 1.12$, and $Q^F < Q^L = 1.25$;

d) yes, and answer c) is incorrect by the following reason: if (as a rule) C < 0 we get $P^F < P^P = 1.12$ and $Q^F > Q^L = 1.25$, for example $P^F = 1.1$ and $Q^F = 1.3$ such that 10% + 30% = 40%..

21	2.4	3

Given the figures of no. 20, that is $V_{0t} = 1.4$, $Q^L_{0t} = 1.25$, and $P^P_{0t} = 1.12$. Calculate

	price indices		quantity indices	
Covariance	Laspeyres	Fisher	Laspeyres	Fisher
- 0.1				
+ 0.1				

22	2.5	1

Given the following two combinations of quantities (two commodities) a utility maximizing household consumes at time 0 and time t (representing the same utility level)

	prices		quantities	
commodity	0	t	0	t
A	12	10	10	15
B	20	15	12	8

Calculate the "true cost of living index" (COLI), and the Paasche- and Laspeyres indices, showing how value change is decomposed into price and quantity component. Result:

	change of value	price component	quantity component
COLI (economic theory)			
traditional approach	$V_{0t} =$	$P^L_{0t} =$	$Q^P_{0t} =$

$P^P_{0t} = Q^L_{0t} =$

Ergänzung: ein Zahlenbeispiel für P^L und schwacher Test der Zeitumkehrbarkeit

Digression: a weak variant of the time reversal test

It is obviously rather restrictive to require an index P_{t0} to be the inverse index P_{0t}. It appears sufficient to postulate:

(2.2.29) $$\text{if } P_{0t} > 1 \text{ then } P_{t0} < 1 \text{ and if } P_{0t} < 1 \text{ then } P_{t0} > 1.$$

This requirement seems to be reasonable and not too ambitious: it is only desired that an increase in the direction $0 \rightarrow t$ should correspond to a decline in the opposite direction $t \rightarrow 0$ and vice versa.

Example 2.2.2 Assume the following prices and quantities

i	p_{i0}	p_{it}	q_{i0}	q_{it}
1	12	15	80	20
2	20	18	10	80

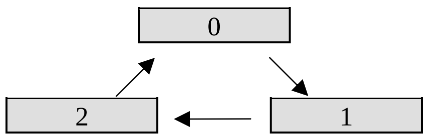
Calculate the following indices P^C (Carli), P^L, P^P, P^{DR} (Drobisch), each in both directions, that is $0 \rightarrow t$ and $t \rightarrow 0$. The results are as follows:

formula	P_{0t} direction $0 \rightarrow t$	P_{t0} direction $t \rightarrow 0$
Carli	$P_{0t}^C = (1.25 + 0.9)/2 = 1.075 > 1$	$P_{t0}^C = 0.9555 < 1$
Laspeyres	$P_{0t}^L = 1380/1160 = 1.1897 > 1$	$P_{t0}^L = 1/P_{0t}^P = 1.0575 > 1$
Paasche	$P_{0t}^P = 1740/1840 = 0.9457 < 1$	$P_{t0}^P = 1/P_{0t}^L = 0.8406 < 1$
Drobisch	$P_{0t}^{DR} = 1.0677 > 1$	$P_{0t}^{DR} = 0.9490 < 1$

Thus both, the Laspeyres- as well as the Paasche formula may fail this weak time reversal test, while the indices of Carli and Drobisch (or Sidgwick) will pass this test *necessarily* (though both indices do *not* satisfy the time reversal test). ♦

The meaning and significance of Fisher's cicular test

(2.2.30) $P_{01}\, P_{12} = P_{02}$, and in connection with identity



(2.2.30a) $P_{01}\, P_{12}\, P_{20} = P_{00} = 1$,

(2.2.30b) $P_{01}\, P_{12}\, P_{23} = P_{03}$.

(2.2.31) $$P_{0t}^{LW} = \frac{\sum p_t q}{\sum p_0 q}$$ (Lowe's price index).

g) A critique of circularity and time reversibility (Pfouts)

Circularity is tantamount to the requirement that a certain matrix **P** of index numbers has to be *singular*. **P** is defined as follows (in the case of T+1 = 4 rows and colums, t = 0,1,...,T)

$$\mathbf{P} = \begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} \\ P_{10} & P_{11} & P_{12} & P_{13} \\ P_{20} & P_{21} & P_{22} & P_{23} \\ P_{30} & P_{31} & P_{32} & P_{33} \end{bmatrix}.$$

Fisher's tests, however, tacitly assume **P** being singular. This can easily be seen since in the case of T = 2 we obtain:

$$\mathbf{P} = \begin{bmatrix} P_{00} & P_{01} & P_{02} \\ P_{10} & P_{11} & P_{12} \\ P_{20} & P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 1 & P_{01} & P_{01}P_{12} \\ 1/P_{01} & 1 & P_{12} \\ 1/P_{01}P_{12} & 1/P_{12} & 1 \end{bmatrix}$$