**Additivity of National Accounts Reconsidered**
(version 1 June 2007)

Bert M. Balk*
RSM Erasmus University
Rotterdam
bbalk@rsm.nl
and
Statistics Netherlands
Voorburg

and

Utz-Peter Reich
Fachhochschule Mainz
University of Applied Sciences
utz.reich@wiwi.fh-mainz.de

**Abstract**

It is well known that the use of chained indices in economic accounting systems leads to volume figures (deflated nominal values) that are inherently non-additive. This inconvenience for users is caused by a mathematical impossibility result. Recognizing this state of affairs, in this paper a deflation method will be developed that uses chained indices of a certain functional form but leads to volume figures that exhibit additivity and can be nicely interpreted.

## 1. Introduction

Though there are exceptions to this rule, users and makers of economic statistics love additivity because it makes the interpretation of accounting systems so easy.

In a single-commodity economy, growth and decline would simply be measured by adding up all the quantities of the commodity which are produced or consumed per period, and comparing these figures through time. In a multi-commodities economy this simplicity is lost. Quantities of distinct commodities cannot be added meaningfully. However, if there is a price system then quantities of distinct commodities can be transformed to values, and these can be added. In a sense a price system serves to transform all the distinct commodities into one uniform commodity, called 'money'. It seems that by using this transformation we are back to the simplicity of the single-commodity economy.

---

* The views expressed in this paper are those of the author and do not necessarily reflect any policy of Statistics Netherlands. The authors thank Malte Knüppel, Karl-Heinz Tödter, Jan van Dalen, Taeke Takema, Piet Verbiest and Alex Lammertsma for helpful comments and/or discussions.

This, however, is unfortunately not true. The price system happens to change through time, so that there are as many systems as there are periods to compare. The change, moreover, has two components: the structure of the system (as given by the relative prices) changes, as well as its level (somehow defined; the price level being an inverse measure of the value of money). It is the simultaneous occurrence of these two phenomena that causes ambiguity in measures of growth and decline.

In this paper the issue of additivity will be reconsidered. Compared to other issues of index number theory, additivity is relatively new on stage. More precisely, it has become an issue, when the direct Laspeyres index, which traditionally valued quantities at prices of a relatively distant, fixed base year was given up in favour of a chained index, where quantities are valued at prices of each previous year. Nominal values ("in current prices") were thus complemented not by real values ("in constant prices") any more, but by volume changes measured relative to the nominal values of each previous year, using "limping prices", so to speak, where each current year drags its predecessor behind, as a base year for growth measurement.

Additivity means that the operations of deflating and aggregating values in national accounts are interchangeable, or commutative. If you aggregate two or more nominal entries and deflate the aggregate, the outcome should be the same as if in reverse order of these operations, you first deflate and then aggregate. The problem did not exist as long as the Laspeyres index was used, because with a constant price system additivity is naturally given. But as the assumption of constancy is counterfactual, at least in the long run, it had to yield to the more realistic assumption of a defined constancy over only one year, realised in the chain index, and for chain indexes, in their traditional form, additivity is lacking. The disadvantage is not a new discovery, and has been known all along, but in between two conflicting goals, realism of the price system, yielding adequate growth rates, has been accorded priority over additivity of values, yielding coherent national accounts. The cost of non-additivity has been generally accepted as the price to be paid for the gain of characteristicity.

The theorem of incompatibility of these two goals is questioned, in this paper. It will be shown that by a small modification of the chained index numbers currently in use one obtains additive results in the accounts. The conceptual tool for interpreting the method is given by distinguishing the relative from the absolute price of a commodity. The absolute price (nominal price) is given by the amount of money paid in exchange for a unit of a commodity. Money itself, however, is not invariable in its purchasing power, but subject to more or less inflation. The variation is measured by the general price level, which is (more or less arbitrarily) defined as an average over the prices of all commodities. The relative price is then the price of a commodity relative to the chosen basket of all commodities, which we may then call the "real" price in analogy to the real wage or the real interest, known in macroeconomic analysis.

The structure of the paper is as follows. Section 2 prepares the ground by providing the necessary definitions. Section 3 treats the construction of real values based on direct indices. Section 4 treats the construction of real values based on chained indices. The advantages and disadvantages of the two systems will be highlighted. Section 5, then, proceeds to the novel approach that is proposed in this paper. Section 6 discusses its working on a simple example, and Section 7 concludes.

## 2. Definitions

We consider an economic aggregate, consisting of a number of (fairly homogeneous) transaction categories which will be called ‘commodities’. For simplicity, it will be assumed that these commodities do not change through time. Each of these commodities has an (average) price $p_n^t$ and a corresponding quantity $q_n^t$, where $n$ = 1,…,$N$ denotes a commodity, and $t$ is an accounting period. The nominal (transaction) value of commodity $n$ in period $t$ is then $v_n^t \equiv p_n^t q_n^t$, and the value of the entire aggregate in period $t$ is $V^t \equiv \sum_{n=1}^{N} p_n^t q_n^t$. It is assumed that the length of an accounting period is such that it is indeed meaningful to add up the values of the transactions that have occurred during this period. The usual length of such a period is a year.[1] It is efficient to use from hereon simple vector notation.[2] Then one can write $V^t = p^t \cdot q^t$.

Suppose that this aggregate can be partitioned into $K$ subaggregates and let (after permutation of commodities) the price and quantity vectors be partitioned as $p^t = (p_1^t,...,p_K^t)$ and $q^t = (q_1^t,...,q_K^t)$ respectively, where $(p_k^t, q_k^t)$ is the subvector corresponding to the subaggregate $k$ = 1,…,$K$. The subaggregate values in period $t$ are $V_k^t \equiv p_k^t \cdot q_k^t$ ($k$ = 1,…,$K$). Consider now the development of the aggregate and its subaggregates through a number of consecutive periods, say $t$ = 0, 1, 2, …., $T$. One obtains sequences of nominal values

$$V^0, V^1, V^2, ..., V^T \quad (1)$$

$$V_k^0, V_k^1, V_k^2, ..., V_k^T \ (k = 1,\ldots,K). \quad (2)$$

Nominal values are additive, that is, $V^t = \sum_{k=1}^{K} V_k^t$. It is clear that nominal value development is driven by price and quantity changes. The problem is about how to disentangle the two components in order to get a picture of these separate forces. The disentanglement is usually executed by price and quantity index numbers.

Suppose we have some (bilateral) price index $P$ and quantity index $Q$, each being a function of 4$N$ variables $(p^t, q^t, p^{t'}, q^{t'}) \equiv (t, t')$, such that

$$V^t / V^{t'} = P(t,t')Q(t,t') \ (t, t' = 1,\ldots,T). \quad (3)$$

Here nominal value change, measured as a ratio, is decomposed as the product of a direct price index and a direct quantity index. A direct index is an index where data for periods lying between the two periods compared are not taken into consideration. If there are such data, then there are alternative decompositions, namely by chained

---

[1] For conceptual reasons it is assumed that the basic data consist of prices and quantities. In practice one usually has to deal with (estimates of) price index numbers $p_n^t / p_n^b$ relative to some reference period $b$ and deflated values $v_n^t / (p_n^t / p_n^b)$.

[2] Hence, $p^t \cdot q^{t'} \equiv \sum_{n=1}^{N} p_n^t q_n^{t'}$, where $t$ and $t'$ denote two, not necessarily different, time periods.

indices, the number of such decompositions depending on the number of intermediate time periods. Using all the intermediate periods, one obtains

$$\frac{V^t}{V^{t'}} = \prod_{\tau=t'+1}^{t} \frac{V^\tau}{V^{\tau-1}} = \prod_{\tau=t'+1}^{t} P(\tau,\tau-1)Q(\tau,\tau-1) = \prod_{\tau=t'+1}^{t} P(\tau,\tau-1) \prod_{\tau=t'+1}^{t} Q(\tau,\tau-1)$$

$$\equiv P^c(t,t')Q^c(t,t') \quad (t,t' = 1,\ldots,T). \tag{4}$$

Let $P_k(t,t')$ be a price index with the same functional form as $P(t,t')$, but with the size of its price and quantity vector variables reduced to the number of commodities which make up subaggregate *k*. Similarly, let $Q_k(t,t')$ be a quantity index with the same functional form as $Q(t,t')$, but applicable to subaggregate *k*. Then the decompositions analogous to (3) and (4) are

$$V_k^t / V_k^{t'} = P_k(t,t')Q_k(t,t') \quad (k = 1,\ldots,K;\ t,\ t' = 1,\ldots,T) \tag{5}$$

and

$$V_k^t / V_k^{t'} = P_k^c(t,t')Q_k^c(t,t') \quad (k = 1,\ldots,K;\ t,\ t' = 1,\ldots,T) \tag{6}$$

respectively. The ‘real’ development of the aggregate and its subaggregates, relative to the base period 0, is usually presented in the form of sequences of deflated nominal values.

## 3. Real values based on direct indices

Using *direct* indices, the ‘real’ development of the aggregate, corresponding to (1), is presented as

$$V^0, V^1/P(1,0), V^2/P(2,0), \ldots, V^T/P(T,0), \tag{7}$$

which can also be written as

$$V^0, V^0Q(1,0), V^0Q(2,0), \ldots, V^0Q(T,0). \tag{8}$$

The ‘real’ development of each subaggregate is likewise presented as

$$V_k^0, V_k^1/P_k(1,0), V_k^2/P_k(2,0), \ldots, V_k^T/P_k(T,0) \quad (k = 1,\ldots,K), \tag{9}$$

or

$$V_k^0, V_k^0Q_k(1,0), V_k^0Q_k(2,0), \ldots, V_k^0Q_k(T,0) \quad (k = 1,\ldots,K). \tag{10}$$

Additivity would be preserved if and only if

$$V^t / P(t,0) = \sum_{k=1}^{K} V_k^t / P_k(t,0) \quad (t = 1,\ldots,T), \tag{11}$$

or

$$V^0Q(t,0) = \sum_{k=1}^{K} V_k^0 Q_k(t,0) \quad (t = 1,\ldots,T). \tag{12}$$

A little bit of rearrangement delivers the following expressions,

$$1/P(t,0)=\sum_{k=1}^{K}(V_k^t/V^t)/P_k(t,0) \quad (t = 1,\ldots,T) \tag{13}$$

and

$$Q(t,0)=\sum_{k=1}^{K}(V_k^0/V^0)Q_k(t,0) \quad (t = 1,\ldots,T), \tag{14}$$

which look more familiar. These two expressions are supposed to hold for all partitions of the aggregate, in particular for those where all the subaggregates consist of a single commodity (thus $K = N$). It is natural to assume that for $N = 1$ price index and quantity index reduce to price relative and quantity relative respectively, that is, $P(t,t')=p^t/p^{t'}$ and $Q(t,t')=q^t/q^{t'}$. Then expression (13) is the definition of the Paasche price index, and expression (14) is the definition of the Laspeyres quantity index.

*Conclusion*: An index pair $(P,Q)$ satisfies the product test (3) and the condition of additivity (11) or (12) if and only if $P$ is the Paasche index and $Q$ is the Laspeyres index.[3]

Then the ‘real’ values corresponding to (1) and (2) are given by

$$p^0\cdot q^0, p^0\cdot q^1, p^0\cdot q^2,\ldots, p^0\cdot q^T \tag{15}$$

$$p_k^0\cdot q_k^0, p_k^0\cdot q_k^1, p_k^0\cdot q_k^2,\ldots, p_k^0\cdot q_k^T \quad (k = 1,\ldots,K), \tag{16}$$

and it is clear that these values are additive. The ‘real’-value system (15)-(16) has undoubtedly the virtue of simplicity. Yet its disadvantages are also well known. The most important of these disadvantages have to do with the measurement and interpretation of price and quantity changes between consecutive periods. Based on (15), the quantity change between periods $t$-1 and $t$ is measured by the Lowe index $p^0\cdot q^t/p^0\cdot q^{t-1}$ rather than by a quantity index of the form $Q(p^t,q^t,p^{t-1},q^{t-1})$. Put otherwise, with the progress of time it becomes less and less obvious to use period 0 prices for measuring the quantity change between remote periods. In addition, the price index which corresponds to the Lowe quantity index (in the sense of satisfying the product test) does not pass the rather fundamental identity test (which says that a price index should equal unity in case of no price changes).

There is another strategy for preserving additivity. Instead of (9), the ‘real’ subaggregate values are presented as

$$V_k^0, V_k^1/P(1,0), V_k^2/P(2,0),\ldots, V_k^T/P(T,0) \quad (k = 1,\ldots,K); \tag{17}$$

that is, the aggregate deflators $P(t,0)$ are used also to deflate subaggregate nominal values $V_k^t$. Evidently, additivity is preserved, since

---

[3] This conclusion is a specific case of Corollary 16 of Balk (1995). See also Balk (2004).

$$V^t / P(t,0) = \sum_{k=1}^{K} V_k^t / P(t,0) \text{ } (t = 1,\ldots,T). \quad (18)$$

The big disadvantage of such a system of 'real' values is that subaggregate quantity change between, say, consecutive periods is measured in a strange way, namely as

$$\frac{V_k^t / P(t,0)}{V_k^{t-1} / P(t-1,0)} = \frac{V_k^0 P_k(t,0) Q_k(t,0) / P(t,0)}{V_k^0 P_k(t-1,0) Q_k(t-1,0) / P(t-1,0)}$$

$$= \frac{Q_k(t,0)}{Q_k(t-1,0)} \frac{P_k(t,0)/P_k(t-1,0)}{P(t,0)/P(t-1,0)} \text{ } (k = 1,\ldots,K), \quad (19)$$

where expressions (3) and (5) were used. The first factor at the right-hand side of expression (19) is indeed a measure of quantity change. The second factor, however, measures relative price change between the subaggregate and the aggregate. The whole expression confounds quantity and (relative) price change. For example, it might be that $Q_k(t,0)/Q_k(t-1,0) > 1$ but $(V_k^t / P(t,0))/(V_k^{t-1} / P(t-1,0)) < 1$.

## 4. Real values based on chained indices

Let us now consider the use of *chained* indices for the presentation of 'real' values. Instead of (7) and (8) one obtains

$$V^0, V^1 / P^c(1,0), V^2 / P^c(2,0), \ldots, V^T / P^c(T,0), \quad (20)$$

and

$$V^0, V^0 Q^c(1,0), V^0 Q^c(2,0), \ldots, V^0 Q^c(T,0). \quad (21)$$

The 'real' development of each subaggregate is likewise presented as

$$V_k^0, V_k^1 / P_k^c(1,0), V_k^2 / P_k^c(2,0), \ldots, V_k^T / P_k^c(T,0) \text{ } (k = 1,\ldots,K), \quad (22)$$

or

$$V_k^0, V_k^0 Q_k^c(1,0), V_k^0 Q_k^c(2,0), \ldots, V_k^0 Q_k^c(T,0) \text{ } (k = 1,\ldots,K). \quad (23)$$

Additivity would be preserved if and only if

$$V^t / P^c(t,0) = \sum_{k=1}^{K} V_k^t / P_k^c(t,0) \text{ } (t = 1,\ldots,T), \quad (24)$$

or

$$V^0 Q^c(t,0) = \sum_{k=1}^{K} V_k^0 Q_k^c(t,0) \text{ } (t = 1,\ldots,T). \quad (25)$$

Chained indices can be conceived as discrete-time approximations of Divisia line-integral indices. It is well known that Divisia indices exhibit the property of consistency-in-aggregation (Balk 2005). However, this property, and *a fortiori* the stronger property of additivity, gets lost in any discrete-time approximation. Put otherwise, 'real' values based on chained indices will as a rule violate additivity. Sometimes this problem is solved by adding a row "additivity discrepancy" to a table of 'real' values, or by distributing the additivity discrepancy over the subaggregates 1,…,*K*.

In return, the comparison of 'real' values between consecutive periods yields unequivocal quantity index numbers. For instance, using expressions (4) and (5), it appears that

$$\frac{V_k^t / P_k^c(t,0)}{V_k^{t-1} / P_k^c(t-1,0)} = \frac{V_k^t / V_k^{t-1}}{P_k(t,t-1)} = Q_k(t,t-1) \;\; (k = 1,\ldots,K). \tag{26}$$

The violation of additivity, however, continues to bother many statisticians. One way out would be to replace all the subaggregate deflators $P_k^c(t,0)$ ($k$ = 1,…,$K$) by the single aggregate deflator $P^c(t,0)$, as suggested by Hillinger (2002). Ehemann, Katz and Moulton (2002) showed that this can easily lead to perverse outcomes. To see this, consider the comparison of 'real' subaggregate values between consecutive periods:

$$\frac{V_k^t / P^c(t,0)}{V_k^{t-1} / P^c(t-1,0)} = \frac{V_k^t / V_k^{t-1}}{P\,(t,t-1)} = Q_k(t,t-1)\frac{P_k(t,t-1)}{P(t,t-1)}, \tag{27}$$

which structurally resembles expression (19). Except when there are no relative price changes across subaggregates, the left-hand side of expression (27) differs from $Q_k(t,t-1)$.

## 5. A novel approach for deflating nominal values

The novel approach considered in this paper consists of the following two building blocks (Reich 2003). First, the nominal value of the *aggregate* at period *t* is deflated by the chained Paasche price index

$$P^*(t,0) \equiv P^{cP}(t,0) \equiv \prod_{\tau=1}^{t} P^P(\tau,\tau-1) \;\; (t = 1,\ldots,T). \tag{28}$$

Second, the nominal value of each *subaggregate* at period *t* is deflated by the following chained price index,

$$P_k^*(t,0) \equiv P_k^P(t,t-1)P^{cP}(t-1,0) \;\; (k = 1,\ldots,K;\; t = 1,\ldots,T). \tag{29}$$

This deflator, then, consists of two parts: the final part of the chain, from *t*-1 to *t*, is the subaggregate-specific Paasche price index, while the remainder, from 0 to *t*-1, is the chained Paasche price index for the aggregate.

The two steps of deflating nominal values, as given by equations (28) and (29), are based on a theoretical refinement of concepts, which is not new in economic theory, but has not found its way into statistical application yet. We distinguish between ”volume”, on the one hand, and “real value”, on the other. The meaning of the distinction is the following. Real values are nominal values corrected only for the change in the purchasing power of money, as measured by the general price level. They still include the change in prices specific to each commodity, as shown in equation (19) and (27). Volumes eliminate the change in price, in addition to the correction for inflation, comprising the pure changes in quantity, or quality, of a commodity.

A change in an individual price $p_n^t$ may occur for two reasons. The first has to do with the particular commodity, and results from a change in supply or demand on the market on which the commodity is traded. A rise in demand, or a shortage in supply, will raise the price, and this price only, in comparison to all the other prices. The second reason of a price change has nothing to do with any particular commodity, but is of a monetary nature, qualifying the means of payment (and measure of value) through which the debt created by the purchase of the commodity is being discharged. In order to separate the two theoretical causes of a change in the nominal price $p_n^t$ of a commodity we may write

$$p_n^t = \Lambda(t,0)\overline{p}_n^t \quad (n = 1,\ldots,N;\ t = 1,\ldots,T) \qquad (30)$$

where $\Lambda(t,0)$ stands for the general price level (relative to a certain period 0) which determines the value of the means of payment of the economy at period $t$, and $\overline{p}_n^t$ describes the relative or, as we may now define, the “real” price of the commodity $n$, which is the price it carries in relation to the other commodities.[4]

Suppose that the aggregate $V^t$ is the GDP of an economy, and the general price level is measured by the implicit GDP deflator, derived from the national accounts and calculated according to equation (28). The sequences of nominal values, as given by equations (1) and (2), are then transformed into what we now define as “real values”, which are additive, but include quantity together with price change (equation (27)). Their purpose is to exclude the general price level change, so that the observed nominal values and prices are comparable over time. Or, saying the same thing in a different way, real values are quantities measured at real prices (prices corrected for inflation). If GDP is the commodity basket chosen for measuring the general price level, its real price is always equal to one, by definition.

One verifies immediately that the method given by expressions (28) and (29) exhibits additivity of volumes, since

$$\sum_{k=1}^{K} \frac{V_k^t}{P_k^*(t,0)} = \left(\sum_{k=1}^{K} \frac{V_k^t}{P_k^P(t,t-1)}\right)\left(\frac{1}{P^P(t-1,t-2)\cdots P^P(1,0)}\right)$$

[4] Reich (2005) relates these ideas back to Neubauer (1974), (1978).

$$= \left( \frac{V^t}{P^P(t,t-1)} \right) \left( \frac{1}{P^P(t-1,t-2) \cdots P^P(1,0)} \right) = \frac{V^t}{P^{cP}(t,0)} = \frac{V^t}{P^*(t,0)}, \quad (31)$$

which corresponds to expression (11) defining additivity. The meaning of the method may be illuminated by looking at the temporal difference between two deflated values. The volume of the *aggregate* can be decomposed as

$$\frac{V^t}{P^*(t,0)} = \sum_{k=1}^{K} \frac{V_k^t}{P_k^*(t,0)} = \sum_{k=1}^{K} \frac{p_k^{t-1} \cdot q_k^t}{P^{cP}(t-1,0)}$$

$$= \sum_{\tau=1}^{t} \sum_{k=1}^{K} \frac{p_k^{\tau-1} \cdot (q_k^{\tau} - q_k^{\tau-1})}{P^{cP}(\tau-1,0)} + \sum_{k=1}^{K} p_k^0 \cdot q_k^0$$

$$= \sum_{k=1}^{K} \left( \sum_{\tau=1}^{t} \frac{p_k^{\tau-1} \cdot (q_k^{\tau} - q_k^{\tau-1})}{P^{cP}(\tau-1,0)} + V_k^0 \right). \quad (32)$$

It then appears that the volume change of the *aggregate* between years *t*-1 and *t*, measured as a difference, is given by

$$\sum_{k=1}^{K} \frac{V_k^t}{P_k^*(t,0)} - \sum_{k=1}^{K} \frac{V_k^{t-1}}{P_k^*(t-1,0)} = \sum_{k=1}^{K} \left( \sum_{\tau=1}^{t} \frac{p_k^{\tau-1} \cdot (q_k^{\tau} - q_k^{\tau-1})}{P^{cP}(\tau-1,0)} - \sum_{\tau=1}^{t-1} \frac{p_k^{\tau-1} \cdot (q_k^{\tau} - q_k^{\tau-1})}{P^{cP}(\tau-1,0)} \right)$$

$$= \sum_{k=1}^{K} \frac{p_k^{t-1} \cdot (q_k^t - q_k^{t-1})}{P^{cP}(t-1,0)}. \quad (33)$$

Here the individual quantity changes between *t*-1 and *t* are valued at the earlier period's prices. In order to make these values comparable over time, they must be deflated to the base period price level by the aggregate price index.

Each term at the right-hand side of expression (33) is linked to a conventional Laspeyres growth rate by

$$g_k^t \equiv \frac{p_k^{t-1} \cdot (q_k^t - q_k^{t-1})}{P^{cP}(t-1,0)} \Bigg/ \frac{p_k^{t-1} q_k^{t-1}}{P^{cP}(t-1,0)} \quad (k = 1,\ldots,K). \quad (34)$$

However, care should be taken with interpreting the *ratio* of subaggregate volumes at periods *t*-1 and *t*. It appears that

$$\frac{V_k^t / P_k^*(t,0)}{V_k^{t-1} / P_k^*(t-1,0)} = \frac{p_k^{t-1} \cdot q_k^t}{p_k^{t-2} \cdot q_k^{t-1}} \frac{1}{P^P(t-1,t-2)}$$

$$= \frac{p_k^{t-1} \cdot q_k^t}{p_k^{t-1} \cdot q_k^{t-1}} \frac{P_k^P(t-1,t-2)}{P^P(t-1,t-2)} \quad (k=1,\ldots,K). \quad (35)$$

As the right-hand side of expression (35) shows, the ratio of subaggregate volumes combines quantity changes (measured by a Laspeyres index) with relative price changes. However, as demonstrated by equation (32), consecutive volumes are defined in an additive way, which implies that the ratio is a less meaningful measurement tool.

It is tempting to expect that the equalities (32) and (33) also hold componentwise. This, however, is in general not so, which is the "price" that one must pay for having additivity of volumes together with chained indices. Equalities (32) and (33) hold componentwise if and only if $P^P(t-1,t-2) = P_k^P(t-1,t-2)$ for $k$ = 1,…,$K$; that is, if and only if going from period $t$-2 to $t$-1 all the subaggregate price index numbers are the same. Then the right-hand side of expression (35) reduces to $p_k^{t-1} \cdot q_k^t / p_k^{t-1} \cdot q_k^{t-1}$, which is the familiar Laspeyres quantity index for subaggregate $k$.

## 6. An illustration

The working of expression (32) may be explained by means of a simple example. Table 1 has been constructed so as to compare the traditional chaining method with its additive counterpart. It follows the typical order of operations employed in building national accounts. The compilation begins in nominal terms, ordinarily, with, let us say, nominal values v0, v1, v2, v3, for four years 0, 1, 2, 3. The economy produces four commodities, two for consumption purposes (C1, C2), two for formation of capital (I1, I2). There are thus four elementary price index numbers, one for each product group. Nominal values can be added within a year, thus for each year, panel 1a yields a GDP in nominal terms. These nominal values cannot be compared between different years. They are made comparable by using additional information about the development of prices, summarised in indexes p0, p1, p2, p3, with year 0 as the base year (p0 = 100), and added to panel 1a as additional data.

The price information is used in panel 1b where the flows are re-valued and expressed in prices of the previous year, except for the base year. Thus for the years following the base year we have expressions p0q1, p1q2, p2q3 which may be added to yield GDP in prices of the previous year. In this way, one analytically separates the volume component from the price component in the observed nominal value change. Thus in prices of the previous year GDP of year 1 equals that of year 0 (=1000), GDP of year 2 is 1060 in prices of year 1.

In addition, using the assumption expressed in equation (3), we divide the volume component compiled in panel 1b into its corresponding nominal value of panel 1a, which yields an implicit Paasche price index number P01, P12, P23 for each period, for each of the sub-aggregates C and I, and for GDP as a whole. For example, the implicit aggregate price index number for consumption goods rises by 14.2 percent between year 0 and year 1, and shrinks to 98.8 percent the year after, to rise again to 121.5 percent of the base year between year 2 and year 3.

On the basis of equation (3), panel 1c supplies the aggregate year-to-year volume index numbers (Q01, Q12, Q23) corresponding to the price index numbers. They are found by dividing the nominal values of panel 1b into the deflated values of the following year in panel 1a. Thus nominal consumption value of 790 in year 2 (panel

1a) divided into deflated consumption of 905 in year 3 (panel 1b) yields a volume index of 114.6 in panel 1c.

We have not constructed any longer time series, so far yet. Each year has been compared with its predecessor, independently of earlier or later years. Panel 1d now introduces chaining. The year-to-year index numbers are multiplied and assigned to the nominal values of the base year, yielding chained volumes in prices of the base year. Violating equation (24), these volumes are not additive. The deflated sub-aggregates C and I (3rd line in panel 1d) do not yield the deflated GDP when summed ($4^{th}$ line). Additivity works between year 0 and year 1, where the chain does not appear, but for year 2 the deflated GDP is 1029 Euros of year 0, while the sum of the sub-aggregates yields 1002 Euros of year 0, and so on for all later years. This is the disadvantage of the traditional chaining method. It consists in chaining quantity changes to nominal values of a base year, which implies that the relative price structure within an aggregate is kept constant. Since prices do change over time there is a discrepancy in valuation which causes the non-additivity.

If instead of multiplying the yearly volume changes one constructs an index by adding them according to equation (32) one arrives at panel 1e. The volume change is measured not as an index number, but as a value figure in Euros of the previous year. From panels 1a and 1b one deducts, for example, that aggregate consumption C has shrunk by 100 Euros of year 0 (600 – 700) between year 0 and year 1, and grown by 115 Euros of year 1, afterwards, and finally by 115 Euros of year 2.

Although the name is the same, Euros of year 1 may not simply be added to Euros of year 2, because there is inflation. Just as the rate of inflation is deducted from the nominal interest rate in order to arrive at the real interest rate, or from the nominal wage rate in order to arrive at the real wage rate, we must deduct it from the observed volume changes in order to be able to add these changes. In this example we use the implicit GDP deflator as the rate of inflation, measuring the rise in the general price level and corresponding loss in general purchasing power of the ruling measure of value (see equation (28)). Dividing the chained GDP deflator from panel 1b into the Euros of panel 1e yields panel 1f.

Finally, panel 1g sums the Euros which are now of the same purchasing power, namely of year 0, and adds them to the nominal values of year 0. This delivers a panel of cumulative volume change for each year's GDP and its sub-aggregates (equation (32)). GDP comes out exactly as it does in panel 1d under the traditional chaining procedure, because it has been chosen as the commodity basket against which all other baskets or individual commodities are compared in their price change. Its price index number measures the general price level so that its relative price is equal to one, by definition.

## 7. Conclusion

Though Peter von der Lippe might still contest this – as he did in his 2001 book –, for good reasons the *SNA 1993* advocates chained indices for measuring price and volume change in a national accounting system. It is well known that by doing so the additivity (of so-called real values, or volumes) must be given up. While recognizing that the mathematics of aggregation and deflation makes a marriage of chaining and additivity impossible, in the foregoing a procedure was developed that pairs the two to the extent possible. The authors believe to have demonstrated that this method is able to deliver useful insights in a handsome way.

## References


Balk, B. M., 1995, "Axiomatic Price Index Theory: A Survey", *International Statistical Review* 63, 69-93.

Balk, B. M., 2004, "Direct and chained indices: a review of two paradigms". Keynote lecture delivered at the SSHRC International Conference on Index Number Theory and the Measurement of Prices and Productivity, Vancouver, 30 June - 3 July 2004.

Balk, B. M., 2005, "Divisia Price and Quantity Indices: 80 Years After", *Statistica Neerlandica* 59, 119-158.

Ehemann, C., A. J. Katz and B. R. Moulton, 2002, "The Chain-Additivity Issue and the US National Economic Accounts", *Journal of Economic and Social Measurement* 28, 37-49.

Hillinger, C., 2002, "Consistent Aggregation and Chaining of Price and Quantity Measures", *Journal of Economic and Social Measurement* 28, 1-20.

Lippe, P. von der, 2001, *Chain Indices: A Study in Price Index Theory.* Spectrum of Federal Statistics, Volume 16 (Metzler-Poeschel, Stuttgart).

Neubauer, W., 1974, "Irreales Inlandsprodukt in konstanten Preisen. Kritisches zur Deflationierung in der Volkswirtschaftlichen Gesamtrechnung", *Allgemeines Statistisches Archiv* 58, 237-271.

Neubauer, W., 1978, "Reales Inlandsprodukt: 'preisbereinigt' oder 'inflationsbereinigt'? Zur Deflationierung bei veränderter Preisstruktur", *Allgemeines Statistisches Archiv* 62, 115-160.

Reich, U.-P., 2003, "Additiver Kettenindex für die Preisbereinigung der Volkswirtschaftlichen Gesamtrechnung: Kritische Überlegungen aus aktuellen Anlass", *Austrian Journal of Statistics* 32, 323-327.

Reich, U. P., 2005, "Irreales Inlandsprodukt in konstanten Preisen"; Ein Rückblick nach 30 Jahren aus Heutiger Perspektive. Mimeo.


**Table 1. Multiplicative and additive volume change computation.**

a) Nominal values and elementary price index numbers

| | Year 0 | | Year 1 | | Year 2 | | Year 3 | |
|---|---|---|---|---|---|---|---|---|
| | v0 | p0 | v1 | p1 | v2 | p2 | v3 | p3 |
| C1 | 400 | 100 | 385 | 110 | 400 | 100 | 540 | 120 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| C2 | 300 | 100 | 300 | 120 | 390 | 130 | 560 | 160 |
| C | 700 | | 685 | | 790 | | 1100 | |
| | | | | | | | | |
| I1 | 200 | 100 | 225 | 90 | 200 | 100 | 165 | 110 |
| I2 | 100 | 100 | 120 | 80 | 70 | 70 | 30 | 60 |
| I | 300 | | 345 | | 270 | | 195 | |
| | | | | | | | | |
| GDP | 1000 | | 1030 | | 1060 | | 1295 | |

b) Values in prices of previous year and implied aggregate year-to-year price index numbers

| | Year 0 | Year 1 | | Year 2 | | Year 3 | |
|---|---|---|---|---|---|---|---|
| | p0q0 | p0q1 | P01 | p1q2 | P12 | p2q3 | P23 |
| C1 | 400 | 350 | | 440 | | 450 | |
| C2 | 300 | 250 | | 360 | | 455 | |
| C | 700 | 600 | 114,2 | 800 | 98,8 | 905 | 121,5 |
| | | | | | | | |
| I1 | 200 | 250 | | 180 | | 150 | |
| I2 | 100 | 150 | | 80 | | 35 | |
| I | 300 | 400 | 86,25 | 260 | 103,8 | 185 | 105,4 |
| | | | | | | | |
| GDP | 1000 | 1000 | 103 | 1060 | 100 | 1090 | 118,8 |

c) Year to year volume index numbers

| | | Q01 | Q12 | Q23 |
|---|---|---|---|---|
| C | | 85,71 | 116,8 | 114,6 |
| I | | 133,3 | 75,36 | 68,52 |
| GDP | | 100 | 102,9 | 102,8 |

d) Chained volumes

| | | | | |
|---|---|---|---|---|
| C | 700 | 600 | 700,7 | 802,7 |
| I | 300 | 400 | 301,4 | 206,5 |
| C+I | | 1000 | 1002 | 1009 |
| GDP | 1000 | 1000 | 1029 | 1058 |

e) Volume change in prices of previous year (Euros)

| | | | |
|---|---|---|---|
| C | -100 | 115 | 115 |
| I | 100 | -85 | -85 |
| C+I | 0 | 30 | 30 |
| GDP | 0 | 30 | 30 |

f) Accounting for inflation (GDP deflator)

| | | | |
|---|---|---|---|
| C | -100 | 111,7 | 111,7 |
| I | 100 | -82,5 | -82,5 |
| GDP | 0 | 29,13 | 29,13 |

g) Volume change accumulated

| | | | | |
|---|---|---|---|---|
| C | 700 | 600 | 711,7 | 823,3 |
| I | 300 | 400 | 317,5 | 235 |
| GDP | 1000 | 1000 | 1029 | 1058 |